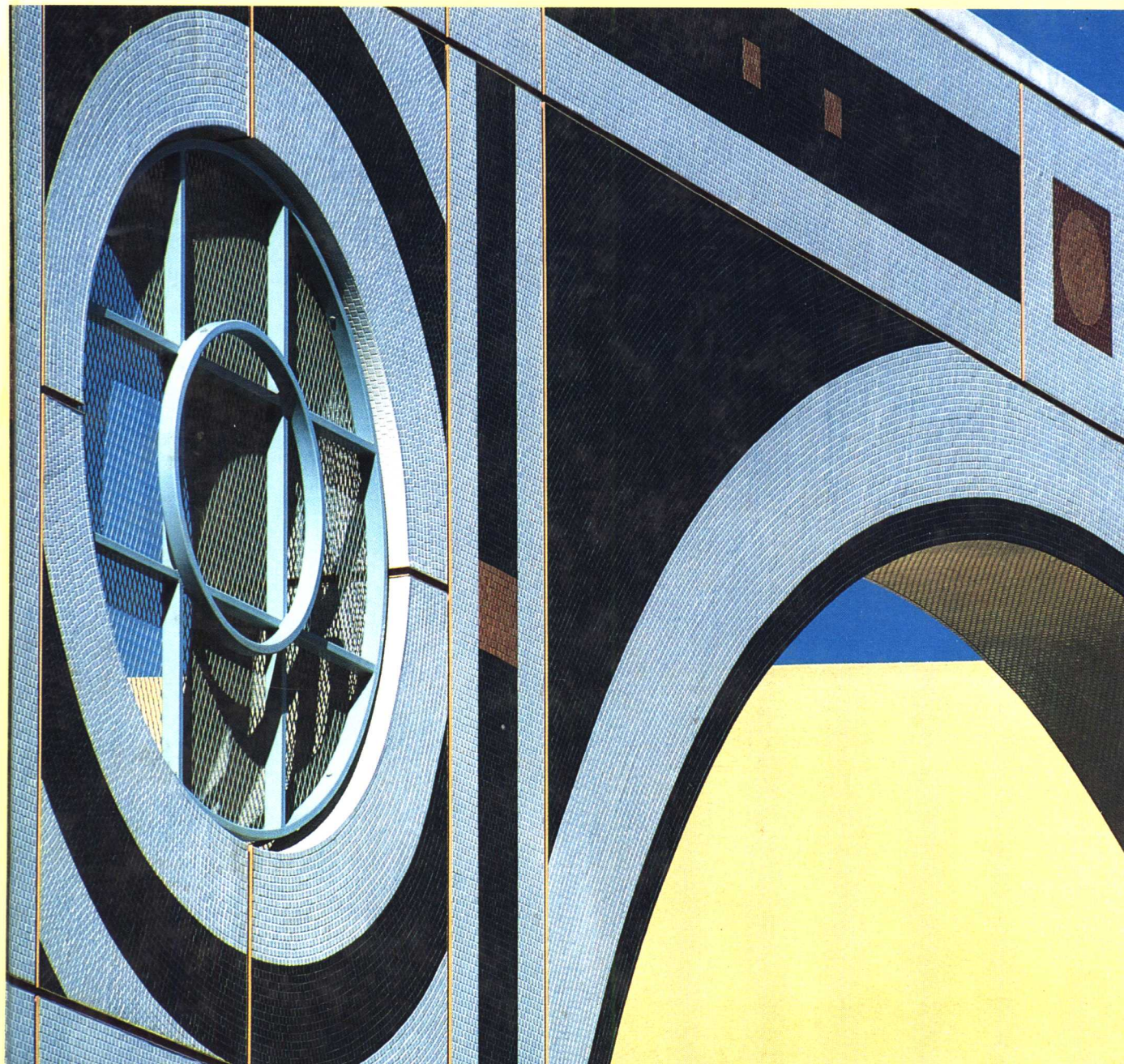


Mathematical Applications

for Management, Life, and Social Sciences

Harshbarger · Reynolds
Third Edition



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Mathematical Applications

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and Social Sciences

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Preface

To paraphrase Alfred North Whitehead, the purpose of education is not to fill a vessel but to kindle a fire. This desirable goal is not always an easy one to realize with students whose primary interest is in an area other than mathematics. The purpose of this text, then, is to present mathematical skills and concepts and to apply them to areas that are important to students in the management, life, and social sciences. The applications included allow students to view mathematics in a practical setting relevant to their intended careers. Almost every chapter of this book includes a section or two devoted to the applications of mathematical topics. An index of these applications on the inside covers demonstrates the wide variety used in examples and exercises. Although intended for students who have completed two years, or the equivalent, of high school algebra, this text begins with a brief review of algebra, which, if covered, will aid in preparing students for the work ahead.

Pedagogical Features

Important pedagogical features that have been retained in this new edition are the following.

Intuitive Viewpoint. The book is written from an intuitive viewpoint, with emphasis on concepts and problem solving rather than on mathematical theory. Each topic is carefully explained and examples illustrate the techniques involved. Exercises stress computation and drill, but there are enough challenging problems to stimulate students.

Flexibility. At different colleges or universities the coverage and sequencing of topics may vary according to the purpose of this course. To accommodate this, the text has a great deal of flexibility in the order of topics.

Chapter Warmups. A Warmup appears at the beginning of each chapter and invites students to test themselves on the skills needed for that chapter. The Warmups present several prerequisite problem types that are taken from parts of upcoming problems. Each prerequisite problem type is keyed to the upcoming section where that skill is needed, and students who have difficulty with any particular skill are directed to specific sections of the text for review. Instructors may find the Warmups useful in creating a syllabus.

Applications. We have found that offering applied topics such as cost, revenue, and profit functions in a separate section brings the preceding mathematical discussions into clear and concise focus. There are 16 such sections in this book. Beyond this, there are over 1450 applied exercises and hundreds of applied examples throughout the text.

Objective Lists. Every section begins with a brief list of objectives that outlines the goals of that section for the student.

Procedure/Example Tables. Sprinkled throughout the text, Procedure/Example tables aid student understanding by giving step-by-step descriptions of important procedures with illustrative examples worked out beside the procedures.

Boxed Information. All important information is boxed for easy reference, and key terms are highlighted in boldface.

Review Exercises. At the end of each chapter, a set of Review Exercises offers the student extra practice on the topics in that chapter. These exercises are annotated with section numbers so that the student having difficulty can turn to the appropriate section for review.

Changes In the Third Edition

A major focus of this revision was to improve the exercise sets. This was achieved by providing the following: (1) a better balance between the odd- and the even-numbered exercises, (2) a smoother progression from easy exercises to difficult exercises, (3) better coverage of the topics within each section, and (4) the inclusion of numerous new applications. In fact, the number of exercises has been increased by more than 20% to over 4250 problems. The number of applications has also been increased by more than 20% to over 1450 applications.

Other significant improvements in the third edition are the following.

The discussion of radicals was expanded and placed in a separate section of Chapter 0.

In Chapter 1, the discussion of functions was expanded to include piecewise-defined functions. In addition, the simultaneous solution of three equations in three variables was added to this chapter.

In Chapter 3, the notation for the simplex method has been standardized so that it is more consistent with other linear programming discussions. The simplex method was extended to cases where infinitely many and no solutions occurred.

Chapter 6 was rewritten with a stronger emphasis on the mathematics of finance. The discussion of sequences is now used to support the development of the mathematics of finance rather than as the central theme of the chapter. Sigma notation was moved from Chapter 6 to Chapter 13, where it is needed to develop the definite integral. Deferred annuities were added to the section on ordinary annuities. The discussion of depreciation has been updated to reflect changes brought about by the Tax Reform Law of 1986. The notation used in this chapter has been made consistent with the notation used in business textbooks and all important formulas have been boxed.

In Chapter 13, a new section on areas between curves follows the definite integral. This gives a firmer foundation to the notions of producer's and consumer's surplus. Also, a discussion of continuous income streams (both total income and present value) replaced and extended the subsection on present value of an annuity.

Throughout the text, exercises that are best worked with a calculator are now highlighted with the symbol .

Supplements

Instructor's Guide. This booklet contains two forms of a test for each chapter of the text, with answers provided. In addition, the answers to all even-numbered exercises of the text are included.

Selected Solutions Guide. In addition to an answer section at the end of the text, the solutions to all odd-numbered exercises are included in this supplementary booklet.

Computerized Testing Program by Engineering Software Associates, Inc. Hardware requirements: IBM PC or compatible, two disk drives, IBM graphics-compatible dot-matrix printer or laser printer.

Items included: One program disk, test item disks, User Manual/Printed Test Item File.

This new, versatile test-generating program allows instructors to customize tests for their own classes. It contains over 1000 test items and offers full graphics capability (including mathematical symbols).

With this program, instructors can preview questions on-screen and then add each item to a test with one keystroke. Random generation of test items by chapter is possible. Additionally, instructors may edit existing items or add new items either to the database or to individual tests. Tests may be saved and then printed in multiple scrambled versions. Answer keys are automatically generated.

Interactive Finite Mathematics and Interactive Applied Calculus by The Math Lab. For use with the IBM PC or Apple II, this software program provides labwork for business, life, and social science majors. A total of 80 labs are provided, allowing instructors to select those labs appropriate for their use. Twenty-two of the labs are related to business and economic applications, while many others model real-life business problems and situations.

Related Texts

This book is one of three covering finite mathematics and applied calculus. All three texts heavily emphasize real-world applications of the mathematics featured as the students in these courses are typically majoring in management or the life or social sciences. The material in *Mathematical Applications* is also available in the following two separate texts.

Finite Mathematics for Management, Life, and Social Sciences, Third Edition.

This text is intended for a one-term course covering sets, matrices, inequalities and linear programming, mathematics of finance, probability, and statistics. A chapter on game theory has been added to this text.

Applied Calculus for Management, Life, and Social Sciences, Third Edition.

This text is designed for a one-term course covering a review of algebra, functions of one variable, derivatives, exponential and logarithmic functions, indefinite and definite integrals, and finally, functions of two or more variables. Sections on numerical methods of integration and double integrals have been added to this text.

Acknowledgements

We would like to thank the many people who have helped us at various stages of this project. The encouragement, criticism, and suggestions that have been offered have been invaluable to us. We are especially indebted to Samuel Laposata, Virginia Electric Power Company, who provided ideas and encouragement, and to Frank Kocher, The Pennsylvania State University, who provided support and reviews throughout the first edition's evolution. Our special thanks are due Gordon Shilling, University of Texas at Arlington, who carefully examined every exercise set; and to Stanley Chadick, Northwestern State University; James Runyon, Rochester Institute of Technology; and Ellen Wood, Stephen F. Austin State University; each of whom reviewed the entire manuscript and made many helpful comments. Survey respondents offered many valuable suggestions; our thanks to Patricia Blitch, Lander College; Louis Bush, San Diego City College; Nancy Fisher, University of Alabama; Mary Guffey, Auburn University; Ronald Rule, Georgia College; and Gary Taka, Santa Monica College. Our thanks also to Bill Bolem, Glenn Ostrowski, Patti Stacey, and Sheryl Rutkowski for their help with manuscript preparation. We would also like to express our appreciation to the editorial staff at D. C. Heath for their continued enthusiasm and support.

Ronald J. Harshbarger
James J. Reynolds

Appendix

Table 1 Exponential Functions

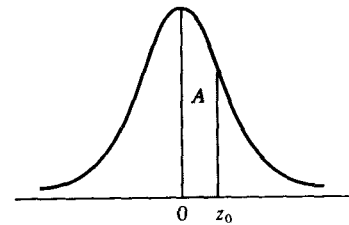
x	e^x	e^{-x}	x	e^x	e^{-x}
0.0	1.000	1.000	3.0	20.09	0.0498
0.1	1.105	0.9048	3.1	22.20	0.0450
0.2	1.221	0.8187	3.2	24.53	0.0408
0.3	1.350	0.7408	3.3	27.11	0.0369
0.4	1.492	0.6703	3.4	29.96	0.0334
0.5	1.649	0.6065	3.5	33.12	0.0302
0.6	1.822	0.5488	3.6	36.60	0.0273
0.7	2.014	0.4966	3.7	40.45	0.0247
0.8	2.226	0.4493	3.8	44.70	0.0224
0.9	2.460	0.4066	3.9	49.40	0.0202
1.0	2.718	0.3679	4.0	54.60	0.0183
1.1	3.004	0.3329	4.1	60.34	0.0166
1.2	3.320	0.3012	4.2	66.69	0.0150
1.3	3.669	0.2725	4.3	73.70	0.0136
1.4	4.055	0.2466	4.4	81.45	0.0123
1.5	4.482	0.2231	4.5	90.02	0.0111
1.6	4.953	0.2019	4.6	99.48	0.0101
1.7	5.474	0.1827	4.7	109.9	0.0091
1.8	6.050	0.1653	4.8	121.5	0.0082
1.9	6.686	0.1496	4.9	134.3	0.0074
2.0	7.389	0.1353	5.0	148.4	0.0067
2.1	8.166	0.1225	5.1	164.0	0.0061
2.2	9.025	0.1108	5.2	181.3	0.0055
2.3	9.974	0.1003	5.3	200.3	0.0050
2.4	11.02	0.0907	5.4	221.4	0.0045
2.5	12.18	0.0821	5.5	244.7	0.0041
2.6	13.46	0.0743	5.6	270.4	0.0037
2.7	14.88	0.0672	5.7	298.9	0.0033
2.8	16.44	0.0608	5.8	330.3	0.0030
2.9	18.17	0.0550	5.9	365.0	0.0027
			6.0	403.4	0.0025
			7.0	1097	0.0009
			8.0	2981	0.0003
			9.0	8103	0.0001

Table II Selected Values of $\text{Log}_e x$

x	$\ln x$	x	$\ln x$	x	$\ln x$
.002	-6.215	1.40	0.336	600	6.397
.004	-5.521	1.50	0.405	700	6.551
.010	-4.605	2	0.693	800	6.685
.015	-4.200	3	1.099	900	6.802
.018	-4.017	5	1.609	1000	6.908
.020	-3.912	7	1.946	1300	7.170
.023	-3.772	10	2.303	1600	7.378
.027	-3.612	20	2.996	2000	7.601
.030	-3.507	30	3.401	2300	7.741
.040	-3.219	50	3.912	2600	7.863
.045	-3.101	80	4.382	3000	8.006
.050	-2.996	100	4.605	3500	8.161
.080	-2.526	110	4.700	4000	8.294
.10	-2.303	130	4.868	4600	8.434
.20	-1.609	150	5.011	5100	8.537
.30	-1.204	170	5.136	5700	8.648
.40	-0.916	200	5.298	6300	8.748
.41	-0.892	230	5.438	6900	8.839
.48	-0.734	250	5.521	7500	8.923
.50	-0.693	280	5.635	8100	9.000
.52	-0.654	300	5.704	8700	9.071
.60	-0.511	350	5.858	9300	9.138
.70	-0.357	400	5.991	10000	9.210
.80	-0.223	410	6.016	11000	9.306
.90	-0.105	460	6.131	12000	9.363
1.00	0.0	500	6.215	13000	9.473
1.30	0.262	520	6.254	14000	9.547
1.35	0.300	580	6.363	15000	9.616

Table III Areas Under the Standard Normal Curve

The value of A is the area under the standard normal curve between $z = 0$ and $z = z_0$, for $z_0 \geq 0$. Areas for negative values of z_0 are obtained by symmetry.



z_0	A	z_0	A	z_0	A	z_0	A
.00	.0000	.06	.0239	.12	.0478	.18	.0714
.01	.0040	.07	.0279	.13	.0517	.19	.0754
.02	.0080	.08	.0319	.14	.0557	.20	.0793
.03	.0120	.09	.0359	.15	.0596	.21	.0832
.04	.0160	.10	.0398	.16	.0636	.22	.0871
.05	.0199	.11	.0438	.17	.0675	.23	.0910

Table III Areas Under the Standard Normal Curve (Continued)

z_0	A	z_0	A	z_0	A	z_0	A
.24	.0948	.70	.2580	1.16	.3770	1.62	.4474
.25	.0987	.71	.2612	1.17	.3790	1.63	.4485
.26	.1026	.72	.2642	1.18	.3810	1.64	.4495
.27	.1064	.73	.2673	1.19	.3830	1.65	.4505
.28	.1103	.74	.2704	1.20	.3849	1.66	.4515
.29	.1141	.75	.2734	1.21	.3869	1.67	.4525
.30	.1179	.76	.2764	1.22	.3888	1.68	.4535
.31	.1217	.77	.2794	1.23	.3907	1.69	.4545
.32	.1255	.78	.2823	1.24	.3925	1.70	.4554
.33	.1293	.79	.2852	1.25	.3944	1.71	.4564
.34	.1331	.80	.2881	1.26	.3962	1.72	.4573
.35	.1368	.81	.2910	1.27	.3980	1.73	.4582
.36	.1406	.82	.2939	1.28	.3997	1.74	.4591
.37	.1443	.83	.2967	1.29	.4015	1.75	.4599
.38	.1480	.84	.2996	1.30	.4032	1.76	.4608
.39	.1517	.85	.3023	1.31	.4049	1.77	.4616
.40	.1554	.86	.3051	1.32	.4066	1.78	.4625
.41	.1591	.87	.3079	1.33	.4082	1.79	.4633
.42	.1628	.88	.3106	1.34	.4099	1.80	.4641
.43	.1664	.89	.3133	1.35	.4115	1.81	.4649
.44	.1700	.90	.3159	1.36	.4131	1.82	.4656
.45	.1736	.91	.3186	1.37	.4147	1.83	.4664
.46	.1772	.92	.3212	1.38	.4162	1.84	.4671
.47	.1808	.93	.3238	1.39	.4177	1.85	.4678
.48	.1844	.94	.3264	1.40	.4192	1.86	.4686
.49	.1879	.95	.3289	1.41	.4207	1.87	.4693
.50	.1915	.96	.3315	1.42	.4222	1.88	.4700
.51	.1950	.97	.3340	1.43	.4236	1.89	.4706
.52	.1985	.98	.3365	1.44	.4251	1.90	.4713
.53	.2019	.99	.3389	1.45	.4265	1.91	.4719
.54	.2054	1.00	.3413	1.46	.4279	1.92	.4726
.55	.2088	1.01	.3438	1.47	.4292	1.93	.4732
.56	.2123	1.02	.3461	1.48	.4306	1.94	.4738
.57	.2157	1.03	.3485	1.49	.4319	1.95	.4744
.58	.2190	1.04	.3508	1.50	.4332	1.96	.4750
.59	.2224	1.05	.3531	1.51	.4345	1.97	.4756
.60	.2258	1.06	.3554	1.52	.4357	1.98	.4762
.61	.2291	1.07	.3577	1.53	.4370	1.99	.4767
.62	.2324	1.08	.3599	1.54	.4382	2.00	.4773
.63	.2357	1.09	.3621	1.55	.4394	2.01	.4778
.64	.2389	1.10	.3643	1.56	.4406	2.02	.4783
.65	.2422	1.11	.3665	1.57	.4418	2.03	.4788
.66	.2454	1.12	.3686	1.58	.4430	2.04	.4793
.67	.2486	1.13	.3708	1.59	.4441	2.05	.4798
.68	.2518	1.14	.3729	1.60	.4452	2.06	.4803
.69	.2549	1.15	.3749	1.61	.4463	2.07	.4808

Table III Areas Under the Standard Normal Curve (Continued)

z_0	A	z_0	A	z_0	A	z_0	A
2.08	.4812	2.54	.4945	3.00	.4987	3.46	.4997
2.09	.4817	2.55	.4946	3.01	.4987	3.47	.4997
2.10	.4821	2.56	.4948	3.02	.4987	3.48	.4998
2.11	.4826	2.57	.4949	3.03	.4988	3.49	.4998
2.12	.4830	2.58	.4951	3.04	.4988	3.50	.4998
2.13	.4834	2.59	.4952	3.05	.4989	3.51	.4998
2.14	.4838	2.60	.4953	3.06	.4989	3.52	.4998
2.15	.4842	2.61	.4955	3.07	.4989	3.53	.4998
2.16	.4846	2.62	.4956	3.08	.4990	3.54	.4998
2.17	.4850	2.63	.4957	3.09	.4990	3.55	.4998
2.18	.4854	2.64	.4959	3.10	.4990	3.56	.4998
2.19	.4857	2.65	.4960	3.11	.4991	3.57	.4998
2.20	.4861	2.66	.4961	3.12	.4991	3.58	.4998
2.21	.4865	2.67	.4962	3.13	.4991	3.59	.4998
2.22	.4868	2.68	.4963	3.14	.4992	3.60	.4998
2.23	.4871	2.69	.4964	3.15	.4992	3.61	.4999
2.24	.4875	2.70	.4965	3.16	.4992	3.62	.4999
2.25	.4878	2.71	.4966	3.17	.4992	3.63	.4999
2.26	.4881	2.72	.4967	3.18	.4993	3.64	.4999
2.27	.4884	2.73	.4968	3.19	.4993	3.65	.4999
2.28	.4887	2.74	.4969	3.20	.4993	3.66	.4999
2.29	.4890	2.75	.4970	3.21	.4993	3.67	.4999
2.30	.4893	2.76	.4971	3.22	.4994	3.68	.4999
2.31	.4896	2.77	.4972	3.23	.4994	3.69	.4999
2.32	.4898	2.78	.4973	3.24	.4994	3.70	.4999
2.33	.4901	2.79	.4974	3.25	.4994	3.71	.4999
2.34	.4904	2.80	.4974	3.26	.4994	3.72	.4999
2.35	.4906	2.81	.4975	3.27	.4995	3.73	.4999
2.36	.4909	2.82	.4976	3.28	.4995	3.74	.4999
2.37	.4911	2.83	.4977	3.29	.4995	3.75	.4999
2.38	.4913	2.84	.4977	3.30	.4995	3.76	.4999
2.39	.4916	2.85	.4978	3.31	.4995	3.77	.4999
2.40	.4918	2.86	.4979	3.32	.4996	3.78	.4999
2.41	.4920	2.87	.4980	3.33	.4996	3.79	.4999
2.42	.4922	2.88	.4980	3.34	.4996	3.80	.4999
2.43	.4925	2.89	.4981	3.35	.4996	3.81	.4999
2.44	.4927	2.90	.4981	3.36	.4996	3.82	.4999
2.45	.4929	2.91	.4982	3.37	.4996	3.83	.4999
2.46	.4931	2.92	.4983	3.38	.4996	3.84	.4999
2.47	.4932	2.93	.4983	3.39	.4997	3.85	.4999
2.48	.4934	2.94	.4984	3.40	.4997	3.86	.4999
2.49	.4936	2.95	.4984	3.41	.4997	3.87	.5000
2.50	.4938	2.96	.4985	3.42	.4997	3.88	.5000
2.51	.4940	2.97	.4985	3.43	.4997	3.89	.5000
2.52	.4941	2.98	.4986	3.44	.4997		
2.53	.4943	2.99	.4986	3.45	.4997		

Table IV AMOUNT OF \$1 AT COMPOUND INTEREST $\left(1 + \frac{i}{k}\right)^{kn}$

<i>Periods</i>	$\frac{1}{2}\%$	$\frac{3}{4}\%$	1%	$1\frac{1}{2}\%$	2%
1	1.005 000	1.007 500	1.010 000	1.015 000	1.020 000
2	1.010 025	1.015 056	1.020 100	1.030 225	1.040 400
3	1.015 075	1.022 669	1.030 301	1.045 678	1.061 208
4	1.020 151	1.030 339	1.040 604	1.061 364	1.082 432
5	1.025 251	1.038 067	1.051 010	1.077 284	1.104 081
6	1.030 378	1.045 852	1.061 520	1.093 443	1.126 162
7	1.035 529	1.053 696	1.072 135	1.109 845	1.148 686
8	1.040 707	1.061 599	1.082 857	1.126 493	1.171 659
9	1.045 911	1.069 561	1.093 685	1.143 390	1.195 093
10	1.051 140	1.077 583	1.104 622	1.160 541	1.218 994
11	1.056 396	1.085 664	1.115 668	1.177 949	1.243 374
12	1.061 678	1.093 807	1.126 825	1.195 618	1.268 242
13	1.066 986	1.102 010	1.138 093	1.213 552	1.293 607
14	1.072 321	1.110 276	1.149 474	1.231 756	1.319 479
15	1.077 683	1.118 603	1.160 969	1.250 232	1.345 868
16	1.083 071	1.126 992	1.172 579	1.268 986	1.372 786
17	1.088 487	1.135 445	1.184 304	1.288 020	1.400 241
18	1.093 929	1.143 960	1.196 147	1.307 341	1.428 246
19	1.099 399	1.152 540	1.208 109	1.326 951	1.456 811
20	1.104 896	1.161 184	1.220 190	1.346 855	1.485 947
21	1.110 420	1.169 893	1.232 392	1.367 058	1.515 666
22	1.115 972	1.178 667	1.244 716	1.387 564	1.545 980
23	1.121 552	1.187 507	1.257 163	1.408 377	1.576 899
24	1.127 160	1.196 414	1.269 735	1.429 503	1.608 437
25	1.132 796	1.205 387	1.282 432	1.450 945	1.640 606
26	1.138 460	1.214 427	1.295 256	1.472 710	1.673 418
27	1.144 152	1.223 535	1.308 209	1.494 800	1.706 886
28	1.149 873	1.232 712	1.321 291	1.517 222	1.741 024
29	1.155 622	1.241 957	1.334 504	1.539 981	1.775 845
30	1.161 400	1.251 272	1.347 849	1.563 080	1.811 362
31	1.167 207	1.260 656	1.361 327	1.586 526	1.847 589
32	1.173 043	1.270 111	1.374 941	1.610 324	1.884 541
33	1.178 908	1.279 637	1.388 690	1.634 479	1.922 231
34	1.184 803	1.289 234	1.402 577	1.658 996	1.960 676
35	1.190 727	1.298 904	1.416 603	1.683 881	1.999 890
36	1.196 681	1.308 645	1.430 769	1.709 140	2.039 887
37	1.202 664	1.318 460	1.445 076	1.734 777	2.080 685
38	1.208 677	1.328 349	1.459 527	1.760 798	2.122 299
39	1.214 721	1.338 311	1.474 123	1.787 210	2.164 745
40	1.220 794	1.348 349	1.488 864	1.814 018	2.208 040

Table IV AMOUNT OF \$1 AT COMPOUND INTEREST $\left(1 + \frac{i}{k}\right)^{kn}$ (Continued)

<i>Periods</i>	3%	4%	5%	6%	7%
1	1.030 000	1.040 000	1.050 000	1.060 000	1.070 000
2	1.060 900	1.081 600	1.102 500	1.123 600	1.144 900
3	1.092 727	1.124 864	1.157 625	1.191 016	1.225 043
4	1.125 509	1.169 859	1.215 506	1.262 477	1.310 796
5	1.159 274	1.216 653	1.276 282	1.338 226	1.402 552
6	1.194 052	1.265 319	1.340 096	1.418 519	1.500 730
7	1.229 874	1.315 932	1.407 100	1.503 630	1.605 781
8	1.266 770	1.368 569	1.477 455	1.593 848	1.718 186
9	1.304 773	1.423 312	1.551 328	1.689 479	1.838 459
10	1.343 916	1.480 244	1.628 895	1.790 848	1.967 151
11	1.384 234	1.539 454	1.710 339	1.898 299	2.104 852
12	1.425 761	1.601 032	1.795 856	2.012 196	2.252 192
13	1.468 534	1.665 074	1.885 649	2.132 928	2.409 845
14	1.512 590	1.731 676	1.979 932	2.260 904	2.578 534
15	1.557 967	1.800 944	2.078 928	2.396 558	2.759 032
16	1.604 706	1.872 981	2.182 875	2.540 352	2.952 164
17	1.652 848	1.947 901	2.292 018	2.692 773	3.158 815
18	1.702 433	2.025 817	2.406 619	2.854 339	3.379 932
19	1.753 506	2.106 849	2.526 950	3.025 600	3.616 528
20	1.806 111	2.191 123	2.653 298	3.207 135	3.869 684
21	1.860 295	2.278 768	2.785 963	3.399 564	4.140 562
22	1.916 103	2.369 919	2.925 261	3.603 537	4.430 402
23	1.973 587	2.464 716	3.071 524	3.819 750	4.740 530
24	2.032 794	2.563 304	3.225 100	4.048 935	5.072 367
25	2.093 778	2.665 836	3.386 355	4.291 871	5.427 433
26	2.156 591	2.772 470	3.555 673	4.549 383	5.807 353
27	2.221 289	2.883 369	3.733 456	4.822 346	6.213 868
28	2.287 928	2.998 703	3.920 129	5.111 687	6.648 838
29	2.356 566	3.118 651	4.116 136	5.418 388	7.114 257
30	2.427 262	3.243 398	4.321 942	5.743 491	7.612 255
31	2.500 080	3.373 133	4.538 039	6.088 101	8.145 113
32	2.575 083	3.508 059	4.764 941	6.453 387	8.715 271
33	2.652 335	3.648 381	5.003 189	6.840 590	9.325 340
34	2.731 905	3.794 316	5.253 348	7.251 025	9.978 114
35	2.813 862	3.946 089	5.516 015	7.686 087	10.676 582
36	2.898 278	4.103 933	5.791 816	8.147 252	11.423 942
37	2.985 227	4.268 090	6.081 407	8.636 087	12.223 618
38	3.074 783	4.438 813	6.385 477	9.154 252	13.079 272
39	3.167 027	4.616 366	6.704 751	9.703 507	13.994 821
40	3.262 038	4.801 021	7.039 989	10.285 718	14.974 458

Table IV AMOUNT OF \$1 AT COMPOUND INTEREST $\left(1 + \frac{i}{k}\right)^{kn}$ (Continued)

Periods	8%	9%	10%	11%	12%
1	1.080 000	1.090 000	1.100 000	1.110 000	1.120 000
2	1.166 400	1.188 100	1.210 000	1.232 100	1.254 400
3	1.259 712	1.295 029	1.331 000	1.367 631	1.404 928
4	1.360 489	1.411 582	1.464 100	1.518 070	1.573 519
5	1.469 328	1.538 624	1.610 510	1.685 058	1.762 342
6	1.586 874	1.677 100	1.771 561	1.870 415	1.973 823
7	1.713 824	1.828 039	1.948 717	2.076 160	2.210 681
8	1.850 930	1.992 563	2.143 589	2.304 538	2.475 963
9	1.999 005	2.171 893	2.357 948	2.558 037	2.773 079
10	2.158 925	2.367 364	2.593 742	2.839 421	3.105 848
11	2.331 639	2.580 426	2.853 117	3.151 757	3.478 550
12	2.518 170	2.812 665	3.138 428	3.498 451	3.895 976
13	2.719 624	3.065 805	3.452 271	3.883 280	4.363 493
14	2.937 194	3.341 727	3.797 498	4.310 441	4.887 112
15	3.172 169	3.642 482	4.177 248	4.784 590	5.473 566
16	3.425 943	3.970 306	4.594 973	5.310 894	6.130 394
17	3.700 018	4.327 633	5.054 470	5.895 093	6.866 041
18	3.996 020	4.717 120	5.559 917	6.543 553	7.689 966
19	4.315 701	5.141 661	6.115 909	7.263 344	8.612 762
20	4.660 957	5.604 411	6.727 500	8.062 312	9.646 293
21	5.033 834	6.108 808	7.400 250	8.949 166	10.803 848
22	5.436 540	6.658 600	8.140 275	9.933 574	12.100 310
23	5.871 464	7.257 874	8.954 303	11.026 267	13.552 347
24	6.341 181	7.911 083	9.849 733	12.239 157	15.178 629
25	6.848 475	8.623 081	10.834 706	13.585 464	17.000 064
26	7.396 353	9.399 158	11.918 177	15.079 865	19.040 072
27	7.988 062	10.245 082	13.109 994	16.738 650	21.324 881
28	8.627 106	11.167 140	14.420 994	18.579 902	23.883 867
29	9.317 275	12.172 182	15.863 093	20.623 691	26.749 931
30	10.062 657	13.267 678	17.449 403	22.892 297	29.959 922
31	10.867 670	14.461 770	19.194 343	25.410 450	33.555 113
32	11.737 083	15.763 329	21.113 777	28.205 599	37.581 726
33	12.676 050	17.182 028	23.225 155	31.308 215	42.091 534
34	13.690 134	18.728 411	25.547 670	34.752 119	47.142 518
35	14.785 344	20.413 968	28.102 437	38.574 852	52.799 620
36	15.968 172	22.251 225	30.912 681	42.818 085	59.135 574
37	17.245 626	24.253 835	34.003 948	47.528 075	66.231 843
38	18.625 276	26.436 680	37.404 344	52.756 163	74.179 664
39	20.115 298	28.815 982	41.144 779	58.559 341	83.081 224
40	21.724 522	31.409 420	45.259 257	65.000 868	93.050 971

Table V Amount of an Ordinary Annuity of \$1 ($s_{\overline{n}|i}$)

Periods	1%	2%	3%	4%	5%	6%
1	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000
2	2.010 000	2.020 000	2.030 000	2.040 000	2.050 000	2.060 000
3	3.030 100	3.060 400	3.090 900	3.121 600	3.152 500	3.183 600
4	4.060 401	4.121 608	4.183 627	4.246 464	4.310 125	4.374 616
5	5.101 005	5.204 040	5.309 136	5.416 323	5.525 631	5.637 093
6	6.152 015	6.308 121	6.468 410	6.632 975	6.801 913	6.975 319
7	7.213 535	7.434 284	7.662 462	7.898 294	8.142 008	8.393 838
8	8.285 670	8.582 969	8.892 336	9.214 226	9.549 109	9.897 468
9	9.368 527	9.754 629	10.159 106	10.582 795	11.026 564	11.491 316
10	10.462 212	10.949 721	11.463 879	12.006 107	12.577 893	13.180 795
11	11.566 834	12.168 716	12.807 796	13.486 351	14.206 787	14.971 643
12	12.682 503	13.412 090	14.192 030	15.025 805	15.917 126	16.869 941
13	13.809 328	14.680 332	15.617 790	16.626 838	17.712 983	18.882 138
14	14.947 421	15.973 939	17.086 324	18.291 911	19.598 632	21.015 066
15	16.096 895	17.293 418	18.598 914	20.023 588	21.578 564	23.275 970
16	17.257 864	18.639 286	20.156 881	21.824 531	23.657 492	25.672 529
17	18.430 443	20.012 072	21.761 588	23.697 512	25.840 366	28.212 880
18	19.614 747	21.412 313	23.414 435	26.645 413	28.132 385	30.905 653
19	20.810 895	22.840 559	25.116 869	27.671 229	30.539 004	33.759 992
20	22.019 004	24.297 371	26.870 375	29.778 079	33.065 954	36.785 592
21	23.239 194	25.783 318	28.676 486	31.969 202	35.719 252	39.992 727
22	24.471 586	27.298 985	30.536 780	34.247 970	38.505 214	43.392 291
23	25.716 301	28.844 964	32.452 884	36.617 889	41.430 475	46.995 829
24	26.973 464	30.421 864	34.426 470	39.082 604	44.501 999	50.815 578
25	28.243 199	32.030 301	36.459 264	41.645 908	47.727 099	54.864 513
26	29.525 631	33.670 907	38.553 042	44.311 745	51.113 454	59.156 384
27	30.820 887	35.344 325	40.709 634	47.084 214	54.669 126	63.705 767
28	32.129 096	37.051 212	42.930 923	49.967 583	58.402 583	68.528 113
29	33.450 387	38.792 236	45.218 850	52.966 286	62.322 712	73.639 800
30	34.784 891	40.568 081	47.575 416	56.084 938	66.438 847	79.058 188
31	36.132 740	42.379 443	50.002 678	59.328 335	70.760 790	84.801 679
32	37.494 067	44.227 031	52.502 759	62.701 469	75.298 829	90.889 780
33	38.869 008	46.111 572	55.077 842	66.209 528	80.063 771	97.343 167
34	40.257 698	48.033 804	57.730 177	69.857 909	85.066 959	104.183 757
35	41.660 275	49.994 480	60.462 082	73.652 225	90.320 307	111.434 783
36	43.076 878	51.994 369	63.275 945	77.598 314	95.836 323	119.120 870
37	44.507 646	54.034 257	66.174 223	81.702 247	101.628 139	127.268 122
38	45.952 723	56.114 942	69.159 450	85.970 336	107.709 546	135.904 209
39	47.412 250	58.237 241	72.234 233	90.409 150	114.095 023	145.058 462
40	48.886 373	60.401 986	75.401 260	95.025 516	120.799 774	154.761 970

Table V Amount of an Ordinary Annuity of \$1 ($s_{\overline{n}|i}$) (Continued)

<i>Periods</i>	7%	8%	9%	10%	11%	12%
1	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000
2	2.070 000	2.080 000	2.090 000	2.100 000	2.110 000	2.120 000
3	3.214 900	3.246 400	3.278 100	3.310 000	3.342 100	3.374 400
4	4.439 943	4.506 112	4.573 129	4.641 000	4.709 731	4.779 328
5	5.750 739	5.866 601	5.984 711	6.105 100	6.227 801	6.352 847
6	7.153 291	7.335 929	7.523 335	7.715 610	7.912 860	8.115 189
7	8.654 021	8.922 803	9.200 435	9.487 171	9.783 274	10.089 012
8	10.259 803	10.636 628	11.028 474	11.435 888	11.859 434	12.299 693
9	11.977 989	12.487 558	13.021 036	13.579 477	14.136 972	14.775 656
10	13.816 448	14.486 563	15.192 930	15.937 425	16.722 009	17.548 735
11	15.783 599	16.645 488	17.560 293	18.531 167	19.561 430	20.654 583
12	17.888 451	18.977 127	20.140 720	21.384 284	22.713 187	24.133 133
13	20.140 643	21.495 297	22.953 385	24.522 712	26.211 638	28.029 109
14	22.550 488	24.214 920	26.019 189	27.974 984	30.094 918	32.392 602
15	25.129 022	27.152 114	29.360 916	31.772 482	34.405 359	37.279 715
16	27.888 054	30.324 283	33.003 399	35.949 730	39.189 949	42.753 281
17	30.840 218	33.750 226	36.973 705	40.544 703	44.500 843	48.883 674
18	33.999 033	37.450 244	41.301 338	45.599 174	50.395 936	55.749 715
19	37.378 965	41.446 263	46.018 458	51.159 091	56.939 489	63.439 681
20	40.995 493	45.761 965	51.160 120	57.275 000	64.202 833	72.052 443
21	44.865 177	50.422 922	56.764 530	64.002 501	72.265 145	81.698 736
22	49.005 740	55.456 756	62.873 338	71.402 750	81.214 310	92.502 584
23	53.436 142	60.893 296	69.531 939	79.543 025	91.147 885	104.602 894
24	58.176 672	66.764 760	76.789 813	88.497 328	102.174 152	118.155 242
25	63.249 039	73.105 940	84.700 896	98.347 061	114.413 309	133.333 871
26	68.676 471	79.954 416	93.323 977	109.181 767	127.998 773	150.333 935
27	74.483 824	87.350 769	102.723 135	121.099 944	143.078 638	169.374 007
28	80.697 692	95.338 831	112.968 217	134.209 938	159.817 288	190.698 888
29	87.346 531	103.965 937	124.135 357	148.630 932	178.397 190	214.582 755
30	94.460 788	113.283 212	136.307 539	164.494 026	199.020 881	241.332 686
31	102.073 043	123.345 869	149.575 217	181.943 428	221.913 178	271.292 608
32	110.218 156	134.213 539	164.036 987	201.137 771	247.323 628	304.847 721
33	118.933 427	145.950 622	179.800 316	222.251 548	275.529 227	342.429 447
34	128.258 767	158.626 671	196.982 344	245.476 703	306.837 442	384.520 981
35	138.236 881	172.316 805	215.710 755	271.024 374	341.589 561	431.663 499
36	148.913 462	187.102 150	236.124 723	299.126 811	380.164 413	484.463 119
37	160.337 405	203.070 322	258.375 948	330.039 493	422.982 498	543.598 693
38	172.561 023	220.315 948	282.629 783	364.043 442	470.510 573	609.830 536
39	185.640 295	238.941 223	309.066 464	401.447 787	523.266 737	684.010 201
40	199.635 116	259.056 521	337.882 446	442.592 566	581.826 078	767.091 425

Table VI Present Value of an Annuity of \$1 $\{a_{\overline{n}|i}\}$

Periods	1%	2%	3%	4%	5%	6%
1	0.990 099	0.980 392	0.970 874	0.961 538	0.952 381	0.943 396
2	1.970 395	1.941 561	1.913 470	1.886 095	1.859 410	1.833 393
3	2.940 985	2.883 883	2.828 611	2.775 091	2.723 248	2.673 012
4	3.901 965	3.807 729	3.717 098	3.629 895	3.545 951	3.465 106
5	4.853 431	4.713 460	4.579 707	4.451 822	4.329 477	4.212 364
6	5.795 476	5.601 431	5.417 191	5.242 137	5.075 692	4.917 324
7	6.728 194	6.471 991	6.230 283	6.002 055	5.786 373	5.582 381
8	7.651 678	7.325 482	7.019 692	6.732 745	6.463 213	6.209 794
9	8.566 017	8.162 237	7.786 109	7.435 332	7.107 822	6.801 692
10	9.471 304	8.982 585	8.530 203	8.110 896	7.721 735	7.360 087
11	10.367 628	9.786 848	9.252 624	8.760 477	8.306 414	7.886 875
12	11.255 077	10.575 342	9.954 004	9.385 074	8.863 252	8.383 844
13	12.133 740	11.348 374	10.634 955	9.985 648	9.393 573	8.852 683
14	13.003 703	12.106 249	11.296 073	10.563 123	9.898 641	9.294 984
15	13.865 052	12.849 264	11.937 935	11.118 387	10.379 658	9.712 249
16	14.717 874	13.577 710	12.561 102	11.652 296	10.837 770	10.105 895
17	15.562 251	14.291 872	13.166 118	12.165 669	11.274 066	10.477 260
18	16.398 268	14.992 032	13.753 513	12.659 297	11.689 587	10.827 604
19	17.226 008	15.678 462	14.323 799	13.133 939	12.085 321	11.158 117
20	18.045 553	16.351 434	14.877 475	13.590 326	12.462 210	11.469 921
21	18.856 983	17.011 210	15.415 024	14.029 160	12.821 153	11.764 077
22	18.660 379	17.658 049	15.936 917	14.451 115	13.163 003	12.041 582
23	20.455 821	18.292 205	16.443 608	14.856 842	13.488 574	12.303 379
24	21.243 387	18.913 926	16.935 542	15.246 963	13.798 642	12.550 358
25	22.023 155	19.523 457	17.413 148	15.622 080	14.093 945	12.783 356
26	22.795 203	20.121 036	17.876 842	15.982 769	14.375 185	13.003 166
27	23.559 607	20.706 898	18.327 032	16.329 586	14.643 034	13.210 534
28	24.316 443	21.281 273	18.764 108	16.663 063	14.898 127	13.406 164
29	25.065 785	21.844 385	19.188 455	16.983 715	15.141 074	13.590 721
30	25.807 708	22.396 456	19.600 441	17.292 033	15.372 451	13.764 831
31	26.542 285	22.937 702	20.000 429	17.588 494	15.592 810	13.929 086
32	27.269 589	23.468 335	20.388 766	17.873 552	15.802 677	14.084 043
33	27.989 692	23.988 564	20.765 792	18.147 646	16.002 549	14.230 230
34	28.702 666	24.498 592	21.131 837	18.411 198	16.192 904	14.368 141
35	28.408 580	24.998 620	21.487 220	18.664 613	16.374 194	14.498 246
36	30.107 505	25.488 843	21.832 253	18.908 282	16.546 852	14.620 987
37	30.799 510	25.969 454	22.167 235	19.142 579	16.711 287	14.736 780
38	31.484 663	26.440 641	22.492 462	19.367 864	16.867 893	14.846 019
39	32.163 033	26.902 589	22.808 215	19.584 485	17.017 041	14.949 075
40	32.834 686	27.355 480	23.114 772	19.792 774	17.159 086	15.046 297