

R. PAUL

MEASUREMENT OF  
TRANSISTOR PARAMETERS

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TRANSISTOR PARAMETERS**

**Translated by Scripta Technica Ltd**

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## ***PREFACE TO THE ENGLISH EDITION***

This translation of Prof. R. Paul's monograph from the original German edition should be of value both as a reference book to practising electrical and electronic engineers and as a textbook for final year undergraduates and postgraduates in physics and engineering.

It is not the intention of the author to give a complete description of the elementary basis of the analysis of the performance of the transistor as a circuit element in an academic way, but rather to extend considerably the coverage of the many elementary texts available on this subject. In particular, as suggested by the title, the main concern of the book is to deal with the principles of measurement of the various transistor leakage, saturation, switching, noise, thermal and quadripole parameters. The description of the methods of measurement is amply illustrated by numerous diagrams showing specific experimental results. The frequency range covered extends from d.c. frequencies to Gc/s frequencies.

As editor, I found this work illuminating in several respects. I think others will find this to be the case also.

D. Llanwyn Jones  
January 1968

## INDEX OF THE MOST IMPORTANT SYMBOLS

### Print

Magnitudes of physical quantities: Italics ( $V, I$ ).

Vectors: Italics in bold type ( $\mathbf{V}, \mathbf{I}$ );  $V^*$  is the conjugate complex number to  $V$  (etc.).

### Voltage, current and power symbols

These symbols usually have subscripts. The instantaneous values, d.c. and a.c. parts are marked as follows.

1. Small (lower case) letters are used for instantaneous values of quantities varying in time ( $v, i$ ) with small letter subscripts for periodic quantities (pure a.c. constituents) ( $v_e, i_e$ ) and with capital (upper case) letter subscripts for non-periodic quantities (pulse quantities) ( $v_{EB}, i_E$ ).

2. Capital (upper case) letters are used for time-constant quantities ( $V, I$ ) with small letter subscripts for the mean values of periodically varying quantities ( $V_e, I_e$ ) and capital letter subscripts for d.c. quantities ( $V_{EB}, I_E$ ).

### Impedances, admittances, capacitances, inductances

Small letters are used for the equivalent circuit quantities corresponding to the transistor itself ( $r_b, r_d, r_{E'E}, c_e$ ); capital letter subscripts when the quantity corresponds to d.c. operation, small letter subscripts when the quantity corresponds to a.c. operation. Conductance  $S$  and several noise parameters are exceptions.

Capital letters are used for the elements lying outside the terminals of the transistor ( $R_{BE}, R_L, R_E$ ).

Symbols generally used for admittances and impedances are  $y = g + jb = g + j\omega c$ ,  $z = r + jx$ ,  $Y = G + jB$ ,  $Z = R + jX$ .

Inner transistor quantities are additionally marked by ( $h'_{21}, y'_{11}$ ).

## List of the most important mathematical symbols

These are the most important symbols used in mathematical expressions. If a symbol is used in several meanings, all its meanings are listed separately.

$A$	cross-section, area
$A_N(A_I)$	d.c. current amplification in grounded base circuit in forward (reverse) operation
$A_{N1}(A_{I1})$	d.c. current transport factor in normal (reverse) operation
$a_v(a_r)$	forward (reverse) voltage transfer
$B, b(B', b')$	as subscripts: base terminal (inner) grounded base circuit (in quadripole parameter symbols)
$B$	collector-base currents ratio
$B_G$	generator susceptance
$B_L$	load susceptance
$B_N$	susceptance of the standard
$B_N(B_I)$	d.c. current amplification in common emitter circuit in forward (reverse) operation
$B_{NO}$	d.c. current amplification $B_N$ at the limit of overdrive
$C, c(C', c')$	as subscripts: collector terminal (inner)
$C_{th}$	thermal capacity
$c$	inner capacitance of the transistor (always with a subscript)
$c$	velocity of light
$c_o, c', c_k$	temperature coefficient of a saturation or leakage current
$c_e, c_b, c_o$	emitter, base, collector capacitance
$c_{ed}, c_{od}$	emitter, collector diffusion capacitance
$c_{eb}, c_{eo}, c_{ob}$	leader capacitances between emitter, base and collector terminals
$c_{es}, c_{os}$	emitter, collector junction capacitances
$D_T$	temperature slope reciprocal
$E, e(E', e')$	as subscripts: emitter terminal (inner), grounded emitter circuit (in quadripole parameter symbols)
$e$	electron charge
$F$	noise factor
$F_e$	additional noise factor
$F_{tot}$	total noise factor
$F_{oo}$	noise factor for white noise spectrum
$f$	frequency
$\Delta f$	bandwidth
$\Delta f_e$	equivalent bandwidth
$f_b$	upper cut-off frequency of $ h_{21b} $
$f_o$	normalising frequency ( $g_o = \omega_o c_o$ )
$f_e$	normalising frequency ( $g_e = \omega_e c_e$ )
$f_{Nb}$	(lower) cut-off frequency of $ h_{21b} $
$f_{Ne}$	cut-off frequency of $ h_{21e} $
$f'_{Nb}, f'_{Ne}$	inner transistor cut-off frequencies $f_{Nb}, f_{Ne}$
$f_o$	centre frequency
$f_o$	normalising frequency
$f_q$	normalising frequency $g_o = \omega_q(c_o + c_{ob})$
$f_{v21}$	$y_{21}$ cut-off frequency

$f_T$	transit frequency
$f_s$	normalising frequency ( $\omega_s r_b c_e = 1 + r_b g_e$ )
$f_1$	cut-off frequency at which $ h_{21e}  = 1$
$f_{\max}$	maximum oscillation frequency
$G$	conductance (general symbol)
$G$	power amplification (general symbol)
$G_G$	generator conductance
$G_L$	load conductance
$G_N$	conductance of the standard
$G_n$	equivalent noise conductance ( $G_n = g_n$ )
$G_t$	transfer gain
$G_a$	available power amplification
$G_{a \max}$	maximum available power amplification
$g$	inner conductance of the transistor (always with a subscript)
$g_b, g_e, g_c$	base, emitter, collector conductance ( $g_b \neq \frac{1}{r_b}$ )
$g_d$	diffusion conductance
$g_{ed}, g_{cd}$	emitter, collector diffusion conductance
$g_{er}$	recombination conductance in the emitter zone
$g_{cr}$	recombination conductance in the collector zone (residual collector conductance)
$g_n$	equivalent noise conductance
$h_{ik}$	$h$ parameter of the transistor (when referred to the transistor circuit configuration, provided with subscript: $b, e, c$ )
$h'_{ik}$	$h$ parameter of the inner transistor
$I$	current
$I$	as subscript: reverse operation
$I_B, I_E, I_C$	base, emitter, collector d.c. current
$I_{BO}, I_{CO}$	base, collector d.c. current at the limit of overdrive
$I_{BS}$	base excess current
$I_R$	leakage (residual) current, general symbol
$I_{EBO}$	emitter leakage current with open-circuited collector
$I_{CBO}$	collector leakage current with open-circuited emitter
$I_{CEO}$	collector leakage current with open-circuited base
$I_{CBK}$	collector leakage current with shorted emitter and base
$I_{CBR}$	collector leakage current with emitter and base joined via a resistance
$I_e, I_c, I_b$	emitter, collector, base a.c. current
$i$	as subscript: intrinsic
$i$	instantaneous value of the current (subscripts as with $I$ )
$i_n$	noise current — general symbol
$k$	abbreviation
$k$	Boltzmann constant
$k$	switching-off factor
$k$	transformation factor
$k_T (k_{T \text{ eff}})$	duty ratio (effective)
$l_d$	inner inductance of the transistor
$l_r$	inner inductance of the transistor, with the Early effect taken into account

$I_b, I_c, I_e$	lead inductance of the electrode shown by the subscript
$M$	multiplication factor (general symbol)
$M_b, M_g, M_x$	mutual inductance
$m$	matching factor
$m$	drift factor
$m$	abbreviation
$m$	overdrive factor
$N, n$	as subscripts: forward (normal) operation
$N_A, N_D$	donor density
$P$	power
$P$	power loss, dissipated power
$P_o$	available generator power
$P_n$	noise power (general symbol)
$P_{ns}$	additional noise power introduced by the noise quadripole
$Q$	amount of heat
$Q_B$	minority carrier charge stored in the base zone
$Q_{BS}$	excess charge
$q$	elementary charge of holes ( $q =  e $ )
$R$	resistance (general symbol)
$R_C, R_E, R_B$	external collector, emitter, base resistance
$R_G$	generator resistance
$R_L$	load resistance
$R_N$	standard resistance
$R_n$	equivalent noise resistance
$R_{th}$	thermal resistance
$R_{thi}, R_{the}$	thermal resistance: inner, external
$R_{thk}$	thermal contact resistance
$R_{thi\ off}$	effective inner thermal resistance
$r$	inner resistances of the transistor (always with subscripts)
$r$	abbreviation
$r$	reflection factor
$r_d$	diffusion resistance
$r_b, r_B$	base resistance
$r'_E, r'_e$	emitter path resistance
$r'_C, r'_c$	collector path resistance
$r_n$	equivalent noise resistance ( $r_n \neq R_n$ )
$r_r$	inner longitudinal resistance of the transistor, taking into account the Early effect
$S_{ab}, S_{ae}$	total conductance in the base and emitter configuration
$S_i$	inner conductance
$S_{io}$	inner conductance at low frequencies
$S_e$	external conductance
$T$	absolute temperature (subscript denotes the actual point as for $\vartheta$ )
$T$	duration of one period
$t$	time
$t_a$	off-time
$t_e$	on-time
$t_d$	delay time



$t_r$	rise time
$t_s$	storage time
$t_f$	fall time
$t_p$	pulse duration
$v$	instantaneous value of the voltage (general symbol)
$v_r$	noise voltage (general symbol)
$V$	voltage (general symbol)
$V_C, V_B, V_E$	original voltage source
$BV_{CEO}$	collector-emitter breakdown voltage with open-circuited base
$BV_{CBO}$	collector-base breakdown voltage with open-circuited emitter
$BV_{CER}$	collector-emitter breakdown voltage with an arbitrary connection between emitter and base
$V_D$	diffusion voltage
$V_{EB}, (V_{CB})$	voltage between the (external) emitter and base terminals (collector and base terminals)
$V_{B'B'F}$	emitter floating voltage
$V_{C'B'F}$	collector floating voltage
$V_{CES}, V_{CER}$	collector-emitter saturation voltage, collector residual voltage
$V_T$	thermal voltage $kT/q$
$V_{pt}$	punch-through voltage
$V_g$	generator voltage
$W$	width of the base
$Y$	external admittance (general symbol)
$Y_e (Y_a)$	input (output) admittance
$Y_G$	generator admittance
$Y_L$	load admittance
$Y_{cor}$	correlation admittance
$y$	inner admittance of the transistor (always with a subscript)
$y_{ik}$	$y$ parameter of the transistor (when pertaining to the transistor circuit configuration, with subscript $b, c, e$ )
$y'_{ik}$	$y$ parameter of the inner transistor
$Z$	external impedance (general symbol)
$Z$	wave impedance, characteristic impedance
$Z_L$	terminating impedance
$Z_{ik}$	$z$ parameter of the transistor (when pertaining to the transistor circuit configuration, with subscript $b, e, c$ )
$\alpha_b (\alpha'_b)$	short-circuit current amplification in the common base circuit in forward operation (inner transistor)
$\alpha_e (\alpha_{oe})$	short-circuit current amplification in the common emitter configuration in forward operation (at low frequencies)
$\alpha_o$	short-circuit current amplification in the common base configuration at low frequencies
$\vartheta$	temperature in degrees Celsius
$\Delta\vartheta$	junction temperature
$\vartheta_a$	ambient temperature
$\vartheta_{j \max}$	maximum permitted junction temperature (limit value)
$\vartheta_o$	case temperature
$\vartheta_o$	reference temperature

$\lambda$	wavelength
$\lambda$	attenuation factor
$\tau_B, \tau_E, \tau_C$	base, emitter, collector time constant (switching time constants, time factors)
$\tau_{CO}$	collector time constant at the limit of overdrive
$\tau_c'$	collector time constant
$\tau_F$	fall time constant
$\tau_L$	delay time constant
$\tau_R$	rise time constant
$\tau_S$	saturation time constant
$\tau_{10}$	general time constant
$\phi$	phase angle
$\omega$	angular frequency (pulsatance)

## **INTRODUCTION**

The great advantages of transistors, as compared with electron tubes, have gained for them a steadily widening field of application, particularly in switching circuits. This makes it even more imperative that not only the manufacturer, but also the user, should be able to measure the electrical properties of these devices. This is necessary not only because of the greater variation of electrical parameters among devices of the same type (which the partisans of conventional electron tubes stress, and which indeed was a difficulty with some early circuits), but mainly because an understanding of the modes of operation of transistors requires knowledge of more parameters than in the case of electron tubes. It is also necessary to protect transistors from overloading. The relatively strong influence of various parameters (operating point, temperature, frequency) upon the characteristics requires many measurements by circuit designers, even though the manufacturing scatter of device parameters is being made narrower by technical progress.

Depending on the objectives, the requirements set for transistor measurements vary to a great extent. The development of devices requires, above all, measurement methods to be as accurate as possible and universally applicable, without regard to the expenditure of time and apparatus. Such methods are known simply as laboratory methods.

Circuit designers may be forced to use such methods, particularly when the circuit details must be based on characteristics that the data sheet produces only in general form, or not at all.

However, most circuit designers will be satisfied by a measurement of typical values in more stringent conditions than usual and by a fair accuracy of measurement, since either the data supplied by transistor manufacturers will be sufficient, or the user will have accumulated over a period of time a sufficiently great degree of experience; in either case, highly accurate measurements may

be superfluous. Ordinary measurements require less time and instrumentation than laboratory methods but they may be applicable only to some specified types of transistors. Such methods will be called service methods.

Finally, in production or acceptance tests of transistors the question revolves not around absolute values of typical parameters but is only whether a certain limiting value, usually given in the data sheet, is exceeded or not achieved. In these tests, the speed of measurement, simple manipulations, constancy of the operating point and simplicity of instrumentation are primary considerations.

Transistor measurement techniques comprise all the above classified methods for determining the characteristics equally important to the manufacturer and to the user. The scope of transistor measurement techniques is not so much the development of new, more generally applicable methods, but rather the adaptation of well-known methods for measurements of active and passive devices to the specific character of transistors. The problems may consist in the use of relatively small amplitudes at very high frequencies of measurement (of the order of Gc/s), in setting the operating point without affecting the measurement result and in strict preservation of supplementary conditions during the measurement (quadripole termination, test jig), etc.

The great diversity of methods that have developed over the years is mainly occasioned by the great number of parameters that may be used for characterising transistor circuit performance for a given operational condition. These parameters can be divided into characteristic and effective classes; characteristic parameters depend solely on the transistor, effective parameters depend on the circuit conditions of application as well. The great advantage of the characteristic parameters is that, by definition, the method of measurement is uniquely determined and there is no need to describe in detail how the measurement is to be carried out. On the other hand, there are quite a few parameters, e.g. large signal parameters, that can be specified only as effective values since the amplitude of the driving signal must be specified. These parameters are little suited to serve as universal data, but are frequently used, nevertheless, particularly in pulse operation.

Some applications require information about the elements of an equivalent circuit and others information relating to thermal parameters, etc. [7.11, 7.21].

In accordance with the mode of operation the parameters can be divided into several groups.

1. Static parameters, which can be in most cases derived from the characteristic curves.
2. Dynamic parameters, comprising, for example, small signal values at low and high frequencies (quadripole parameters), cut-off and characteristic frequencies, and the elements of equivalent circuits.

3. Pulse parameters, which describe switching times and switching time constants;
4. Thermal parameters, which include the temperature of the junction, thermal resistances and a thermal time constant;
5. Noise parameters.

This classification is applied for describing appropriate measurement methods in Chapters 2 and 6; Chapter 1 describes the most important parameters and the limiting values of the junction transistor.

There will be no attempt to discuss in detail how some particular [7.27] physical and constructional parameters are determined, e.g. base width [1.100], diffusion constant, effective lifetime [1.40, 3.166, 3.119, 3.175], impurity density, drift factor [1.40, 3.174], junction surface area, etc. Such quantities are, in principle, included in the elements of an equivalent circuit, but specific points relating to the construction and type are also of importance and have to be taken into account in each case; see for example the following literature references: [3.57, 3.46, 3.93, 3.165, 1.40, 3.120, 3.172, 3.174, 3.173, 3.83, 3.54].

All the discussion in the following text refers, unless otherwise stated, to transistors.

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# 1

## TRANSISTOR PARAMETERS

### 1.1 STATIC PARAMETERS

All applications of transistors are based, in one way or another, on their d.c. operation, be it by the setting of the operating point, in the amplification of very large signals with relatively slow variations, or in switching operations. The d.c. operation of an idealised transistor model (inner transistor) having current paths of no resistance ( $r_B \approx r_{C'C} \approx r_{E'E} \approx 0$ ) can be described by the following system of equations which may be inferred by physical discussion [1.1, 1.4, 1.110]; directions of current flow are as shown in Fig. 1.1:

$$\begin{aligned} I_E &= -\frac{I_{EBO}}{1 - A_N A_I} \left( \exp \frac{V_{E'B'}}{V_T} - 1 \right) - \frac{A_I I_{CBO}}{1 - A_N A_I} \left( \exp \frac{V_{C'B'}}{V_T} - 1 \right) \\ I_C &= -\frac{A_N I_{EBO}}{1 - A_N A_I} \left( \exp \frac{V_{E'B'}}{V_T} - 1 \right) - \frac{I_{CBO}}{1 - A_N A_I} \left( \exp \frac{V_{C'B'}}{V_T} - 1 \right) \end{aligned} \quad (1.1)$$

$I_{EBO}$  - emitter leakage current (see Section 1.1.3);

$I_{CBO}$  - collector leakage current (see Section 1.1.3);

$A_N$  - d.c. amplification of the normal common-base circuit;

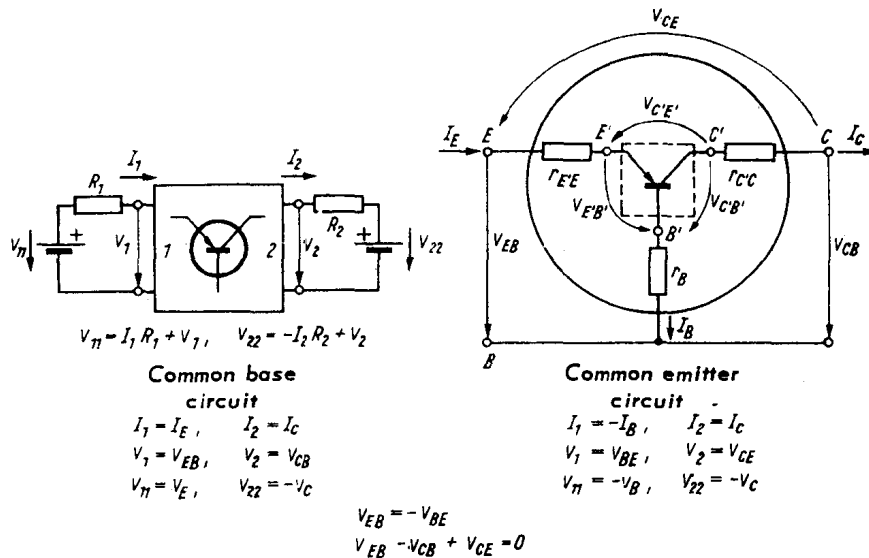
$A_I$  - d.c. amplification of the inverse common-base circuit;

$V_T = kT/q$  - which may be termed the 'thermal voltage'.

The quantities above,  $I_{EBO}$ ,  $I_{CBO}$ ,  $A_N$  and  $A_I$ , will be regarded in our considerations as basic intrinsic properties of the transistor and will not be analysed further.

The 'thermal voltage'  $V_T$  has a value of about 26 mV at room temperature.

In modern transistors (mesa and planar types) and at the high loading which is frequent with these types, (1.1) are valid only when used with some additional fairly important limitations due to specific physical effects [1.1]. It is, therefore, more useful to



In a normally conducting *pnp* transistor  $I_E, I_B, I_C, V_{EB}$  and  $V_E$  have positive numerical values and  $V_{CE}, V_{CB}, V_C$  and  $V_B$  negative values. In *npn* transistors all numerical current and voltage values have signs opposite to those of *pnp* transistors

Fig. 1.1 Conventional directions of current flow in a transistor

start directly from the characteristic curves of the external transistor, i.e. from the graphical representation of the general relationships

$$I_E = f_1(V_{EB}, V_{CB})$$

$$I_C = f_2(V_{EB}, V_{CB})$$

with an implicit inclusion of these specific effects, and to consider such typical parameters and limiting values which describe and

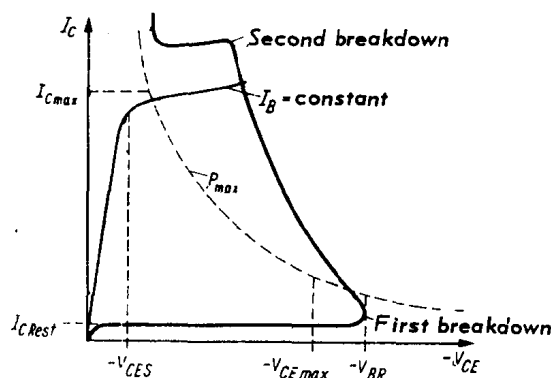


Fig. 1.2 Collector characteristics of transistor operating in common emitter circuit. (For further details, see text)



delineate the complete properties of a transistor [1.1].

Limiting values are dependent upon the operating point and can be given, as power, current or voltage limits. Since the power loss takes place mainly in the cut-off diode path (two-terminal path), i.e. in active normal operation in the collector diode, when the usual active operational range is considered, the effective parameters and limiting values can easily be found and interpreted by using the collector characteristics. An example of these for the common emitter circuit is given in Fig. 1.2.

### 1.1.1 POWER DISSIPATION. MAXIMUM CURRENT

The maximum permissible power dissipation is given by

$$P_{\max} = [I_E V_{EB} + I_C (-V_{CB})]_{\max} \approx I_C (-V_{CB})_{\max}$$

and depends upon the maximum junction temperature  $\vartheta_{j\max}$  which can be allowed and on the heat dissipation conditions which can be described in terms of a thermal resistance  $R_{th}$  where

$$\begin{aligned} P_{\max} &= \frac{\vartheta_{j\max} - \vartheta_a}{R_{th}} \\ &= \frac{\vartheta_{j\max} - \vartheta_c}{R_{thi}} \end{aligned} \quad (1.2)$$

In principle, it makes no difference whether in the above formula one uses the thermal resistance  $R_{th}$  (determined by the construction of the transistor, its envelope and cooling conditions, and the ambient temperature  $\vartheta_a$ ), or the inner thermal resistance  $R_{thi}$  where the case temperature  $\vartheta_c$  is treated as the reference point.

The maximum power dissipation is determined by the manufacturer as a limiting value, and is not one of those parameters that are being checked by the user. The power limit is given either

1. directly as a function of the case ( $\vartheta_c$ ) or ambient temperature ( $\vartheta_a$ ) as a graph pertaining to certain thermal conditions (heat-sink surface-area geometry and heat-sink material; see Section 1.4.1),
2. indirectly, by producing figures for the thermal resistance  $R_{th}$ , or  $R_{thi}$ , maximum temperature of the junction,  $\vartheta_{j\max}$ , and the highest permissible temperature difference between the junction and the environment, or casing.

In the region of large current operation, the limit is given by manufacturer's maximum currents ( $I_{C\max}$ ,  $I_{B\max}$ ,  $I_{E\max}$ ). These values apply at a reference temperature,  $\vartheta_0$  (see Section 1.4). The values of maximum currents are fixed by the manufacturer mainly from thermal considerations in conjunction with reliability trials.