SIGNAL PROCESSING
The Model-Based Approach

James V. Candy

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PREFACE

This text is designed primarily for the first year graduate student or practicing engineer. It is more advanced than Schwarz and Shaw [1], but not as detailed (in its proofs) as Anderson and Moore [2] or Maybeck [3]. Only the essential information is presented; detailed proofs are referenced. The "practice" of estimation is emphasized, and concepts are substantiated through detailed simulations using state-of-the-art software [4, 5]. The required background is a course in stochastic processes and basic linear system theory (z-transforms and state space). The main theme of the text concerns fundamental concepts underlying model-based signal processing. Two popular stochastic models—the autoregressive moving-average model with exogeneous inputs (ARMAX) and the state-space model—are employed in schemes that lead to solutions for both known and unknown model cases. Emphasis is on the practical design of these processors using popular techniques. Each processor is developed in the unified "model-based" framework, and examples as well as computer simulations are heavily employed as a teaching aide.

The first chapter discusses the concepts of model-based signal processing and lays the basic framework for future developments. The idea of a recursive form is developed and used throughout the text.

In Chapter 2 we discuss the modeling of stochastic processes by first examining various representations and then introducing the ARMAX and Gauss–Markov (state-space) models. It is shown how the models can be used to simulate stochastic processes. Fundamental theorems proven in Astrom [6] and Jazwinski [7] are discussed in terms of their practical applications. This chapter coupled with Appendixes A (probability) and B (state space) provide the minimum background for further developments.

Chapter 3 discusses estimation theory and means of assessing estimator performance. The general techniques of minimum variance, least squares, maximum a posteriori, and maximum likelihood estimation are developed. This chapter is based primarily on the paper by Rhodes [8].

State-space (Kalman) filters are developed theoretically in Chapter 4 using the innovations approach developed by Kailath [9]. The derivation of the processor is coupled with more classical approaches using the Gauss–Markov and bayesian techniques. Appendixes C (matrices) and E (gaussian vectors) provide the required background information.

The practical aspects of Kalman filter design are discussed in depth in Chapter 5. A well-founded "cookbook" approach is developed and applied to simulated data sets. For those only interested in filter design, this chapter provides the minimum required information.

Chapter 6 discusses the extensions of the Kalman filtering technique to solve problems it was not directly designed to solve. It is shown how to use the existing approach with minor modifications. Simulated examples are discussed. Nonlinear estimation using the linearized and extended Kalman filters is developed as well. Appendix D (*U-D* factorization [7]) discusses the preferred numerical approach to implementing the Kalman algorithm.

In Chapter 7 the classical Wiener filter is linked back to the Kalman estimator. Classical Wiener filter design is discussed in terms of the innovations approach, and then it is shown how many of the current "identification" techniques (e.g., linear prediction, recursive extended least squares [10]) can be considered Wiener filter design techniques. Finally, the systems theory approach of stochastic realization for scalar systems is discussed. This chapter should prove interesting to signal-processing specialists.

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CONTENTS

CHAPTER

ONE

INTRODUCTION

In this chapter we introduce the concepts of signal processing from the filtering of deterministic data to sophisticated model-based signal processing. First, we investigate conceptually the difference between processing deterministic and random data. We discuss a procedure for processor design and then develop the concepts behind model-based processing.

1.1 BACKGROUND

In many applications, engineers and scientists are frequently faced with the problem of measuring a quantity to directly predict physical phenomenology, control some mechanism, or infer information about a quantity not directly measurable. For example, a geophysicist may make measurements of seismic signals and attempt to predict the subsurface structure of the earth for oil exploration. A control engineer may use measurements of rotor speed to control a turbine generator in a power plant. A chemical engineer may make measurements of temperature and pressure to determine the density of a particular liquid stored in a tank. In all these cases, a measurement is used to predict or infer specific information about some phenomenon. Heuristically, if the measured signal is free from extraneous variations and is repeatable, then it is termed *deterministic*, but if it varies extraneously and is not repeatable, then it is *random*. Figure 1.1-1 depicts these relationships.

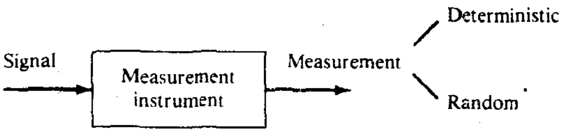

Figure 1.1-1 Signal-measurement relationship.

The process of extracting the useful information from a signal and discarding the extraneous is called (loosely) *signal processing.* Signal processing takes many forms depending primarily on the a priori knowledge or guesses of the underlying phenomenology generating the signal. If it is felt that the signal is deterministic, then techniques such as filtering (analog or digital) can be used to remove unwanted disturbances. However, if the signal is random, then more sophisticated filters must be used to extract the pertinent information. Normally, the filtering of random signals is referred to as *estimation,* because most estimation filters are statistical and estimation is a well-defined statistical technique. One could also argue that estimation is rooted in optimization theory because, in general, the "best" or optimal estimate is desired.

For example, consider the data and corresponding spectrum shown in Fig. 1.1-2*a*. From a priori knowledge of the process, it is known that the desired signal has no frequencies greater than 15 Hz. The raw spectrum indicates a disturbance at 20 Hz. Since the data are deterministic, a low-pass filter with cutoff frequency of 12.5 Hz is designed to extract the signal (information) at 10 Hz and reject the disturbance. The data are processed through the filter and the results are shown in Fig. 1.1-2*b*. The processor has extracted (passed) the desired information (10-Hz signal) and rejected the undesired (20-Hz disturbance).

This text is concerned with the development of signal-processing techniques to extract pertinent signal information from random signals utilizing any a priori information available. We call these techniques *signal estimation,* and we call a particular algorithm a *signal estimator* or just *estimator.* Sometimes estimators are called filters (e.g., Wiener filter) because they perform the same function as a deterministic filter except for random signals; i.e., they remove unwanted disturbances. Figure 1.1-3 depicts the operation of a typical signal estimator. Noisy measurements are processed by the estimator to produce "filtered" data.

Consider the random data and noisy spectrum depicted in Fig. 1.1-4*a*. An estimation filter is designed to extract the signal and remove the noise. The data are processed through the filter, and the results are shown in Fig. 1.1-4*b*.* The processor has extracted the desired signal (pulse) and rejected the undesired (noise).

Estimation can be thought of as a procedure made up of three primary parts:

1. The criterion function
2. Models
3. The algorithm

* The hat notation ˆ is used to define an estimate throughout this text.

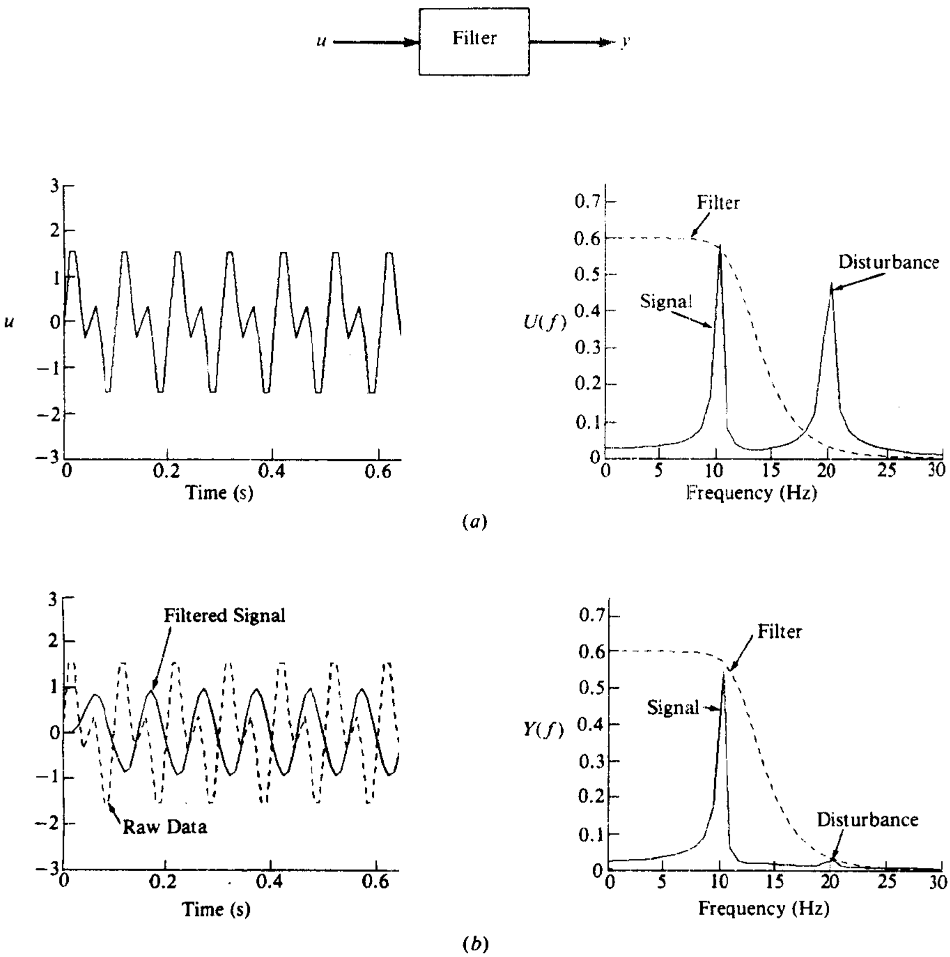

Figure 1.1-2 Processing of a deterministic signal and disturbance. (*a*) Raw data and spectrum. (*b*) Processed data (signal) and spectrum.

The criterion function can take many forms and can also be classified as deterministic or stochastic. For example, consider the well-known squared error criterion

$$J = (\text{error})^2$$

or, if the error is interpreted as random, then

$$J = \text{av}\ (\text{error})^2$$

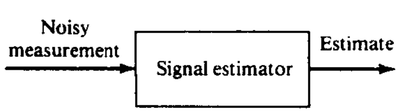

Figure 1.1-3 Typical signal estimator structure.

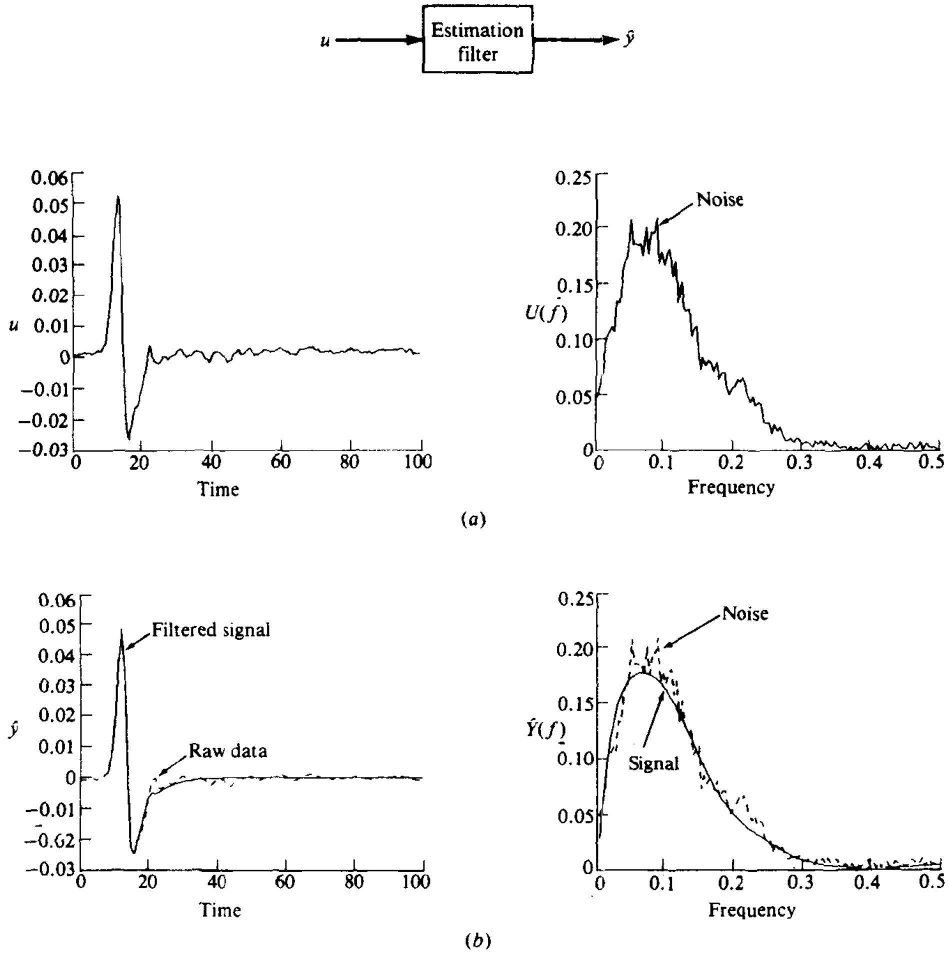

Figure 1.1-4 Processing of a random signal and noise. (*a*) Raw data and spectrum. (*b*) Processed data (signal) and spectrum.

Models represent a broad class of information formalizing the a priori knowledge about the process generating the signal, measurement instrumentation, noise characterization, underlying probabilistic structure, etc. For example, a standard signal-processing model is that of a signal in additive noise

$$\text{Measurement} = \text{signal} + \text{noise}$$

where the noise statistics are specified as well as the signal structure.

Finally, the algorithm or technique chosen to minimize (or maximize) the criterion can take many different forms depending on (1) the models, (2) the criterion, and (3) the choice of solution. For example, one may choose to solve the well-known least-squares problem recursively or with a numerical-optimization algorithm. Another important aspect of most estimation algorithms is that they provide a "measure of quality" of the estimator. Usually what this means is that the estimator also predicts vital statistical information about how well it is performing. To formalize these ideas further, consider the following example.

Example 1.1-1 Consider the design of an estimator to minimize the squared-error criterion and extract a deterministic constant from noisy measurements. Suppose we define the following models of our process from a priori information, that is,

$$\text{Measurement} = y$$

$$\text{Signal} = s \text{ (deterministic)}$$

$$\text{Noise} = n$$

The criterion function is given by

$$J = E(s - \hat{s})^2$$

We model the measurement by

$$y = s + n$$

and the noise as random (uncorrelated) and zero-mean with variance R_n. (See App. A for a review of probability concepts.) A linear estimator of the form

$$\hat{s} = ky$$

is desired and the appropriate weight k must be found. By minimizing J with respect to k, setting the derivative to zero, and solving for k, that is,

$$\frac{dJ}{dk} = 0$$

we find that

$$k = \frac{E\{sy\}}{E\{y^2\}}$$

or, in terms of the noise statistics,

$$k = \frac{s^2}{s^2 + R_n}$$

The quality of the estimate is given by $\hat{J}$, where

$$\hat{J} = E\{s^2\} - \frac{E^2\{sy\}}{E\{y^2\}}$$

Consequently, for these statistics we obtain

$$\hat{J} = \frac{s^2 R_n}{s^2 + R_n}$$

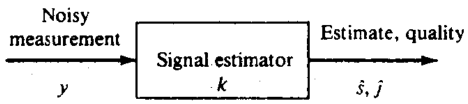

Figure 1.1-5 Estimator for unknown signal in random noise.

The structure of this estimator and process is shown in Fig. 1.1-5.

Thus estimation can be thought of as filtering random data. In the next section we discuss the estimation procedure in more detail.

1.2 ESTIMATION PROCEDURE

Intuitively, we can think of the estimation procedure as

1. The specification of a criterion
2. The selection of models from a priori knowledge
3. The development and implementation of an algorithm

Criterion functions are usually selected on the basis of information that is meaningful about the process or the ease with which an estimator can be developed. Criterion functions that are useful in estimation can be classified as deterministic and probabilistic. Some typical functions are as follows:

Deterministic:

Squared error
Absolute error
Integral absolute error
Integral squared error

Probabilistic:

Maximum likelihood
Maximum a posteriori (bayesian)
Maximum entropy
Minimum (error) variance

Models can also be deterministic as well as probabilistic; however, here we prefer to limit their basis to knowledge of the process phenomenology (physics) and the underlying probability density functions as well as the necessary statistics to describe the functions. Phenomenological models fall into the usual classes defined by the type of underlying mathematical equations and their structure, i.e., linear or nonlinear, differential or difference, ordinary or partial, time invariant or