

Porous Media

Fluid Transport and Pore Structure

F. A. L. DULLIEN

52.423
D 883
C.2

Porous Media

Fluid Transport and Pore Structure

F. A. L. DULLIEN

*Faculty of Engineering
Department of Chemical Engineering
University of Waterloo
Waterloo, Ontario, Canada*

2635/16

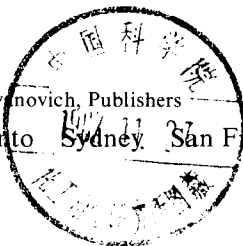


1979

ACADEMIC PRESS

A Subsidiary of Harcourt Brace Jovanovich, Publishers

New York London Toronto Sydney San Francisco



**COPYRIGHT © 1979, BY ACADEMIC PRESS, INC.
ALL RIGHTS RESERVED.**

**NO PART OF THIS PUBLICATION MAY BE REPRODUCED OR
TRANSMITTED IN ANY FORM OR BY ANY MEANS, ELECTRONIC
OR MECHANICAL, INCLUDING PHOTOCOPY, RECORDING, OR ANY
INFORMATION STORAGE AND RETRIEVAL SYSTEM, WITHOUT
PERMISSION IN WRITING FROM THE PUBLISHER.**

ACADEMIC PRESS, INC.
111 Fifth Avenue, New York, New York 10003

United Kingdom Edition published by
ACADEMIC PRESS, INC. (LONDON) LTD.
24/28 Oval Road, London NW1 7DX

Library of Congress Cataloging in Publication Data

Dullien, F. A. L.
Porous media

Includes bibliographies.

1. Transport theory. 2. Porous materials.

I. Title.

QC175.2.D84 532'.053 79-52794

ISBN 0-12-223650-5

PRINTED IN THE UNITED STATES OF AMERICA

79 80 81 82 9 8 7 6 5 4 3 2 1

Porous Media
Fluid Transport and Pore Structure

*To my friend and colleague
Howard Brenner
for his encouragement and inspiration*

Preface

The unique property of a porous medium, the one that distinguishes it from other solid bodies on the one hand, and from simple conduits on the other, is its complicated pore structure. The vast majority of porous media contain an interconnected three-dimensional network of capillary channels of nonuniform sizes and shapes, commonly referred to as “pores.”

Fluid flow, diffusion, and electrical conduction in porous media take place within extremely complicated microscopic boundaries that make any rigorous solution of the equations of change in the capillary network practically impossible. This is one reason why some of the brilliant and successful practitioners in the field of “flow through porous media” have tried, as much as possible, to stick with the continuum approach in which no attention is paid to pores or pore structure. The other reason is that the continuum approach is often adequate for the phenomenological description of macroscopic transport processes in porous media. The continuum approach fails to provide a clue to help explain any of a multitude of observations that depend on the properties of the microscopic channels and the behavior of the fluids on the microscopic scale. The desire of scientists and engineers to be able to understand and then explain their observations has always been a powerful driving force for progress. Therefore there have been numerous attempts over the past fifty years or so to explain the flow phenomena in terms of the microscopic structure as accurately as possible. The results have

seldom been entirely satisfactory, but with every step a further penetration into an immensely complicated territory has been achieved. There is a great deal of information available in the technical literature on the role played by pore structure in determining transport phenomena in pore spaces.

This book has been written with the primary purpose of presenting in an organized manner the most pertinent information available on the role of pore structure and then putting it to use in the interpretation of experimental data and the results of model calculations.

Pore structure is inseparable from the convective, diffusive, and interfacial effects that take place in the pores; these effects are all interrelated so that there is little point in trying to evaluate their relative merits with the aim of deciding which of them is most important.

Existing books on "flow through porous media" have been written with an emphasis on the fluid mechanical aspects. Interfacial effects, such as interfacial tension and wettability, have been under intensive and productive investigation for quite some time. In this book the author has made an attempt to show that there are benefits to be gained by trying to think about the phenomena in porous media in terms of interactions among the three main factors, i.e., transport phenomena, interfacial effects, and pore structure. The book contains many examples of applications of this concept, and it is the hope of the author that many readers will find this approach useful as well as an inspiration and motivation to do more fundamental research on the role played by pore structure.

The author wishes to acknowledge an operating grant from the National Research Council of Canada. A number of persons have contributed their invaluable time and energy to the preparation of this book, and without their help it could not have been written. Therefore the author takes great pleasure in acknowledging the perfect typing and dedication of Nancy Stade; the outstanding professional artwork of Rinze Koopmans; the helpful comments offered by Professor D. R. Spink who read the Preface and the Introduction; the prompt and competent assistance of Baiba Gomes with the Bibliography; the innumerable discussions conducted with P. K. Shankar and I. Chatzis; and last but not least, the author is deeply indebted to his wife, Ruth, for putting up with him while he worked on the book and for the loving help she gave with the organization of the Bibliography.

List of Symbols

Latin Letters

a	speed of sound; edge length of cube; cross-sectional area; Turner-structure parameter; narrow half-width; medium dispersivity; activity
a'	dynamic medium dispersivity
a''	geometric medium dispersivity
a_{ijkl}	component of medium dispersivity tensor
a_j	$\equiv l_j/D_j$
A	area of plane; cross-sectional area of pore or sample; area of circle; constant parameter
A_A	fractional area of phase A in the plane of section
b	constant parameter; number of branches; Turner-structure parameter
B	constant parameter; hydraulic conductance of channel
c	adjustable parameter; mass concentration
C	constant parameter; connectivity; factor defined by Eq. (3.2.12)
C'	"shift factor" [Eq. (4.7.4)]
C_A	molar concentration of A
C_p	specific heat at constant pressure
C_v	specific heat at constant volume
D	pore (capillary) diameter; sphere diameter
Da	$\equiv \Delta\mathcal{P}/\alpha\mu\nu L$ (Darcy number)
D_b	bulge diameter
D'_{ct}	coefficient of hydrodynamic dispersion in a capillary
D_e	pore entry diameter

\mathcal{D}_{eff}	effective diffusion coefficient
D_{f}	fiber diameter
D_{H}	$\equiv 4r_{\text{H}}$ = hydraulic diameter
D_{ij}	component of dispersivity tensor
D_1	diameter of large capillary
D_{n}	neck diameter
D_{p}	particle diameter
\bar{D}_{p}	effective average particle diameter
$\bar{D}_{\text{p}2}$	surface average sphere diameter
\bar{D}_{pm}	average particle diameter by weight
\bar{D}_{pr}	$\equiv \int_0^\infty D_{\text{p}} D'_{\text{p}} N(D_{\text{p}}) dD_{\text{p}} / \int_0^\infty D'_{\text{p}} N(D_{\text{p}}) dD_{\text{p}}$
$D_{\text{p}50}$	median particle diameter by weight
D_{s}	diameter of narrow capillary; equivalent surface diameter [Eq. (3.2.2)]
D_{T}	container diameter
D_{v}	equivalent volume diameter [Eq. (4.2.12)]; volume average diameter of periodically constricted capillary
D_{v}^*	$\equiv D_{\text{v}}/\lambda$
D_2	length of longest intercept with three-dimensional object
D_3	equivalent spherical void diameter [Eq. (3.2.51)]
D'	coefficient of hydrodynamic dispersion
D'_l	dispersion coefficient in the direction of flow
D'_T	dispersion coefficient in the transverse direction
\bar{D}'	hydrodynamic dispersion tensor
\mathcal{D}_{AB}	$\equiv \mathcal{D}$ = molecular diffusion coefficient
\mathcal{D}_{KA}	Knudsen diffusion coefficient of A
\mathcal{D}_{m}	moisture diffusivity
\mathcal{D}	$\equiv (1/\bar{D}_{\text{c}}) - (1/\bar{D})$ = "structural difficulty index"
E	modulus of elasticity; effective viscosity ratio [Eq. (7.3.110)]
E_{D}	efficiency of conversion of work to creation of surface
f	separation in reverse osmosis [Eq. (5.7.10)]
f_{ϕ}	friction factor of non-Newtonian fluid [Eq. (4.7.9)]
$f(A, l, \alpha)$	probability density of A , l , and α
f_{d}	$\equiv N_{\text{d}} f_2(\phi)$ [Eqs. (4.3.31) and (4.3.32)]
f_{p}	friction factor in porous media
f_r	relative frequency of pores (capillaries) of type r
f_v	friction factor [Eq. (4.4.3)]
f_w	fractional flow function [Eq. (6.4.43)]
$f(\phi)$	porosity function
$f(k)$	probability density of k
$f(R)$	probability density distribution of pore volume by pore radius
F	formation or resistivity factor; bulk flow portion of molar flux
F_{c}	effective formation factor at partial saturation
F_w	$\equiv Q_{\text{w}}/Q_{\text{T}} $ = fractional flow
$F(D_2)$	probability density function of number of objects of diameter D_2
$F(t)$	effect of surface forces of pore walls on thermodynamic potential of adsorbed multilayer
$F(\tau_{\phi})$	$\equiv 2v/\phi D_{\text{H}}$ [Eq. (4.7.2)]
$F(\tau_w)$	$\equiv 2v/D$ [Eq. (4.7.1)]

g	acceleration due to gravity
$g(r^*)$	probability density of r^*
$g(\delta; \theta, \phi, \Omega, D_2)$	shape function, i.e., relationship between distance z , measured from the plane section characterized by δ_{\max} , and δ for various orientations (θ, ϕ, Ω) of an object of diameter D_2
$\bar{g}(\delta)$	shape function averaged over all orientations of object
G	Gibbs free energy; genus; molar flow rate
G_m	mass flow rate
h	height (measured upward); pressure head; half-width of channel
h_c	$\equiv \psi \equiv -(P_w/\rho_w g) =$ capillary pressure head or moisture tension or suction head
H	heterogeneity factor
\bar{H}	mean tangent height or mean caliper diameter
I	$\equiv F_e/F =$ resistivity index
$J(S_w)$	Leverett J function
\tilde{j}_i	mass flux of i with respect to volume average velocity
k	specific permeability or "permeability"
k'	Kozeny constant
\bar{k}	permeability tensor
k_{CK}	permeability defined by Eq. (4.2.9)
k_D	permeability used in gas permeametry [Eq. (4.6.3)]
k'_D	$\equiv \mathcal{D}_{\text{eff}} =$ diffusive permeability coefficient
k_{ei}	effective permeability of i
k_{ri}	relative permeability of i
k_H	hydraulic conductivity
k_M	$\equiv \mathcal{D}K_H =$ permeability coefficient used with membranes
k_{ij}	component of permeability tensor
k'_n, k''_n	directional permeabilities
k_0	shape factor; permeability of infinitely dilute bed
k_1, k_2, k_3	"principal" permeabilities
k_1, k_2, k_3, k_4, k_5	constant parameters
\bar{k}	"pure water permeability constant" [Eq. (5.7.15)]
k	$\equiv \mathcal{D}/l =$ mass transfer coefficient
K	numerical constant; K factor [Eq. (7.3.10)]
K_H	Henry's law constant
l	length; length of capillary; length of a straight line; length of a step in random walk; directional cosine of channel axis
l_{ij}	cumulative length of segments of diameter D_j in cell i
l_1	length of a large capillary
l_s	length of a narrow capillary
l^*	$\equiv l/L =$ dimensionless ratio
L	length of sample; chord or intercept length; greatest pore length in sample
L_A	length of chords in a plane per unit test area
L_e	average effective path length of fluid
L_m	mixing length
L'_m	$\equiv DL_m/2R^2u_m =$ dimensionless mixing length
L_L	lineal fraction of chords contained within two-dimensional features in the section plane
L_V	length of chords in space per unit test volume

$L(R)$	probability density of total length of pores of radius R in the sample
\bar{L}_3	$\equiv 4V/S$ = mean intercept length
$(L_3)_i$	chord length from a random penetration of a three-dimensional object by a straight test line
m	mass; cementation exponent; mobility ratio $= k_{2r}\mu_1/k_{1r}\mu_2$
M	molecular weight; Mach number
$M(\phi)$	“effective viscosity” [Eq. (4.5.3)]
n	number of parallel lines crossing unit area; number of moles; number of nodes; total number of pores in model; mass flux w.r.t. stationary coordinates; adjustable parameter; number of steps taken; number of particles per unit volume
n_i	number of repeating capillary units in cell i
\mathbf{n}	outward unit normal vector; unit vector
$n(D)$	photomicrographically determined probability density of pore sizes expressed as equivalent sphere diameters
$n(D_p)$	probability density of particle diameters
$n'(D)$	photomicrographically determined probability density of pore sizes expressed as equivalent cylinder diameters
$n(L)$	probability density of chord length L per unit area of test section
$n(\delta)$	probability density of two-dimensional features of size δ per unit area of plane section
N	Avogadro number; index denoting mixing cell; number of intervals; rate of transport [Eq. (5.2.14)]
N_A	number of two-dimensional features per unit area of section plane; molar diffusion flux of A
N_B	molar diffusion flux of B
N_{Ca}	$\equiv \mu_w v_w / \phi \sigma$ = “capillary number”
N_{AS}	diffusion flux of A based on cross section of sample
N_d	“deflection number”
N_e	effective pore number
N_j	number of capillaries of size j
N_p	number of pores per unit section of filter
N_L	number of interceptions of features per unit length of test line in the plane of section
N_{Re}	$\equiv Re_p / (1 - \phi)$ = Reynolds number
N'_{Re}	Reynolds number for non-Newtonian fluids [Eq. (4.7.11)]
N_s	number of spheres per unit volume of bed
N_{sAA}	“tertiary oil recovery number” [Eq. (6.4.64)]
N_T	total molar flux
N_v	number of three-dimensional objects per unit test volume; volume flux relative to membrane [Eq. (5.2.1)]
$N(D)$	$\equiv N(D_p)$ = probability density of sphere or particle diameters
$N(D_k)$	photomicrographically determined fraction of pores with diameters $D \geq D_k$
p	probability; probability density of tube radius distribution
\bar{p}	random variable [Eq. (7.3.58)]
p_k	fraction of capillaries with diameters $D \geq D_k$ in network model, i.e., probability that a randomly chosen capillary will have diameter greater than D_k
$p(b)$	probability of bond being open

$p(u, t/u_0, t_0)$	probability that a molecule has a velocity u at time t , if it had a velocity u_0 at time t_0
P	hydrostatic pressure; fraction of pore space which is accessible through pores with diameters less than or equal to a given value
Pe	$\equiv \bar{v}_p \bar{D}_p / D$ = Peclet number
Pe'	$\equiv \bar{v}_p \bar{D}_p / D'$ = dynamic Peclet number
Pe_{ct}	Peclet number in a capillary [Eqs. (7.2.6) and (7.2.7)]
Pe_{rT}	Peclet number [Eq. (7.3.55)]
P_c	capillary pressure
P_{cb}	breakthrough capillary pressure or bubbling pressure
P'_{cb}	$\equiv P_{cb}/4\sigma \cos \theta$ = reduced breakthrough capillary
P_m	arithmetic mean pressure
P_0	vapor pressure over plane surface; hydrostatic pressure at the datum level
P_p	$\equiv P_a/P_T$ = fractional point count
P_c^*	$\equiv l/r^*$ = dimensionless capillary pressure
P_r''	vapor pressure over hemispherical meniscus of radius of curvature r
\mathcal{P}^*	$\equiv \mathcal{P}/\rho v^2$ or $\equiv \mathcal{P}/\rho v_0^2$ = dimensionless group
P'	absolute value of the macroscopic pressure gradient
P_A	number of points (intersections) generated per unit area of section plane with lines in space
P_L	number of points (intersections) generated per unit length of test line with traces of surfaces in the section plane
P_T	total number of grid points
P_V	number of points of intersection between surfaces and lines per unit test volume
P_α	number of grid points falling over phase α
\mathcal{P}	piezometric pressure
q	probability that a randomly chosen pore will not exceed a given size, i.e., number fraction of pores smaller than a given size; volume flow in a tube; $\equiv l/t$ (velocity of particle along a step)
q_i	i th size distribution parameter
q_k	fraction of "pseudo-dead-end capillaries" in network model when D_k is the diameter of the smallest penetrated capillary
$q(D_c, D'_c)$	probability density of volume flow rates [Eq. (4.3.15)]
Q	volume flow rate or discharge
r	radial coordinate or distance; radius of capillary or pore; number of dimensions used in averaging particle diameters; rate of production; ratio of true to apparent surface area; dimensionless distance between centers of spheres
r_m	mean radius of curvature
r_H	hydraulic radius
r^*	$\equiv r/R$ = dimensionless ratio
r_1, r_2	principal radii of curvature of surface, inner and outer radius of rotation, respectively; smallest and largest radius, respectively, or periodically constricted tube
rD_p	interparticle distance
$r(c, t)$	rate of production per unit volume of solution
R	resistivity; greatest pore radius in sample; radius of capillary or grain; electrical resistance of network; universal gas constant

R	force resisting motion [Eq. (4.9.29)]
R_p	cumulative oil recovery in pore volume units
Re	$\equiv \rho v \sqrt{k}/\mu$ = Reynold's number
Re''	$\equiv \beta \rho v / \alpha \mu$ = Reynolds number
Re_c	$\equiv D_t v_p \rho / \mu$ = Reynolds number
Re_f	$\equiv D_t v \rho / \mu$ = "superficial" Reynolds number for fibrous beds
Re_k	$\equiv v_{DF} \sqrt{k}/v$ = Reynolds number
Re_p	$\equiv D_p v \rho / \mu$ = "superficial" or "particle" Reynolds number
Re_v	$\equiv D_v v_m / v$ = Reynolds number
Re_λ	$\equiv \lambda v_0 / v$ = Reynolds number
Re_{s0}	$\equiv D_{p50} v_{DF} / v$ = Reynolds number
R_n	radius defined by Eq. (3.2.10)
R_0	resistivity of saturated sample
R_w	resistivity of electrolyte solution
R_1	constriction radius
R_2	bulge radius
R_1, R_2	radii of curvature of surface
R_1^*	$\equiv R_1 / \lambda$
R_2^*	$\equiv R_2 / \lambda$
s	length measured along axis of channel
s_j	photomicrographically determined volume fraction of capillaries of diameter D_j
S	spreading coefficient; saturation; surface; specific surface per unit mass of sample; standard deviation
Sc	$\equiv v/D$ = Schmidt number
S_i	fraction of pore volume occupied by capillaries in cell i ; saturation of fluid phase i
S_k	saturation with respect to a nonwetting phase, predicted by network analysis, when D_k is the diameter of the smallest penetrated capillary
S_0	specific surface per unit volume of solids
S_p	particle surface area; pore surface area
S_t	specific surface area based on Eq. (3.2.20)
S_v	surface area contained in unit test volume or surface area per unit bulk volume
S_{BET}	specific surface area based on BET method
S'	"constrictedness factor" [Eq. (4.9.25)]
$S(D_k)$	photomicrographically determined volume fraction of pores with diameters $D \geq D_k$ (cumulative pore size distribution)
t	time; multilayer thickness; duration of a step in random walk
T	absolute temperature; "tortuosity"; time it takes to collect one pore volume of effluent; length of a time interval
T', T''	correction factors
\mathcal{T}_{ij} and \mathcal{T}'_{ij}	"tortuosity tensors"
u_r	r velocity component of microscopic point velocity
u_z	z velocity component of microscopic point velocity
\mathbf{u}	microscopic point velocity
\mathbf{u}^*	$\equiv u/v$ = dimensionless velocity
u_r^*	$\equiv u_r/v_0$, or $\equiv u_r/v$ = dimensionless velocity component
u_z^*	$\equiv u_z/v_0$ = dimensionless velocity component
U	velocity component of a marked particle relative to coordinate axes moving with the mean velocity [Eq. (7.3.75)]
\mathbf{v}	$\equiv (\delta Q / \delta A) \mathbf{n}$ = filter velocity; specific discharge or superficial velocity

v_m	$\equiv 4Q/\pi D_v^2$ = volumetric mean velocity
v_0	mean velocity at the entrance of a periodically constricted tube
v_p	“pore velocity” [Eqs. (4.3.14) or (4.3.24)]
v_{pn}	pore velocity in a pore parallel to macroscopic pressure gradient
v_{DF}	$\equiv v/\phi$ = average pore velocity according to Dupuit–Forchheimer assumption
\bar{v}	mean molecular speed
v_D	random part of velocity due to dispersion process
\bar{v}_p	average pore velocity; interstitial velocity or “seepage” velocity
\mathbf{v}^*	mass-averaged velocity [Eq. (4.9.29)]
\mathbf{v}^v	$\equiv \mathbf{v}_{DF}$ = volume average velocity of mixture with respect to the solid matrix
V	volume; volume of vapor adsorbed; velocity component of marked particle in the transverse direction
V_a	adsorbed volume of adsorbate as ATP gas per gram of adsorbent
V_f	fluid volume
V_m	STP volume of gas adsorbed at complete monolayer point
V_0	STP number (22.414 liters)
V_p	particle volume; pore volume
V_s	volume of absorbate at saturation vapor pressure
V_v	fractional volume of a phase
V_{ij}	$\equiv V(D_e, D) \times 100$ = percent pore volume contributed by pores of diameter D_j such that they are accessible through pores of diameter D_i
V_B	bulk volume
V_T	total pore volume
$V(D)$	volume of a capillary (pore) characterized by diameter D
$V(D_e, D)$	probability density of pore volume by pore diameters D_e and D (bivariate pore size distribution: BPSD)
V'	molar volume of liquid
V''	molar volume of vapor
V_i	partial molar volume of i
x	x coordinate; mole fraction; dimensionless ratios $\equiv P''/P$; or $\equiv D_1/D_s$
x'	moving x coordinate
X	distance measured from center of a sphere; electrical tortuosity factor; x coordinate of displacement of a marked particle
y	y coordinate; dimensionless ratios $\equiv v_l/v_s$, or $\equiv l_l/l_s$; mole fraction in gas phase
y_k	number-based saturation with respect to nonwetting phase capillaries in network such that $D > D_k$, when D_k is the smallest penetrated capillary diameter
Y	y coordinate of displacement of marked particle
Y_k	number-based saturation of network with respect to wetting phase when D_k is smallest penetrated capillary diameter
z	z coordinate; distance measured vertically upward; $\equiv g(\delta; \theta, \phi, \Omega, D_2)$
Z	coordination number; z coordinate of displacement of marked particle

Greek Letters

α	angle; reciprocal of “permeability”; $\equiv 1 + N_B/N_A$
$\alpha(D)$	probability density function of pore volume by pore diameters according to the bundle of capillary tubes model

$\alpha(D_e)$	probability density function of pore volume by pore entry diameters
α	orientation vector
$\alpha(t; D)$	probability density of number of chords obtained by intercepting spheres of diameter D per unit length of test line
β	layer spacing parameter; angle; "inertia parameter"; parameter of second-order memory fluid; constant parameter; inverse compaction factor [Eq. (5.1.4)]
βD_p	layer spacing
$\beta(\delta; D)$	probability density of number of circle diameters δ per unit area of plane of section obtained by intersecting spheres of diameter D
$\beta(\delta; D_2)$	probability density of number of features of size δ per unit area of sectioning plane, obtained by intersecting objects of diameter D_2
γ	ratio of specific heats; constant parameter
Δ	difference of two quantities
δ	Dirac delta function; circle diameter; longest intercept with two-dimensional feature in the y direction
$\bar{\delta}$	$\equiv \bar{R}/l$ = ratio of average channel radius to average channel length
ε_i	$\equiv (v_{DF})_i/v_{DF}$ [Eq. (7.3.107)]
θ	contact angle; polar angle; dip angle
κ	$\equiv R_0/R_i$ = dimensionless ratio
λ	mean-free path; "wave length" of periodically constricted tube; constant parameter; "pore size distribution index" [Eq. (6.3.13)]
Λ	characteristic length
μ	chemical potential; viscosity
μ_{jk}	number of capillaries of diameter D_j penetrated in the network by the nonwetting phase when D_k is the diameter of the smallest penetrated capillary
μ_ϕ	Darcy viscosity
ν	kinematic viscosity
ν_l	number of large capillaries
ν_p	number of pore volumes
ν_s	number of narrow capillaries
ν_{jk}	number of "pseudo dead-end capillaries" of diameter D_j when D_k is the diameter of the smallest penetrated capillaries
ζ	constant parameter
$\tilde{\zeta}_i$	statistical weight; local coordinates
π	$\equiv \sigma/\mu v$ = dimensionless group; osmotic pressure
ρ	radius; density; electrical resistance of individual resistor
γ	angle; constant parameter [Eq. (4.4.15)]; $\equiv b/a$ = dimensionless ratio
$\gamma(D_e)$	probability density function of pore entry diameters
Γ	shear rate
$(\Sigma m)_r$	number of pores connected to a pore of type r
σ	surface tension; standard deviation; length along streamline; retention [Eq. (5.1.1)]
σ^0	area of a molecular site
τ	$\equiv tD/R^2$ = dimensionless time; shear stress; tortuosity vector; $\equiv V/Q$ = residence time
τ_w	wall shear stress in capillary
τ_ϕ	wall shear stress in packed bed

ϕ	porosity; angle: azimuthal angle; half-angle of cone; piezometric head; volume fraction
ϕ_c	$\equiv z + \frac{P_w}{\rho_w g} \equiv z - h_c =$ capillary head
ϕ_e	effective porosity [Eq. (4.2.23)]
ϕ_i	$\equiv \phi S_i$
ϕ_o	overall porosity corresponding to a given D_p/D_T
ϕ_s	“sphericity,” i.e., the ratio of surface area of the hypothetical sphere of the same volume as the particle to the actual surface area of the particle
ϕ'	local porosity
$\bar{\phi}$	local mean porosity relative to a reference point or plane
ϕ'_A	local area porosity
ψ	capillary pressure head; arbitrary point function; “viscosity level parameter”
ψ_i	i th particle shape parameter
Ω	angle of rotation about an axis
ω	angular velocity
η	apparent viscosity of non-Newtonian fluid in “Poiseuille flow” [Eq. (4.7.8)]; collection efficiency of an individual fiber [Eq. (5.1.9)]
η_ϕ	“Darcy viscosity” of non-Newtonian fluid [Eq. (4.7.9)]

Subscripts

a	apparent; air; adsorbate; adsorption branch
A	advancing
b	breakthrough; bulge
B	bulk
BET	refers to Brunauer, Emmet, and Teller
c	capillary
ct	capillary tube
c, cr	critical
d	desorption branch
e	entry; equilibrium
eff	effective
f	front
g	gas
h	hysteresis function (= functional)
i	irreducible; inner; initial
i	refers to cell i ; refers to capillary i
j	refers to capillary j
k	refers to smallest penetrated capillary
l	liquid
L	longitudinal
m	mean value; monolayer; molar
max	maximum
min	minimum
mt	multilayer
n	neck; narrow, smallest pore or capillary

nwr	residual nonwetting phase
nw	nonwetting
o	saturated; initial; outer
p	pore
P	constant pressure
r	residual
r	r type of pore
R	receding
s	solid; saturation; solvent
s.s.	steady state
t	refers to t curve
T	transverse; total; constant temperature
v	vapor
w	wetting; water; wide electrolyte solution
w	electrolyte solution
wi	irreducible water
x	x component

Superscripts and Overlines

'	reduced; on the convex side of interface; moving coordinate; distinguished mark
*	refers to tracer
"	on the concave side of the interface
+	positive direction from the xy plane
^	reduced macroscopic variable
—	negative direction from xy plane
=	tensor
--	ensemble average
i	"in"
o	"out"

Special Symbols

∇	"del" or "nabla" operator
∇^*	$D\nabla$
∇^{*2}	$D^2\nabla^2$
δ	denotes a small quantity