GAUGE FIELDS

INTRODUCTION TO QUANTUM THEORY

L. D. FADDEEV

A. A. SLAVNOV

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L. D. FADDEEV *Academy of Sciences U.S.S.R.*
Steclov Mathematical Institute, Leningrad

A. A. SLAVNOV *Academy of Sciences U.S.S.R.*
Steclov Mathematical Institute, Moscow

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L. D. Faddeev and A. A. Slavnov

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EDITOR'S FOREWORD

The problem of communicating in a coherent fashion recent developments in the most exciting and active fields of physics seems particularly pressing today. The enormous growth in the number of physicists has tended to make the familiar channels of communication considerably less effective. It has become increasingly difficult for experts in a given field to keep up with the current literature; the novice can only be confused. What is needed is both a consistent account of a field and the presentation of a definite "point of view" concerning it. Formal monographs cannot meet such a need in a rapidly developing field, and, perhaps more important, the review article seems to have fallen into disfavor. Indeed, it would seem that the people most actively engaged in developing a given field are the people least likely to write at length about it.

FRONTIERS IN PHYSICS has been conceived in an effort to improve the situation in several ways. Leading physicists today frequently give a series of lectures, a graduate seminar, or a graduate course in their special fields of interest. Such lectures serve to summarize the present status of a rapidly developing field and may well constitute the only coherent account available at the time. Often, notes on lectures exist (prepared by the lecturer himself, by graduate students, or by postdoctoral fellows) and are distributed in mimeographed form on a limited basis. One of the principal purposes of the FRONTIERS IN PHYSICS Series is to make such notes available to a wider audience of physicists.

It should be emphasized that lecture notes are necessarily rough and informal, both in style and content; and those in the series will prove no exception. This is as it should be. The point of the series is to offer new, rapid, more informal, and, it is hoped, more effective ways for physicists to teach one another. The point is lost if only elegant notes qualify.

The publication of collections of reprints of recent articles in very active fields of physics will improve communication. Such collections are themselves useful to people working in the field. The value of the reprints will, however, be enhanced if the collection is accompanied by an introduction of moderate length which will serve to tie the collection together and, necessarily, constitute a brief survey of the present status of the field. Again, it is appropriate that such an introduction be informal, in keeping with the active character of the field.

The informal monograph, representing an intermediate step between lecture notes and formal monographs, offers an author the opportunity to present his views of a field which has developed to the point where a summation might prove extraordinarily fruitful but a formal monograph might not be feasible or desirable.

Contemporary classics constitute a particularly valuable approach to the teaching and learning of physics today. Here one thinks of fields that lie at the heart of much of present-day research, but whose essentials are by now well understood, such as quantum electrodynamics or magnetic resonance. In such fields some of the best pedagogical material is not readily available, either because it consists of papers long out of print or lectures that have never been published.

The above words written in August 1961 seem equally applicable today. The development during the past decade of a quantum theory of gauge fields represents a significant contribution to our understanding of elementary particles and their interactions.

The present volume is intended to introduce the reader to the methods of quantum gauge field theory; the authors both set forth the basic elements of the theory and provide illustrative applications.

Ludwig Faddeev and Andrei Slavnov are especially well qualified to write such a book because they have been among the key participants in the development of the theory.

It is a pleasure to welcome them to the ranks of contributors to this Series. I would like also to take this opportunity to thank Dr. Edward Witten of Harvard University for his invaluable assistance in reviewing and editing the English translation of this volume.

DAVID PINES

PREFACE TO THE ORIGINAL (RUSSIAN) EDITION

Progress in quantum field theory, during the last ten years, is to a great extent due to the development of the theory of Yang–Mills fields, sometimes called gauge fields. These fields open up new possibilities for the description of interactions of elementary particles in the framework of quantum field theory. Gauge fields are involved in most modern models of strong and also of weak and electromagnetic interactions. There also arise the extremely attractive prospects of unification of all the interactions into a single universal interaction.

At the same time the Yang–Mills fields have surely not been sufficiently considered in modern monographical literature. Although the Yang–Mills theory seems to be a rather special model from the point of view of general quantum field theory, it is extremely specific and the methods used in this theory are quite far from being traditional. The existing monograph of Konoplyova and Popov, "Gauge Fields," deals mainly with the geometrical aspects of the gauge field theory and illuminates the quantum theory of the Yang–Mills fields insufficiently. We hope that the present book to some extent will close this gap.

The main technical method, used in the quantum theory of gauge fields, is the path-integral method. Therefore, much attention is paid in this book to the description of this alternative approach to the quantum field theory. We have made an attempt to expound this method in a sufficiently self-consistent manner, proceeding from the fundamentals of quantum theory. Nevertheless, for a deeper understanding of the book it is desirable for the reader to be familiar with the traditional methods of quantum theory, for example, in the volume of the first four chapters of the book by N. N. Bogolubov and D. V. Shirkov, "Introduction to the Theory of Quantized Fields." In particular, we shall not go into details of comparing the Feynman diagrams to the terms of the perturbation-theory expansion, and of the rigorous substantiation of the renormalization procedure, based on the *R*-operation. These problems are not specific for the Yang–Mills theory and are presented in detail in the quoted monograph.

There are many publications on the Yang–Mills fields, and we shall not go into a detailed review of this literature to any extent. Our aim is to introduce the methods of the quantum Yang–Mills theory to the reader. We shall not discuss alternative approaches to this theory, but shall present in detail that approach, which seems to us the most simple and natural one. The applications dealt with in the book are illustrative in character and are not the last word to be said about applications of the Yang–Mills fields to elementary-particle models. We do this consciously, since the phenomenological aspects of gauge theories are developing and changing rapidly. At the same time the technique of quantization and renormalization of the Yang–Mills fields has already become well established. Our book is mainly dedicated to these specific problems.

We are grateful to our colleagues of the V. A. Steclov Mathematical Institute in Moscow and Leningrad for numerous helpful discussions of the problems dealt with in this book.

We would especially like to thank D. V. Shirkov and O. I. Zav'yalov who read the manuscript and made many useful comments, and E. Sh. Yegoryan for help in checking the formulas.

Moscow-Leningrad-Kirovsk L. D. FADDEEV, A. A. SLAVNOV

PREFACE TO THE ENGLISH EDITION

This book was written in the spring of 1977 and published in Russian in 1978. By that time, the perturbation theory for quantum Yang–Mills theory had been completed and its relevance for elementary particle theory also had been generally accepted. We hope that our book covers all essential features of this development.

Since publication of the Russian edition, several new and exciting ideas were proposed dealing mainly with the nonperturbative approach and the problem of quark confinement. In particular, the problem of gauge ambiguity, lattice formulation of gauge theories, the role of instanton solutions in quantum dynamics, have been widely discussed. However, it is our opinion that none of these approaches can be considered as definitive. For this reason, we decided that it would be premature to provide any additions to the present English-language version.

We are grateful to Dr. D. B. Pontecorvo for prompt and faithful translation into English.

Aspen, Colorado,
August, 1979

L. D. FADDEEV, A. A. SLAVNOV

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CHAPTER 1

INTRODUCTION

1.1 BASIC CONCEPTS AND NOTATION

At first sight, the theory of gauge fields which we shall discuss in this book describes a rather narrow class of quantum-field-theory models. However, the opinion is becoming more and more popular that this theory has a chance to become the basis of the theory of elementary particles. This opinion is based on the following facts:

First, the only theory (quantum electrodynamics) completely confirmed by experiments is a particular case of the gauge theory.

Second, phenomenological models of weak interactions have acquired an elegant and self-consistent formulation in the framework of gauge theories. The phenomenological four-fermion interaction has been replaced by the interaction with an intermediate vector particle, the quantum of the Yang–Mills field. Existing experimental data along with the requirement of gauge invariance led to the prediction of weak neutral currents and of a new quantum number for hadrons (charm).

Third, it seems that phenomenological quark models of strong interactions also have their most natural foundation in the framework of gauge theories. Gauge theories give a unique possibility of describing, in the framework of quantum field theory, the phenomenon of asymptotic freedom. These theories also afford hopes of explaining quark confinement, although this question is not quite clear.

Finally, the extension of the gauge principle may lead to the gravitational interaction also being placed in the general scheme of Yang–Mills fields.

So the possibility arises of explaining, on the basis of one principle, all the hierarchy of interactions existing in nature. The term

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unified field theory, discredited some time ago, now acquires a new reality in the framework of gauge field theories. Independently of the question to what extent all these great expectations will be realized, the theory of Yang–Mills fields is today an essential method in theoretical physics and without doubt will have an important place in a future theory of elementary particles.

In the formation of this picture a number of scientists took part. Let us mention some of the key dates.

In 1953 Yang and Mills, for the first time, generalized the principle of gauge invariance of the interaction of electric charges for the case of interacting isospins. In their paper, they introduced a vector field, which later became known as the Yang–Mills field, and within the framework of the classical field theory its dynamics was developed.

In 1967 Faddeev and Popov, and de Witt, constructed a self-consistent scheme for the quantization of massless Yang–Mills fields. In the same year, Weinberg and Salam independently proposed a unified gauge model of weak and electromagnetic interactions, in which the electromagnetic field and the field of the intermediate vector boson were combined into a multiplet of Yang–Mills fields. This model was based on the mechanism of mass generation for vector bosons as a result of a spontaneous symmetry breaking, proposed earlier by Higgs and Kibble.

In 1971 G. 't Hooft showed that the general methods of quantization of massless Yang–Mills fields may be applied, practically without any change, to the case of spontaneously broken symmetry. Thus, the possibility was discovered of contructing a self-consistent quantum theory of massive vector fields, which are necessary for the theory of weak interactions and, in particular, for the Salam–Weinberg model.

By 1972 the construction of the quantum theory of gauge fields in the framework of perturbation theory was largely completed. In papers by A. Slavnov, by J. Taylor, by B. Lee and J. Zinn-Justin, and by G. 't Hooft and M. Veltman, various methods of invariant regularization were developed, the generalized Ward identities were obtained, and a renormalization procedure was constructed in the framework of the perturbation theory. This led to the construction of a finite and unitary scattering matrix for the Yang–Mills field.

Since then on, the theory of gauge fields has developed rapidly, both theoretically and phenomenologically. The history of this development may be illustrated by the rapporteurs' talks presented at international conferences on high-energy physics (B. Lee, 1972, Batavia; J. Illiopoulos, 1974, London; A. Slavnov, 1976, Tbilisi).

From the above short historical survey we shall pass on to the description of the Yang–Mills field itself. For this, we must first recal some notation from the theory of compact Lie groups. More specifically, we shall be interested mainly in the Lie algebras of these groups. Let Ω be a compact semisimple Lie group, that is, a compact group which has no invariant commutative (Abelian) subgroups. The number of independent parameters which characterize an arbitrary element of the group (that is, the dimension) is equal to n. Among the representations of this group and its Lie algebra, there exists the representation of $n \times n$ matrices (adjoint representation). It is generated by the natural action of the group on itself by the similarity transformations

$$h \to \omega h \omega^{-1}; \qquad h, \omega \in \Omega. \tag{1.1}$$

Any matrix $\mathcal{T}$ in the adjoint representation of the Lie algebra can be represented by a linear combination of n generators,

$$\mathcal{T} = T^a \alpha^a. \tag{1.2}$$

For us it is essential that the generators T^a can be normalized by the condition

$$\operatorname{tr}(T^a T^b) = -2\delta^{ab}. \tag{1.3}$$

In this case the structure constants t^{abc} which take part in the condition

$$[T^a, T^b] = t^{abc} T^c, \tag{1.4}$$

are completely antisymmetric. The reader unfamiliar with the theory of Lie groups may keep in mind just these two relationships, which are actually a characterizing property of the compact semisimple Lie group.

A compact semisimple group is called simple if it has no invariant Lie subgroups. A general semisimple group is a product of simple groups. This means that the matrices of the Lie algebra in the adjoint rerpresentation have a block-diagonal form, where each block corresponds to one of the simple factors. The generators of the group can be chosen so that each one has nonzero matrix elements only within one of the blocks. We shall always have in mind exactly such a choice of generators, in correspondence with the structure of the direct product.

The simplest example of such a group is the simple group $SU(2)$. The dimension of this group equals 3, the Lie algebra in the adjoint representation is given by the antisymmetric 3 × 3 matrices; as generators the matrices

$$T^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}; \quad T^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}; \quad T^3 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \tag{1.5}$$

can be chosen; the structure constants t^{abc} in this base coincide with the completely antisymmetric tensor ε^{abc}.

Besides semisimple compact groups, we shall also deal with the commutative (Abelian) group $U(1)$. The elements of this group are complex numbers, with absolute values equal to unity. The Lie algebra of this group is one-dimensional and consists of imaginary numbers or of real antisymmetric 2 × 2 matrices.

The Yang–Mills field can be associated with any compact semisimple Lie group. It is given by the vector field $\mathscr{A}_\mu(x)$, with values in the Lie algebra of this group. It is convenient to consider $\mathscr{A}_\mu(x)$ to be a matrix in the adjoint representation of this algebra. In this case it is defined by its coefficients $A_\mu^a(x)$:

$$\mathscr{A}_\mu(x) = A_\mu^a(x)\, T^a \tag{1.6}$$

with respect to the base of the generators T^a.

In the case of the group $U(1)$ the electromagnetic field $\mathscr{A}_\mu(x) = i\,A_\mu(x)$ is an analogous object.

We shall now pass on to the definition of the gauge group and its action on Yang–Mills fields. In the case of electrodynamics the gauge transformation is actually the well-known gradient transformation

$$\mathscr{A}_\mu(x) \to \mathscr{A}_\mu(x) + i\partial_\mu \lambda(x). \tag{1.7}$$

Let us recall its origin in the framework of the classical field theory. The electromagnetic field interacts with charged fields, which are described by complex functions $\psi(x)$. In the equations of motion the field $\mathscr{A}_\mu(x)$ always appears in the following combination:

$$\nabla_\mu \psi = (\partial_\mu - \mathscr{A}_\mu)\,\psi = (\partial_\mu - iA_\mu)\,\psi. \tag{1.8}$$