

# **ADAPTIVE SIGNAL PROCESSING**

**Bernard Widrow**

**Samuel D. Stearns**



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# **ADAPTIVE SIGNAL PROCESSING**

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# Preface

This book has grown out of nearly three decades of research and teaching in the field of adaptive signal processing. It is designed primarily to be a basic text on adaptive signal processing and, at the time of its publication, it is believed to be the only basic text on the subject, or at least the only textbook covering the breadth of subject matter shown in the table of contents.

The book is based on class notes for a one- or two-semester senior or graduate level course in adaptive signal processing taught at Stanford University, the University of New Mexico, and Sandia National Laboratories. Every chapter except Chapter 1 has exercises at the end, and these are considered to be an essential part of any course using the text. The exercises are often used to complete the reader's understanding of a concept or to present different applications of ideas in the text.

Referring to the table of contents, the reader can see that the book is divided into four main parts. The first three parts—General Introduction, Theory of Adaptation with Stationary Signals, and Adaptive Algorithms and Structures—make up a little less than half of the text. The material in these parts is considered basic theory and would normally be included in any first course on adaptive signal processing. The fourth part—Applications—consists of six chapters on various engineering applications of adaptive signal processing. In this part the instructor may wish to concentrate on subjects of special interest. However, even in a one-semester course, the instructor will probably wish to include at least the first portion of each chapter.

For prerequisites, we assume that the student has at least senior-level academic experience in engineering and mathematics, and has the ability to write and run computer programs. The latter is essential for doing many of the exercises. A course in linear systems analysis, particularly in discrete systems with the use of the  $z$ -transform, would provide a very useful (if not essential) background. Also, a course in engineering statistics or probability, or the equivalent, provides a helpful background.

In the first part of the text, Chapter 1 introduces the concept of adaptation as a property or characteristic of certain systems in engineering. Chapter 2 introduces the

adaptive linear combiner, which is the simplest and most widely used adaptive structure. Chapter 2 also describes a geometric "performance surface" which is useful in the analysis of all adaptive systems.

Part II, Theory of Adaptation with Stationary Signals, contains an analysis of the performance surface and its properties. The analysis begins in Chapter 3, and in Chapter 4 adaptation is viewed as the process of searching the performance surface for its minimum. Chapter 5 contains a statistical analysis of gradient estimation on the performance surface and a comparison of search methods.

In Part III, Adaptive Algorithms and Structures, the least mean squares (LMS) algorithm is introduced and discussed in Chapter 6. In Chapter 7 basic signal processing concepts that are required for the rest of the book are introduced. These include primarily the  $z$ -transform relationships linking the time and frequency domains. To conclude Part III, Chapter 8 introduces adaptive algorithms other than the LMS algorithm and adaptive structures other than the adaptive linear combiner, including the adaptive lattice structure. The latter is considered, at the time of this writing, to be a rapidly developing area, and our introduction to it is therefore less comprehensive than we would wish it to be.

Finally, Part IV covers the major application areas of adaptive signal processing. Once the basics in Chapters 1–8 have been learned, subjects can be chosen selectively from Part IV. In Chapters 9 and 10, forward and inverse adaptive modeling are introduced and applied to areas such as multipath communication, geophysical exploration, digital filter design, and telephone channel equalization. Adaptive control systems are introduced in Chapter 11, and Chapter 12 introduces adaptive interference canceling, with several examples of application. Chapters 13 and 14 cover adaptive arrays and beamformers.

While writing this text, the authors have had the benefit of critiques, comments, and suggestions from many talented colleagues. We are very grateful for the reviews and ideas we have received, and thankful for the friendships engendered and increased through this work. We especially wish to acknowledge the help of Robert D. Fraser, Dennis R. Morgan, Dae H. Youn, Eugene Walach, Richard Gooch, Ruth A. David, Sharon K. Fletcher, Claude S. Lindquist, Daksheesh Parikh, Delores M. Etter, Edward S. Angel, Lloyd J. Griffiths, Nasir Ahmed, John R. Treichler, C. Richard Johnson, Jr., Michael G. Larimore, Glenn R. Elliott, John M. McCool, John M. Cioffi, and T. C. Hsia. The book would not be in its present form without the contributions of these special friends.

We also wish to thank all of the students who took the adaptive signal processing courses mentioned above. In effect, they have edited and corrected the text far beyond our ability to do so. We thank all of these students for their patience, interest, and enthusiasm.

The only ones with more patience and perseverance than our students have been the talented ladies who have typed and retyped this text, Debra Shepperd at Sandia and Mieko Parker at Stanford. We also acknowledge their help with gratitude.

**Bernard Widrow  
Samuel D. Stearns**

# List of Symbols

SYMBOL	USE(S) IN THIS BOOK
$a$	(1) forward weight in a linear filter (2) bit in genetic algorithm
$b$	(1) recursive weight in a linear filter (2) bit in genetic algorithm
$c$	(1) plant output signal (2) signal propagation velocity
$d$	(1) desired response (2) antenna element spacing
$e$	natural logarithmic base, 2.71828...
$f( )$	continuous function of
$g$	plant output signal
$h$	impulse response
$j$	$\sqrt{-1}$
$k$	sample number
$l$	(1) weight number (2) element spacing
$n$	(1) general index (2) noise sample value
$p$	total white input noise power
$r$	(1) convergence ratio in gradient search algorithms (2) uniform random number in (0, 1) (3) reference input signal
$s$	(1) signal in lattice filter (2) input signal
$t$	continuous time



$u$	(1) inverse of $z$ (2) plant input signal
$v$	translated weight, $w - w^*$
$v'$	weight value in principal axis coordinate system
$w$	weight value
$x$	input signal
$y$	output signal
$z$	variable in the $z$ -transform
$z^{-1}$	inverse of $z$ (unit delay)
$A$	(1) $z$ -transform of $a$ (2) amplitude gain
$B$	$z$ -transform of $b$
$C$	(1) function used in lattice conversion (2) constant signal amplitude
$D$	signal distortion
$E[ \ ]$	expected (mean, average) value of
$F$	transfer function
$G$	transfer function
$H$	transfer function
$I$	the identity matrix, $\text{diag} [1 \ 1 \ 1 \dots 1]$
$J$	(1) transfer function (2) jamming signal
$K$	number of beamforming elements
$L$	index of the last filter weight, $w_L$
$M$	(1) misadjustment (2) number of feedback weights
$N$	(1) number of samples per cycle (2) number of error samples taken with perturbed weights (3) number of discrete frequencies
$\mathbf{N}$	gradient noise, $\hat{\nabla} - \nabla$
$\mathbf{N}'$	$\mathbf{N}$ in principal axis coordinate system
$P$	(1) perturbation due to derivative measurement (2) estimated signal power (3) plant transfer function
$\mathbf{P}$	correlation vector of input and desired signals
$PS$	transfer function of pseudofilter
$Q$	filter quality factor
$\mathbf{Q}$	(1) eigenvector matrix of $\mathbf{R}$ (2) scaled estimated $\mathbf{R}$ matrix

$\mathbf{R}$	correlation matrix of input signal, $x$
$\mathbf{S}$	(1) matrix used in SER algorithm (2) signal vector in adaptive arrays
$\mathbf{T}$	(1) transpose of a vector or matrix (2) time step between samples in seconds
$T$	time constant of adaptation
$\mathbf{U}$	augmented signal vector
$\mathbf{V}$	translated weight vector, $\mathbf{W} - \mathbf{W}^*$
$\mathbf{V}'$	weight vector in principal axis system
$\mathbf{W}$	weight vector
$X$	z-transform of $x$
$\mathbf{X}$	input signal vector
$Y$	z-transform of $y$
$Z^{-1}[\ ]$	inverse z-transform of
$\alpha$	(1) exponential decay constant (2) output signal derivative (3) forgetting factor in SER and lattice algorithms
$\alpha_r$	$r$ th moment of $\varepsilon_k$
$\hat{\alpha}_r$	estimate of $\alpha_r$
$\beta$	(1) output signal derivative (2) adjustable gain factor
$\gamma$	(1) performance penalty (2) leakage factor
$\delta$	(1) small perturbation in a weight value (2) translated lattice filter weight (3) beam-steering delay
$\varepsilon, \epsilon$	error signal
$\kappa$	lattice filter weight
$\lambda$	(1) eigenvalue (2) wavelength
$\mu$	convergence parameter in gradient search algorithms
$\nu$	(1) convergence parameter (2) lattice filter weight
$\nu^2$	input noise power
$\xi$	mean-square error (MSE) performance function
$\hat{\xi}$	estimate of $\xi$
$\pi$	3.14159265...
$\rho$	signal-to-noise ratio
$\sigma^2$	signal variance or power

$\tau$	time constant of weight convergence
$\phi$	(1) average random signal power (2) correlation function
$\omega$	angular frequency in rad (sampling freq. = $2\pi$ )
$\Delta$	delay value
$\Theta$	phase angle (rad)
$\theta$	signal arrival angle
$\Lambda$	eigenvalue matrix, diag [ $\lambda_0 \lambda_1 \dots \lambda_L$ ]
$\Phi$	power density (z-transform of $\phi$ )
$\psi$	(1) input signal (2) signal arrival angle
$\Omega$	angular frequency in rad/s (sampling freq. = $2\pi/T$ )
*	denotes an optimal value, as in $\mathbf{W}^*$
$\nabla$	gradient vector of the performance function
$\hat{\nabla}$	estimate of $\nabla$

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