ICRU REPORT 28

Basic Aspects of High Energy Particle Interactions and Radiation Dosimetry

INTERNATIONAL COMMISSION
ON RADIATION UNITS
AND MEASUREMENTS

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For detailed information on the availability of this and other ICRU Reports see page 71

Preface

Scope of ICRU Activities

The International Commission on Radiation Units and Measurements (ICRU), since its inception in 1925, has had as its principal objective the development of internationally acceptable recommendations regarding:

(1) Quantities and units of radiation and radioactivity,

(2) Procedures suitable for the measurement and application of these quantities in clinical radiology and radiobiology,

(3) Physical data needed in the application of these procedures, the use of which tends to assure uniformity in reporting.

The Commission also considers and makes similar types of recommendations for the radiation protection field. In this connection, its work is carried out in close cooperation with the International Commission on Radiological Protection (ICRP).

Policy

The ICRU endeavors to collect and evaluate the latest data and information pertinent to the problems of radiation measurement and dosimetry and to recommend the most acceptable values and techniques for current use.

The Commission's recommendations are kept under continual review in order to keep abreast of the rapidly expanding uses of radiation.

The ICRU feels it is the responsibility of national organizations to introduce their own detailed technical procedures for the development and maintenance of standards. However, it urges that all countries adhere as closely as possible to the internationally recommended basic concepts of radiation quantities and units.

The Commission feels that its responsibility lies in developing a system of quantities and units having the widest possible range of applicability. Situations may arise from time to time when an expedient solution of a current problem may seem advisable. Generally speaking, however, the Commission feels that action based on expediency is inadvisable from a long-term viewpoint; it endeavors to base its decisions on the long-range advantages to be expected.

The ICRU invites and welcomes constructive comments and suggestions regarding its recommendations and reports. These may be transmitted to the Chairman.

Current Program

The Commission has divided its field of interest into eleven technical areas and has assigned one or more members of the Commission the responsibility for identification of potential topics for new ICRU activities in each area. A body of consultants has been constituted for each technical area to advise the Commission on the need for ICRU recommendations relating to the technical area and on the means for meeting an identified need. Each area is reviewed periodically by its sponsors and consultants. Recommendations of such groups for new reports are then reviewed by the Commission and a priority assigned. The Technical areas are:

Radiation Therapy
Radiation Diagnosis
Nuclear Medicine
Radiobiology
Radioactivity
Radiation Physics – X Rays, Gamma Rays and Electrons
Radiation Physics – Neutrons and Heavy Particles
Radiation Protection
Values of Factors – *W*, *S*, etc.
Theoretical Aspects
Quantities and Units

The actual preparation of ICRU reports is carried out by ICRU report committees. One or more Commission members serve as sponsors to each committee and provide close liaison with the Commission. The currently active report committees are:

Average Energy Required to Produce an Ion Pair
Computer Uses in Radiotherapy

- Dose Specification for Reporting
- Dosimetry of Pulsed Radiation
- Electron Beam Dosimetry
- Fundamental Quantities and Units
- Measurement of Low-Level Radioactivity in Humans
- Methods of Assessment of Dose in Tracer Investigations
- Microdosimetry
- Photographic Dosimetry in External Beam Therapy
- Radiobiological Dosimetry
- Scanning
- Stopping Power

ICRU Reports

In 1962 the ICRU, in recognition of the fact that its triennial reports were becoming too extensive and in some cases too specialized to justify single-volume publication, initiated the publication of a series of reports, each dealing with a limited range of topics. This series was initiated with the publication of six reports:

ICRU Report 10a,	*Radiation Quantities and Units*
ICRU Report 10b,	*Physical Aspects of Irradiation*
ICRU Report 10c,	*Radioactivity*
ICRU Report 10d,	*Clinical Dosimetry*
ICRU Report 10e,	*Radiobiological Dosimetry*
ICRU Report 10f,	*Methods of Evaluating Radiological Equipment and Materials*

These reports were published, as had been many of the previous reports of the Commission, by the United States Government Printing Office as Handbooks of the National Bureau of Standards.

In 1967 the Commission determined that in the future the recommendations formulated by the ICRU would be published by the Commission itself. This report is published by the ICRU pursuant to this policy. With the exception of ICRU Report 10a, the other reports of the "10" series have continuing validity and, since, except in the case of ICRU Report 10e, none of the reports now in preparation is designed specifically to supersede them, they will remain available until the material is essentially obsolete. All future reports of the Commission, however, will be published under the ICRU's own auspices. Information about the availability of ICRU Reports is given on page 71.

ICRU's Relationships With Other Organizations

In addition to its close relationship with the International Commission on Radiological Protection, the ICRU has developed relationships with other organizations interested in the problems of radiation quantities, units and measurements. Since 1955, the ICRU has had an official relationship with the World Health Organization (WHO) whereby the ICRU is looked to for primary guidance in matters of radiation units and measurements and, in turn, the WHO assists in the world-wide dissemination of the Commission's recommendations. In 1960 the ICRU entered into consultative status with the International Atomic Energy Agency. The Commission has a formal relationship with the United Nations Scientific Committee on the Effects of Atomic Radiation (UNSCEAR), whereby ICRU observers are invited to attend UNSCEAR meetings. The Commission and the International Organization for Standardization (ISO) informally exchange notifications of meetings and the ICRU is formally designated for liaison with two of the ISO Technical Committees. The ICRU also corresponds and exchanges final reports with the following organizations:

- Bureau International des Poids et Mesures
- Commission of the European Communities
- Council for International Organizations of Medical Sciences
- Food and Agriculture Organization
- International Council of Scientific Unions
- International Electrotechnical Commission
- International Labor Office
- International Union of Pure and Applied Physics
- United Nations Educational, Scientific and Cultural Organization

The Commission has found its relationship with each of these organizations fruitful and of substantial benefit to the ICRU program. Relations with these other international bodies do not affect the basic affiliation of the ICRU with the International Society of Radiology.

Operating Funds

In the early days of its existence, the ICRU operated essentially on a voluntary basis, with the travel and operating costs being borne by the parent organizations of the participants. (Only token assistance was originally available from the International Society of Radiology.) Recognizing the impracticability of continuing this mode of operation on an indefinite basis, operating funds were sought from various sources.

Financial support has been received from the following organizations:

- B.A.T. Cigaretten-Fabriken GMBH
- Commission of the European Communities
- Council for International Organizations of Medical Sciences
- Eastman Kodak Company
- E. I. duPont de Nemours and Company
- Ford Foundation
- General Electric Company
- International Atomic Energy Agency
- International Radiation Protection Association

International Society of Radiology
Japan Industries Association of Radiation Apparatus
John och Augusta Perssons stiftelse
National Cancer Institute of the U. S. Department of Health, Education and Welfare
N.V. Philips Gloeilampenfabrieken
Picker Corporation
Radiological Society of North America
Rockefeller Foundation
Siemens Corporation
Society of Nuclear Medicine
Statens laegevidenskabelige Forskningsrad
U. S. Bureau of Radiological Health of the Food and Drug Administration
World Health Organization

In recognition of the fact that its work is made possible by the generous support provided by these organizations, the Commission expresses it deep appreciation.

HAROLD O. WYCKOFF
Chairman, ICRU

Washington, D.C.
1 August 1977

Basic Aspects of High Energy Particle Interactions and Radiation Dosimetry

1. Introduction

1.1 Scope

This report deals with fundamental considerations underlying the dosimetry of radiations having energies in excess of about 10^8 eV (100 MeV). There is no intent to set an upper limit, but information on the physical processes taking place is scarce at energies beyond those presently attainable with accelerators (about 10^{11} eV). Cosmic ray particles can have energies of one joule (6.24×10^{18} eV) and more, but the flux density of these particles is very low and our understanding of their interactions is fragmentary. Throughout this report the term *high-energy radiation* will refer to primary radiations having energies above 10^8 eV. Particles and interactions at energies lower than this limit will be discussed in the context of the *secondary* particles produced in interactions of the high-energy primary particles with matter. These lower-energy particles have obvious significance in many applications, e.g., where massive shielding is present to absorb or attenuate the higher energy radiation.

In the very broad energy range under consideration, all types of nuclear processes can be initiated and all known unstable particles can be created. Such interactions frequently result in a chain of subsequent reactions that can be very complex. Indeed, if the energy of any primary particle exceeds 10^{11} eV, there is an appreciable probability for the production of most presently identified directly and indirectly ionizing radiations with a spectrum that may extend over a 10^{10}-fold range of energy.

High-energy radiation, therefore, is the most demanding topic in dosimetry. It is nevertheless not as difficult as the complexity of physical processes might imply, since many of the reactions and particles can often be ignored with little or no effect on the accuracy of dose determinations. Some dosimetric techniques developed for more conventional radiations can be extended to very much higher energies, particularly when appropriate modifications or corrections are applied. This, however, can only be done with adequate understanding of the physical processes involved. Consequently, a substantial portion of this report (Section 2) deals with the physics of high-energy radiation, with particular emphasis on dosimetric aspects.

High-energy radiation originates from two types of sources under distinctly different conditions. As already mentioned, the radiation energies attained by accelerators are less than those that can be encountered in cosmic radiation. Accelerator-produced radiations, however, are usually much more intense and radiation shielding is almost invariably very bulky. Section 3 contains information on the radiation environment surrounding various types of accelerators. Cosmic radiation both in the vicinity of the earth and in interplanetary space extends to much higher energies, but, with the comparatively rare exception of solar outbursts, flux densities are low. They are of practical importance only for planes flying in or above the stratosphere, or in spacecraft outside the earth's atmosphere, under conditions where shielding is minimal. As a result of space exploration, information on cosmic radiation is continuing to develop. A brief review with reference to more comprehensive treatments is given in Section 4.

Section 5 of this report contains an analysis of the problem of dose equivalent specification, and a survey of absorbed dose and dose equivalent measurement techniques. Since these subjects have been analyzed in considerable detail in ICRU Reports 19S (ICRU, 1973) and 25 (ICRU, 1976b) and in ICRU Report 20 (ICRU, 1971b), respectively, this treatment is also brief.

The remainder of this introduction deals with a description of radiation quantities and a discussion of the applications of dosimetry.

1.2 Radiation Quantities

ICRU Report 19 (ICRU, 1971a) contains a comprehensive list of radiation quantities and their units, and the reader is referred to this document for detailed considerations and precise definitions. The material presented here, however, should be adequate for most practical purposes and for an understanding of dosimetric terms used in the remainder of this report.

The quantities specified in Report 19 are sufficiently general to apply to all radiations; however, some of them (particularly kerma and exposure) have little utility at high energies. On the other hand, there may be a need to formulate new quantities to deal with certain dosimetric aspects of high-energy radiation. The most important example relates to the complex track pattern generated by heavy nuclei moving near relativistic velocities. No attempt is made here to develop the appropriate concept.

1.2.1 Fundamental Quantities

The most fundamental description of a radiation field is in terms of the number of particles that traverse an infinitesimal volume located at the point of interest, per unit cross-sectional area and per unit time. This quantity, the *flux density* or *fluence rate*, ϕ, can refer to *directly* ionizing radiation (i.e., charged particles) or *indirectly ionizing radiations* (e.g., photons, neutrons, neutral mesons, etc.). Increasing detail of this characterization is provided by specifying ϕ with respect to particle type, energy, and direction. The *fluence*, Φ, is the time integral of ϕ.

The above quantities apply to radiation with no reference to its interaction with matter. They are therefore of less immediate concern to dosimetry than others that encompass the interaction with material of specified composition. An example of this is quantification of the process whereby indirectly ionizing radiation gives rise to directly ionizing radiation. The applicable quantity is the *kerma*, K, which is the sum of the kinetic energies of the charged particles ejected per unit mass of irradiated medium.

In traversing matter, charged particles dissipate their energy in various processes. The energy loss of a charged particle per unit distance in a medium is the *linear stopping power*, S. If ρ is the density of the medium, the *mass stopping power* is (S/ρ). This can be subdivided into *collision mass stopping power* $(S/\rho)_{\mathrm{col}}$, *radiative mass stopping power* $(S/\rho)_{\mathrm{rad}}$ and, at least in principle, into other components. It is sometimes necessary to consider "local" energy deposition in which only energy losses or transfers up to some energy Δ are included. In this case, we use the term *restricted linear collision stopping power* or *linear energy transfer* (LET), L_Δ. In this notation, $L_\infty = S_{\mathrm{col}}$.

The central quantity of dosimetry, the *absorbed dose*, D, is the mean energy imparted per unit mass of material at the point of interest. As in the case of stopping power and linear energy transfer, it cannot be specified for a point in a vacuum.

All of these quantities are physical point functions that vary continuously in space and time. The discrete nature of radiation necessitates the use of *stochastic quantities* that do not meet these criteria but represent some particular value of the quantity in question for a small region of the irradiated material. Only one such quantity, the *specific energy*, z, will be mentioned here. Its nature is best explained by comparisons with the corresponding non-stochastic quantity, the absorbed dose, D.

When a medium is uniformly irradiated, D has the same value throughout. However, even for uniform irradiation, the energy ϵ deposited in a small mass, m, varies randomly with location, and if D and m are small enough, the specific energy z, which equals ϵ/m, may be zero, or many times larger than D. The distribution $F(z)$ denotes the probability that the specific energy is less than the value of z. It is a function of D and m and of the charged particle type and energy. D is the expectation value of z. It is also of evident importance to radiation action, since it must be the actual, rather than the expectation value, of the energy density that determines the radiation effect.

The units of all physical radiation quantities are based on units of the International System: the kilogram, meter, second, and ampere. Other units derived from these are also used. Thus the absorbed dose may be expressed in J/kg (joules per kilogram). The unit J/kg has been given a special name, the gray (symbol, Gy). There are, however, other units outside the International System that are commonly employed. These are the rad (0.01 Gy) for absorbed dose, absorbed dose index, kerma, and specific energy; and keV/μm (1.6×10^{-10} J/m) for linear stopping power and LET.

1.2.2 Quantities for Radiation Protection

The biological effect of radiation depends not only on absorbed dose but also on radiation type and energy, and a host of other factors. In order to facilitate the task of radiation protection, several simplifying assumptions are made. The most important of these are that all subsidiary factors are neglected except for radiation quality, and that the effect of irradiation correlates better with the product of absorbed dose, D, and the quality factor, Q, than with absorbed dose alone. Q is given as a function of LET in water. It ranges from 1 at low LET ($L_\infty < 3.5$ keV μm^{-1}) to 20 at high LET ($L_\infty > 175$ keV μm^{-1}). ICRU Report 19 (ICRU, 1971a) gives the recommended dependence. The product of Q, D and N^1 is the dose equivalent, H; this quantity H is expressed in rem when D is in rad. Because of the underlying simplifications, Q and H are intended for use only in radiation protection and then only for comparing the actual dose equivalent received with the recommended limits.

In dosimetric considerations, particularly those involving normally unoccupied areas such as space, it is often desirable to consider the absorbed dose that would be received at a given location. In virtually all radiation fields, the absorbed dose depends on the orientation of the human body as well as the location of the point of interest inside it. In order to eliminate the first of these factors and to specify a single quantity that is sufficiently conservative, the *absorbed dose index*, D_I, is defined as the maximum absorbed dose in a 30-cm-diameter tissue-equivalent sphere centered at the point of interest. Its unit is the gray (or the rad).

An analogous quantity of even more immediate applicability to radiation protection is the *dose equivalent index*, H_I, defined as the maximum dose equivalent in a 30-cm-diameter tissue-equivalent sphere centered at the point of interest. Its unit is the rem.

ICRU Report 25 (ICRU, 1976b) contains a more comprehensive discussion of these concepts.

1.3 Dosimetric Applications

Few accelerators are capable of producing radiations of the energies under discussion in this report, and the size of the population exposed near these devices is small compared to that irradiated at conventional energies. The number of persons exposed to space radiation is very much smaller still. While these numbers may be expected to increase, it does not appear at present that the increase will be dramatic. An appreciable fraction of the population might travel in high-flying supersonic aircraft in the near future, but it seems well established that, with rare exceptions, the dose received will be quite low. However, the very basis for this statement involves dosimetry, and regardless of the number of persons and the magnitude of the doses involved, protection against radiation of high energies is a subject that must be dealt with effectively.

Radiation protection considerations at all energies must be based on a series of suitable approximations. It is customary to avoid exposure in excess of the maximum permissible dose equivalent (MPD) by provision of such extensive protective features that consideration of individual work habits, with a few exceptions, is unnecessary. Personnel monitoring is then carried out primarily to provide assurance that appropriate protection standards are being met. Because of the low levels encountered, it is normally sufficient to achieve an accuracy of measurement such that the uncertainty of assessing the upper limits to the annual dose equivalent does not exceed 50 percent. For values less than 2 rem, an uncertainty of 1 rem is acceptable (ICRP, 1969). It is thus usually unnecessary to construct a 30-cm-diameter tissue equivalent sphere and to perform detailed measurements within such a phantom. One or a few well designed measurements with devices of appropriate wall thickness will often suffice. Discussion of the determination of dose equivalent index is included in ICRU Report 25 (ICRU, 1976b).

Greater accuracy is evidently required when the MPD for radiation workers is approached or even exceeded. The latter may occur either because of unforseen conditions (accidents) or it may be deliberately brought about because of other overriding considerations (e.g., on extended space missions). Such situations must be met by increasingly refined measurements and values of Q are not applicable.

Since Q is a function of L_∞, a determination of the dose equivalent involves an integration, i.e.,

$$H = D\bar{Q},$$
$$\text{where } \bar{Q} = \frac{1}{D}\int_0^\infty QD_{L_\infty}\mathrm{d}L_\infty,$$

where $D_{L_\infty}\mathrm{d}L_\infty$ is the absorbed dose in the interval L_∞ to $L_\infty + \mathrm{d}L_\infty$. Because of the inherent limitation of H, D_{L_∞} is likely to provide more useful dosimetric information for purposes of radiation protection. It must be noted that D_{L_∞} depends on location in the body. Often this function needs only to be determined at the location of maximum absorbed dose. In some instances, however, it may be necessary to know it in a particular organ.

Another important application of dosimetry is in radiobiology. Research in this discipline is rarely performed at high energies because of the complexity of the radiations encountered. This complexity is often greatly increased by interactions taking place in the irradiated tissues. Consequently, even if intrinsic biological differences are ignored, the extrap-

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[1] N is the product of any other modifying factors and can be set equal to unity for external radiation, which is exclusively discussed in this report.

olation from animal (or tissue) to man is complicated by differences in absorbed dose distribution at very high energies, as well as at very low ones.

Radiobiological research having significant quantitative aspects requires refined dosimetry, and accuracy is usually essential. It is equally necessary to provide exhaustive information on radiation quality. D_{L_∞} may not be sufficient and microdosimetric information, such as $F(z)$, may be needed.

Careful dosimetry in radiotherapeutic applications is extremely important because the radiosensitivity of normal tissues adjacent to or near the tumor site is usually the limiting factor. Absorbed dose determinations to within 5 percent accuracy are often desired by the radiotherapist. ICRU Report 24 (ICRU, 1976a) discusses problems connected with absorbed dose determination within the patient.

In its broader sense, the term *dosimetry* applies not only to biological material, but also to all other kinds of material as, for instance, those involved in radiation damage studies. Because of the variability of the information required, this aspect is not explicitly covered in this report but many of the general statements should be applicable.

2. Basic Interactions of High-Energy Particles with Matter

2.1 The Particles and Their Interactions

2.1.1 Introduction

When high-energy particles are incident on matter, a wide variety of processes occurs before all nuclear fragments and newly created particles have been absorbed or have decayed and the kinetic energy deposited in the absorber is degraded to thermal values. The processes important in energy deposition can involve three of the four known types of interaction: the nuclear (strong) interaction, the electromagnetic interaction, and the weak interaction. Only the fourth, the gravitational interaction, plays no significant role.

2.1.2 The Particles

The known particles arising from these interactions are classified into three groups:

a. Photons
b. Leptons: muons, electrons, positrons and neutrinos
c. Hadrons: protons, neutrons, and all other strongly interacting particles.

Table 2.1 lists these groups, indicating the most important members of each in dosimetry, their masses, and where in dosimetry their significance lies.

The *photon* has zero rest mass and travels with the speed of light. It has a spin of 1 and can interact via the electromagnetic or nuclear interaction. The term photon encompasses both x and gamma radiation.

The *lepton* group consists of all of the particles of spin ½ that are lighter than the proton. The group has eight members: four neutrinos, the electron and positron, and the positive and negative muon. Both the electron and positron are stable, but they may annihilate:

$$e^+ + e^- \rightarrow \gamma + \gamma$$
$$\rightarrow \gamma + \gamma + \gamma.$$

The first reaction is more common. Normally, annihilation occurs from a bound state of the electron-positron system and, therefore, each photon has an energy very nearly equal to mc^2 (511 keV). The annihilation radiation often appears as a sharp peak at 511 keV in a continuum of radiation that results from other processes. The muon is unstable and decays into an electron or positron, depending on its charge, and two neutrinos, with a mean life time of 2.2×10^{-6}s. The electrons and muons carry an electric charge and so can interact via the electromagnetic interaction with other electrically charged particles, or indeed with those neutral particles, e.g., neutrons, that possess a magnetic moment. Neutrinos are uncharged, massless, and carry no magnetic moment, and so interact only via the weak interaction.

The *hadron* group contains those particles that interact via the nuclear interaction. It can conveniently be divided into two subgroups, the baryons and the mesons. *Baryons* are particles with mass equal to or greater than that of the proton and have half-integral spin. *Mesons* consist of the strongly interacting particles that have integral (or zero) spin. The mesons of dosimetric significance have masses less than that of the protons. The majority of all known particle types are hadrons. They interact with each other via the strong (or nuclear) interaction, but only when their distance of separation is less than about 10^{-13} cm, the range of the nuclear force. At larger distances, hadrons can interact through the electromagnetic interaction, for example, in the scattering of protons in traversing matter, or in energy-loss by ionization. The weak interaction can also play a role for unstable hadrons, causing

TABLE 2.1—*Classification of the fundamental particles*

Class	Members of the class most important in dosimetry	Rest energy/MeV	Dosimetric significance
Photons	γ	0	Bremsstrahlung, induced radioactivity, electromagnetic cascades, scattered photons, annihilation photons
Leptons	$e^{\pm}$ $\mu^{\pm}$	0.511 105.66	Primary beams, induced radioactivity, cosmic rays, secondary beams, forward shielding
Hadrons			
Mesons	$\pi^{\pm}$	139.57	Secondary beams, radiation therapy
Baryons	p	938.28	Primary beams, shielding leakage, cosmic rays
	n	939.57	Secondary beams, shielding leakage, skyshine, radiation therapy, cosmic rays

various relatively slow decay processes, such as the beta-decay of radioactive nuclei. The proton has the smallest mass of the baryons and is stable in isolation. The neutron is very slightly heavier than the proton, and, as a consequence, when free, is unstable, although its mean lifetime is quite long (approximately 1000 seconds). A neutron decays into a proton, electron, and antineutrino. The time interval between the emission of a low energy (several MeV) neutron from an interaction or disintegration and its subsequent slowing down and capture is of the order of a few milliseconds in all solids and liquids. This figure is much shorter than the mean life for decay, so that decay of a free neutron is rare in these media. However, in the upper atmosphere, at an altitude of say 20 km, the slowing-down time and time for capture is of the order of a few seconds; thus, a few percent of the neutrons produced by interaction processes will decay in flight. Those neutrons that travel outward from the top of the atmosphere have a long path, and an appreciable number decay, contributing a portion of both the proton and electron components of the trapped radiation belts.

The most important mesons are the pions. They are produced copiously in high-energy interactions, and the charged pions play a large part in the propagation of the hadronic cascade discussed in Section 2.5. In air or vacuum they tend to decay into muons, while in condensed matter they have a greater probability of coming to rest, where the positive pions will decay and the negative pions will be captured, forming pi-mesic atoms. In the latter case, the atoms will quickly decay to the ground state, emitting characteristic x rays, and the pions will be captured by the nucleus and interact with it, causing nuclear breakup and emission of low energy protons, alpha particles, and nuclear fragments of a high linear energy transfer. This property has lead to research toward the use of negative pion beams in cancer radiotherapy (Raju and Richman, 1972). Heavier mesons and baryons are also produced in high energy interactions, but are not as important to dosimetry because the probability for their production is significantly less than that for the production of the particles already mentioned.

2.1.3 The Interactions

The *electromagnetic* interaction is better understood than either the nuclear or weak interactions, and can be divided into two categories:

(a) the direct electromagnetic interaction, which occurs between particles with charge or magnetic moment and is of long range, and
(b) interactions in which photons are emitted or absorbed.

The most important direct interaction results from the Coulomb force between charged particles. Sequential electromagnetic interactions of a moving charged particle with the electrons and atoms in a medium result in the important processes of ionization and excitation loss, in which the largest part of the incident energy is often transferred to the absorber. Various aspects of this interaction will be discussed in Sections 2.2, 2.3, and 2.4.

The *nuclear* interaction occurs only between hadrons or a photon and a hadron. It is the strongest of the interactions, but is of extremely short range ($\sim 10^{-13}$ cm) and is responsible for the binding of protons and neutrons in atomic nuclei. The characteristics of this interaction will be dealt with extensively throughout this report, particularly in Sections 2.5 and 2.6.

The *weak* interaction manifests itself most importantly through the decay of particles that have been produced in the strong hadronic interactions under circumstances where the particles persist long enough for the decays to occur, e.g., in the beta-decay of neutrons or radioactive nuclei and the decay in air or vacuum of charged mesons into muons and of muons into electrons.

Although the nuclear interaction occurs only between hadrons or a photon and a hadron, the other two types of interaction may occur between particles of all categories.

2.1.4 The Relation of Fundamental Particles and Their Interactions to Dosimetry

At high energies, radiation environments may be extremely complex since many or all of the fundamental particles may be created. The dosimetric importance of fundamental particles is determined both by the probability with which they are produced and by the probability of their interaction with matter. Since the major concern in dosimetry is with the amount of energy deposited in matter, it is clear that particles that interact only weakly—no matter how copiously they are produced—are of little importance in depositing absorbed dose. Thus, for example, neutrinos interact with matter *only* through the weak interaction. The probability for the neutrino-nucleon interaction is so small that the interaction mean free path is on the order of 10^{18} g cm^{-2}. Neutrinos, therefore, are of no practical concern in radiation dosimetry. Similarly, particles which are produced only rarely, e.g., mesons heavier than pions, are of little or no concern.

Primary photons with energies above 100 MeV interact with matter principally through electron pair production and photonuclear processes. Reaction products of the resulting electromagnetic cascade (see Section 2.4) have lower energy, and, as the energy of the photons is degraded, they interact with increasing probability via the Compton scattering and photoelectric absorption processes. Thus, in many cases, practical high-energy photon dosimetry may in fact be very similar to the more familiar dosimetry of photons in the MeV region. The energy deposited in matter by photons may be calculated from the mass energy absorption coefficients, and these are discussed in Section 2.3.

It is ultimately by the process of ionization (i.e., through the electromagnetic interaction) that the available energy of the fundamental particles is transferred to matter. Thus, for example, it is largely by the production of electrons or positrons that photons deposit energy, or by the production of recoil protons that neutrons deposit a great part of their energy in tissue.

At higher energy there is a larger number of energy deposition modes available. At very high energies, electrons and hadrons each generate both electromagnetic and hadronic cascades; these are discussed in Sections 2.4 and 2.5.

Although all charged particles deliver absorbed dose in tissue, the particles of lower energy and greater mass are attenuated more quickly in shielding by ionization losses. Thus, outside of shielding, uncharged particles—photons and neutrons—in many cases dominate the radiation environment, and it is their subsequent interactions in tissue that deposit energy.

Primary charged particles are responsible for substantial dose deposition in unshielded or thinly shielded regions, as, for example, in outer space, and in or near accelerator beams. For example, primary protons and heavy ions are the dominant source of radiation exposure to astronauts, as discussed in Section 4. Pion and heavier ion dosimetry are of increasing importance at accelerators, where charged-particle beams are being used in radiobiology and radiotherapy.

Muons contribute about one third the absorbed dose caused by the natural radiation environment at sea level and are also significant sources of radiation at high-energy particle accelerators in the tens-of-GeV energy region and above. Their persistence is due to their lack of a strong interaction, so they can travel a considerable distance in air at high energy before they decay.

2.1.5 Summary

A wide variety of processes can occur when any high-energy particle is incident on matter; these will usually involve three forms of interaction and all types of particle. A whole chain of events is initiated by the incidence of even one particle. In order to follow out the details of this chain and understand the mechanisms, the interactions must be studied in some detail.

2.2 Electromagnetic Interactions of Charged Particles

2.2.1 Energy Loss by Ionization and Direct Pair Production

When fast electrically charged particles pass through matter, they interact via their electromagnetic field and impart energy to the electrons in the atoms and, to a lesser extent, to the nuclei of the atoms through which they pass. The encounters with atomic electrons can be conveniently divided into two categories:

(1) hard collisions, where the energy imparted is much greater than the binding energy of the electron.

(2) soft collisions, where the energy given to the electron is similar in magnitude to its binding energy (Rossi, 1952).

The formulae for energy loss are derived assuming that the incident particle is moving at a speed v_p much greater than the median velocity of the electrons in their atomic orbits, i.e., $v_p >> Z^{1/3} c/137$, where Z is the atomic number of the atoms of the medium and c is the velocity of light.

For the hard collisions, the energy transferred is very large compared to the electron binding energy; thus, the atomic electrons are considered initially at rest and unbound. From conservation of momentum and energy, an upper limit $T_{\max}$ can be set to the kinetic energy that can be imparted to an electron in a head-on collision by a particle of rest mass M and total energy E. The value of $T_{\max}$ is given by the expression (Rossi, 1952)

$$T_{\max} = 2mc^2 \frac{p^2c^2}{m^2c^4 + M^2c^4 + 2mc^2E}, \qquad (2.1)$$

where m is the electron rest mass, c is the velocity of light, and p is the momentum of the particle.[2] This expression for $T_{\max}$ takes various limiting forms. When M is much greater than m, as is the case for mesons or protons, and, in addition, when $pc \ll (M/m)Mc^2$, the expression simplifies to

$$T_{\max} \approx 2mc^2 \frac{\beta^2}{1-\beta^2} \qquad (2.2)$$

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[2] The relationship between the total energy, E, of a particle of mass M and its momentum p is: $E^2 = p^2 \cdot c^2 + M^2 \cdot c^4$.

where $T_{\max}$ is very much less than E. Here, β is the particle velocity relative to that of light. At very high energies, $T_{\max}$ approaches pc or E, regardless of the value of M; that is, there is a small probability that the knock-on electron can carry off almost all the kinetic energy of the incident primary particle.

Expressions have been obtained for the energy spectrum of knock-on electrons for hard collisions as a function of incident mass, kinetic energy, and spin for singly charged incident particles with spin 0 and spin 1/2. In an infinitesimal traversal dx, expressed in mass per unit area, one obtains the following expression for $N(E, T)\,\mathrm{d}T\mathrm{d}x$, the number of knock-on electrons with kinetic energy between T and $T + \mathrm{d}T$ produced by a singly charged particle with energy E and velocity βc, in the case of spin 0:

$$N(E, T)\,\mathrm{d}T\,\mathrm{d}x = 2\pi N_{\mathrm{A}} \frac{Z}{A} r_{\mathrm{e}}^2 \frac{mc^2}{\beta^2} \frac{\mathrm{d}T}{T^2} \left(1 - \beta^2 \frac{T}{T_{\max}}\right) \mathrm{d}x \qquad (2.3)$$

and in the case of spin 1/2:

$$N(E, T)\,\mathrm{d}T\,\mathrm{d}x = 2\pi N_{\mathrm{A}} \frac{Z}{A} r_{\mathrm{e}}^2 \frac{mc^2}{\beta^2} \frac{\mathrm{d}T}{T^2} \left(1 - \beta^2 \frac{T}{T_{\max}} + \frac{T^2}{2E^2}\right) \mathrm{d}x, \qquad (2.4)$$

where $r_{\mathrm{e}} = e^2/mc^2$, e is the charge on the electron, and the remaining symbols refer to the medium, which has atomic number Z and molar mass A. N_{A} is the Avogadro constant.

It is of interest to compute the total energy dE transferred to these knock-on electrons for which T is greater than some arbitrary energy Δ, chosen so that the expressions for hard collisions are valid. If the spin 0 case is selected as being sufficiently accurate, and is generalized to include multiply-charged incoming ions with charge z, the following expression for the energy loss, dE, is obtained:

$$\mathrm{d}E(T > \Delta) = 2\pi z^2 N_{\mathrm{A}} \frac{Z}{A} r_{\mathrm{e}}^2 \frac{mc^2}{\beta^2} \left[\ln\left(\frac{T_{\max}}{\Delta}\right) - \beta^2\right] \mathrm{d}x. \qquad (2.5)$$

The distribution of energy transfers for the low energy collisions is considerably more difficult to calculate. The total energy lost in these collisions is:

$$\mathrm{d}E(T < \Delta) = 2\pi z^2 N_{\mathrm{A}} \frac{Z}{A} r_{\mathrm{e}}^2 \frac{mc^2}{\beta^2} \cdot \left[\ln\left(\frac{2mc^2\beta^2\Delta}{(1-\beta^2)I^2}\right) - \beta^2\right] \mathrm{d}x, \qquad (2.6)$$

where I is the effective ionization potential of the atom and is empirically equal to ~ 10Z eV. The linear rate of energy loss dE/dx ($T < \Delta$) transferred in soft collisions given by Eq. (2.6) is known as the linear energy transfer (LET), L_Δ, of the particle. This is known as the "restricted energy loss" in the context of track formation and ion displacement in media other than biological tissue. ICRU Report 16 (ICRU, 1970) contains a comprehensive discussion of linear energy transfer. The value of Δ = 100 eV has been used in biological work. The L of charged particles, or some similar measure of local density of energy deposition, is considered to be important in determining biological effectiveness.

Returning now to the general case for the total energy transferred to electrons, Eqs. (2.5) and (2.6) can be added to obtain an expression for the total energy transferred in collisions with atomic electrons. For the case of massive particles, we substitute expression (2.2) for $T_{\max}$ in Eq. (2.5), giving

$$\mathrm{d}E = 4\pi z^2 N_{\mathrm{A}} \frac{Z}{A} r_{\mathrm{e}}^2 \frac{mc^2}{\beta^2} \cdot \left[\ln\left(\frac{2mc^2\beta^2}{(1-\beta^2)I}\right) - \beta^2\right] \mathrm{d}x. \qquad (2.7)$$

The expression for the ratio of dE to dx obtained from Eq. (2.7) is the total linear rate of energy loss from electronic and atomic collisions along the track of the charged particle. It is the well-known Bethe-Bloch formula for the ionization energy-loss or mass collision stopping power (S_{col}/ρ) of a fast charged particle in matter. It should be stressed that in integrating expression (Eq. 2.3) to obtain the collision stopping power, only an average is obtained; there will be statistical fluctuations of the individual particle energies, which will be discussed in the next section.

In addition, at very high energy, the approximation for $T_{\max}$ from Eq. (2.2) cannot be used; instead, the full expression given in Eq. (2.1) must be substituted. The collision stopping power has a maximum at low velocities where $T_{\max}$ is approximately 2 I. Above the maximum, it decreases steadily with increasing velocity and reaches a minimum at $\beta \sim 0.95$, before rising slowly again. The capture and loss of electrons reduces the apparent charge at low velocity and decreases the magnitude of the maximum value of the collision stopping power and increases the energy at which it occurs. This is discussed in Section 2.2.2.

The Bethe-Bloch expression only accounts for energy imparted to atomic electrons of the medium that the particle penetrates. Three effects have been overlooked and must be taken into account for the ionization loss of very high energy particles: (a) the polarization effect; (b) at even higher energies, direct pair production; and (c) bremsstrahlung.

The Bethe-Bloch expression overestimates the energy loss of charged particles by ignoring the effects of polarization. Polarization reduces the logarithmic rise in collision stopping power at high energies in dense media, since the soft collisions, which occur at a distance greater than the atomic separation, are reduced in effectiveness by intervening atoms. The effect, therefore, is to reduce the relativistic rise contribution given by Eq. (2.6). The total energy loss, however, continues to rise slowly with energy from the hard-collision contribution, with $T_{\max}$ given by Eq. (2.1). In gases at normal temperature and pressure, polarization effects occur only at extremely high energy.

A second electromagnetic process not accounted for in the Bethe-Bloch formula is *direct pair production*. This process is the direct production of an electron-positron pair by the incoming particle in the field of the target nucleus and is important only at very high values of E/Mc^2. The differential energy imparted in this process is

$$dE_{\text{pair}} = \frac{20\, N_A z^2}{\pi(137)^2}\,\frac{Z(Z+1)}{A}\, r_e^{2}\, \frac{mE}{M} \ln\left(\frac{183}{Z^{1/3}}\right) dx. \qquad (2.8)$$

By introducing the radiation length X_0 of the medium, this expression simplifies to

$$dE_{\text{pair}} = \frac{5\, z^2}{137\pi}\,\frac{mE}{M\, X_0}\, dx, \qquad (2.9)$$

$$\text{where } \frac{1}{X_0} = \frac{4}{137}\,\frac{N_A}{A}\, Z(Z+1) r_e^{2} \ln\left(183/Z^{1/3}\right). \qquad (2.10)$$

X_0, the *radiation length*, is a function of the properties of the medium. Values for the radiation length are tabulated, for example, by Tsai (1974). More discussion of radiation length, with tabulated values, will be found in Section 2.4. Most of this energy loss occurs in transfers which result in the production of electron pairs of energy within a factor of 10 of the value of $(m/M)E$. Thus, an appreciable number of such energy transfers contributes to the loss of energy by this process, so that fluctuations are not severe. The energy loss by this process exceeds not only the slow logarithmic rise expressed in the Bethe-Bloch formula; it exceeds the normal ionization loss at very high primary particle energy since it is proportional to the total particle energy and depends only on the ratio of energy to mass of the incoming particle. For all nuclear interacting particles, the nuclear interaction dominates and energy loss by pair production is not significant. For electrons, energy loss by bremsstrahlung is much more important than direct pair production. Thus pair production is important only for muons at high energy.

A third process, bremsstrahlung, is also important at high energy for muons. It will be discussed in Sections 2.2.5 and 2.2.6.

In the very high energy range, i.e., above 10 GeV, the magnitude of the logarithmic term is reduced by the polarization effect so that the energy loss for muons including pair production can be approximated by the simple expression

$$\left(\frac{dE}{dx}\right)_{\text{total}} = \text{a} + \text{b}E. \qquad (2.11)$$

A good approximation results when a is set equal to dE/dx at 10 GeV and b is set equal to the reciprocal of $9 \times 10^3\, X_0$. Figures 2.1 and 2.2 show the energy loss of muons as a function of energy in representative media, and, in Figure 2.2, the departures from the Bethe-Bloch formulation are indicated.

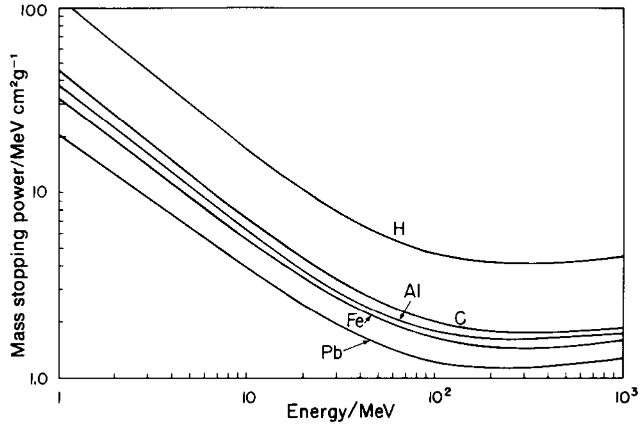

Fig. 2.1. Mass stopping powers for muons in hydrogen, carbon, aluminum, iron, and lead.

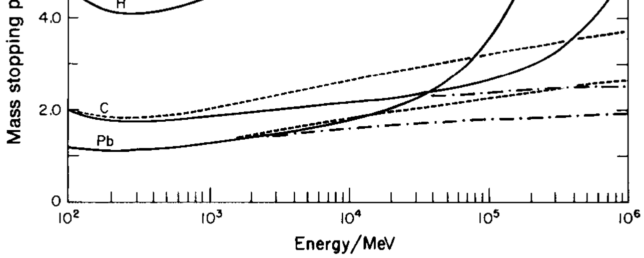

Fig. 2.2. Mass stopping powers for muons in hydrogen, carbon, and lead, including modifications due to polarization and pair production at high energies.

2.2.2 Range-Energy Relations and Straggling

According to the Bethe-Bloch expression, one expects a particle of a given charge, mass, and velocity to be brought to rest by ionization loss, and the mean range can be obtained from integrating the reciprocal of the expression (Eq. 2.7) over the energy:

$$\bar{R} = \int_0^{\bar{R}} \mathrm{d}x = \int_0^E \frac{\mathrm{d}E}{S_{\mathrm{col}}/\rho}. \tag{2.12}$$

The result may be written

$$\bar{R} \propto \frac{M}{z^2} \mathrm{f}(\beta) \tag{2.13}$$

since $\mathrm{d}E/\mathrm{d}x$ is proportional to z^2 and independent of the mass of the particle. It is important to note that the mean range of particles of a given speed is proportional to their mass M and varies as the inverse square of their charge. There is no analytic expression for $\mathrm{f}(\beta)$, but for a considerable range of β (and hence energy):

$$\bar{R} = \frac{\mathrm{k}}{z^2} \frac{E^{1.73}}{M^{0.73}} \text{ for } 10^{-3} < \frac{E}{Mc^2} < 0.8, \tag{2.14}$$

where k depends only slightly on the nature of the medium if $\bar{R}$ is expressed in mass per unit area. Range-energy relations for protons in various media are tabulated in a number of sources (Bichsel, 1968; Barkas and Berger, 1964; Janni, 1966; ICRU, 1970).

The expression for the energy loss of muons at very high energy (Eq. 2.11) can be directly integrated to give the range-energy relation. It is convenient to do this, since muons are the most penetrating charged particles known and often dictate the thickness of shielding required. The relation is:

$$\bar{R} = \frac{1}{\mathrm{b}} \ln\left(1 + \frac{\mathrm{b}E}{\mathrm{a}}\right). \tag{2.15}$$

The value of b should include those contributions from bremsstrahlung (Section 2.2.6) that involve the emission of an appreciable number of photons of energy $h\nu \lesssim 0.1E$. The larger energy losses cause a small fraction of the muons to have a shorter range. The best value of b is about twice that from direct pair production alone, and this has been taken into account in the value of b presented after Eq. (2.11).

As indicated earlier, for a multiply-charged incident particle (charge ze), the energy loss is increased by a factor of z^2 over that given by the Bethe-Bloch expression for a singly charged particle. This is correct only at high energies. As the charged particle decreases in velocity, it is likely to pick up electrons. The velocity β_{K} at which one K electron is picked up is given by the approximate expression $\beta_{\mathrm{K}} \approx z/137$, and empirically the effective charge of the ion, z_{eff}, is given by

$$z_{\mathrm{eff}} = z\left[1 - \exp\left(\frac{-125\beta}{z^{2/3}}\right)\right]. \tag{2.16}$$

At low velocity, the effective charge z_{eff} is appreciably less than z, so that the ionization produced by such a particle is reduced and its range somewhat extended. Figures 2.3 and 2.4 give stopping power-range and stopping power-energy relations for various heavy ions in water (Steward, 1968), and tabulated data are available in ICRU Report 16 (ICRU, 1970). Stopping power data from Janni (1966) for protons above 1 MeV in various substances of interest to dosimetrists are given in the Appendix of the present report.

2.2.3 Statistical Fluctuations of Energy Loss

As indicated in Section 2.2.2, Eqs. (2.3) and (2.4) apply only to the average number of knock-ons. Statistical fluctuations in the energy-loss process, known as Landau fluctuations, affect the pulse height distribution obtained when fast particles pass through thin proportional counters or scintillators. These, along with fluctuations of energy imparted to the sensitive region, can severely reduce the accuracy with which the mean stopping power of an individual particle can be measured. The most probable energy loss can be significantly less than the average energy loss at high energies (Rossi, 1952). This is caused by the unsymmetrical energy loss distribution having a tail at high-energy losses. The fluctuations also result in an r.m.s. spread R in the actual range of individual monoenergetic particles. This effect is

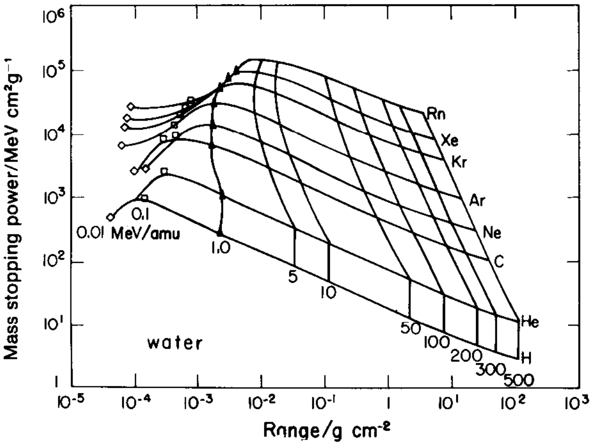

Fig. 2.3. Mass stopping-power curves as a function of residual range for several ions in water. The quotient of kinetic energy and mass of the ion in units of MeV/amu is designated by symbols (for 0.01 to 1.0 MeV/amu) or by curves of constant ion velocity. (From Steward, 1968.)

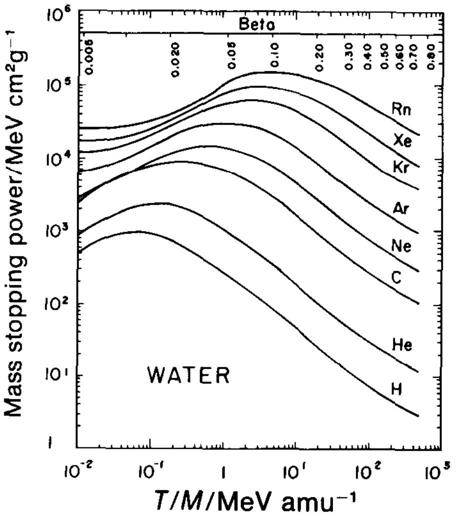

Fig. 2.4. Mass stopping power curves as a function of the quotient of kinetic energy and mass for various ions in water. At the top of the figure, values of $\beta = v_p/c$ are displayed. (From Steward, 1968.)

known as "range straggling" and limits the accuracy with which the observed range of a particle can yield a measure of its initial energy. The fractional spread in R is given by:

$$\frac{\Delta R}{R} = \left(\frac{m}{M}\right)^{1/2} g\left(\frac{E}{Mc^2}\right) \tag{2.17}$$

and is most important for light particles. The quantity $g(E/Mc^2)$ is a slowly varying function with a minimum of 0.4 at $E/Mc^2 \approx 3$ and is below 0.60 for $0.06 < E/Mc^2 < 30$, as computed for iron. Curves for other stopping media may be found in the American Institute of Physics Handbook (Bichsel, 1963).

Calculations have been extended to include thick absorbers (Payne, 1969), and large and extremely large energy losses have been treated (Tschalär, 1968a; 1968b).

2.2.4 Elastic Scattering of Charged Particles

When a charged particle passes in the neighborhood of a nucleus, it undergoes a change in direction; that is, it is scattered. Because of the relatively small probability that a photon is emitted with energy comparable to the kinetic energy of the charged particle, the scattering process is generally considered to be an elastic one. In addition, it is assumed that the nucleus is very much heavier than the incident particle and thus does not acquire significant kinetic energy.

The differential scattering probability is defined as follows:

$\Xi(\Theta)d\omega\ dx$ = probability that a charged particle of momentum p and relative velocity β traversing a thickness dx (expressed in mass per unit area) undergoes a collision that deflects the trajectory of the particle into the solid angle $d\omega$ about Θ (from its original direction).

Various formulae have been derived for $\Xi(\Theta)d\omega\ dx$, which depend on the nature of the medium as well as the charge and spin of the particle. If shielding of a point charge, Ze, by the atomic electrons is neglected, and if the Born approximation[3] is used, the following expressions for heavy singly charged particles (Rossi, 1952) can be obtained:

$$\Xi(\Theta)d\omega = \frac{1}{4} N_A \frac{Z^2}{A} z^2 r_e^2 \left(\frac{mc}{p\beta}\right)^2 \frac{d\omega}{\sin^4(\Theta/2)}, \tag{2.18}$$

for spin zero particles (e.g., alpha particles and pions),

where

N_A = the Avogadro constant
m = mass of electron
r_e = e^2/mc^2 = classical electron radius
p = momentum of the particle
β = the ratio of the particle velocity to that of light
z = charge of the particle
Z = atomic number of the scattering medium
A = molar mass of the scattering medium;

$$\Xi(\theta)d\omega = \frac{1}{4} N_A \frac{Z^2}{A} z^2 r_e^2 \left(\frac{mc}{p\beta}\right)^2 \frac{d\omega}{\sin^4(\Theta/2)} [1 - \beta^2 \sin^2(\Theta/2)]. \tag{2.19}$$

for spin one-half particles (e.g., protons and muons). This formula is called the Mott scattering formula for heavy particles. Mott (1929) derived the elastic scattering cross section for electron scattering from nuclei of charge Ze by employing relativistic Dirac theory with the Born approximation. By expanding Mott's exact formula in powers of αZ, the following expression is obtained:

$$\Xi(\Theta)d\omega = \frac{1}{4} N_A \frac{Z^2}{A} z^2 r_e^2 \left(\frac{mc}{p\beta}\right)^2 \frac{d\omega}{\sin^4(\Theta/2)} \times \{1 - \beta^2 \sin^2(\Theta/2) + \pi\beta\alpha Z[1 - \sin(\Theta/2)]\sin(\Theta/2)\}, \tag{2.20}$$

where α = fine structure constant.

This formula is valid only for high velocities ($\beta \approx 1$) and for rather low Z materials ($\alpha Z \leqslant 0.2$ and $Z \leqslant 27$).

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[3] The Born approximation assumes that the entire potential energy of interaction can be regarded as a small perturbation; quantum mechanical perturbation theory is applied, and calculations are carried only to first order.