PROCEEDINGS OF THE FIRST

MARCEL GROSSMANN MEETING ON GENERAL RELATIVITY

REMO RUFFINI EDITOR

PROCEEDINGS OF THE FIRST

MARCEL GROSSMANN MEETING ON GENERAL RELATIVITY

organized and held at the

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS, TRIESTE

and

ISTITUTO DI FISICA TEORICA, UNIVERSITY OF TRIESTE
7-12 July 1975

Edited by

REMO RUFFINI

Department of Physics
University of Rome

1977

NORTH-HOLLAND PUBLISHING COMPANY
AMSTERDAM · NEW YORK · OXFORD

North-Holland ISBN: 0 7204 0707 9

Published by:
NORTH-HOLLAND PUBLISHING COMPANY – AMSTERDAM • NEW YORK • OXFORD

Distributors for the U.S.A. and Canada:
Elsevier North-Holland, Inc.
52 Vanderbilt Avenue
New York, N.Y. 10017

Library of Congress Cataloging in Publication Data

Marcel Grossmann Meeting on General Relativity, 1st, Trieste, 1975.
Proceedings of the First Marcel Grossmann Meeting on General Relativity.

Bibliography: p.
Includes index.
1. General relativity (Physics)--Congresses.
I. Ruffini, Remo. II. International Centre for Theoretical Physics. III. Triesto. Università. Istituto di fisica teorica.
QC173.6.M37 1975 530.1'1 76-57709
ISBN 0-7204-0707-9

PRINTED IN GREAT BRITAIN

PROCEEDINGS OF THE FIRST
MARCEL GROSSMANN MEETING ON GENERAL RELATIVITY

MARCEL GROSSMANN

9 April 1878 Budapest — 7 September 1936 Zurich

Marcel Grossmann studied mathematics at the Zurich Polytechnikum and earned his doctorate in 1912. He was appointed professor of descriptive geometry at the Eidgenössische Technische Hochschule in 1907; he was a teacher of outstanding ability and gave to many mathematicians their training in geometry.

Marcel Grossmann was Albert Einstein's classmate. When Einstein sought to formulate mathematically his ideas on the general theory of relativity he turned to Grossmann for assistance. Grossmann introduced Einstein to the differential calculus, started by Elwin Bruno Christoffel (1864) and fully developed at the University of Padova by Gregorio Ricci Curbastro and Tullio Levi Civita (1901). The collaboration between Einstein and Grossmann is significantly documented in their article "Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation" in Zeit. für Mathem. und Phys. 62, 3 (1913).

By allowing the encounter of the mathematical achievements of the Italian geometers and the profound physical insight of Einstein, Marcel Grossmann facilitated the unique synthesis of mathematical and theoretical physics reached by Albert Einstein in the most elegant and powerful field theory of physics: The General Theory of Relativity.

PREFACE

The Marcel Grossmann meeting was conceived with the scope of reviewing recent advances in gravitation and general relativity with major emphasis on their mathematical foundations and their physical predictions. The main objective was to elicit contributions, deepening our understanding of space-time structure, as well as review the status of experiments verifying Einstein's theory of gravitation. The volume is a record of this meeting.

The contributions range from alternative approaches to the quantization of gravity, to the theoretical significance of gravitational experiments; from recent advances in vacuum polarization in strong gravitational fields, to alternative classical microphysical structure of space-time.

The first day of the meeting was dedicated to the quantization of gravitation. Four different treatments of this problem are presented in these proceedings: the covariant approach (P. van Nieuwenhuizen), the super-space, or rather the total of configuration space approach (D. Christodoulou), the string model approach (T. Regge and C. Teitelboim) and finally the geometric of symplectic formalism approach (J.M. Souriau). More on these topics is presented in the communications section: the possible advantage of using null co-ordinates formalism (C. Aragone), a review of the traditional canonical quantization programme (M. Pilati and C. Teitelboim) and more details on the symplectic structure approach (W. Szczyrba). Related communications deal with possible new unified approach to electromagnetism and gravitation (L. Halpern), on strong gravity (A. Inomata) and Segal's cosmology (J. Tarski).

From the wealth of the approaches presented, it may be concluded that this profound and fundamental problem of quantization of gravitation is still very much at the frontier of the theoretical understanding. No consensus has yet been reached either on the approach or the techniques to be followed. Each one of the contributions presented contains some desirable feature either in respect of the direct understanding of the physics involved, or in respect of elegance and simplicity of the formalism.

The second day and part of the third day of the meeting were devoted to the progress made in our understanding of general properties of solutions of Einstein's field equations and of space-time structure. The reports cover, once again, a vast variety of topics: general properties of singularities, conjectures and their entropy and on their connection to the arrow of time (R. Penrose), the description of the angular momentum, spin and gyromagnetic ratio of a relativistic system in the manifold of complex asymptotically shear free light cones (E. Newman), a new formulation of the asymptotic conditions of an isolated system (B. Schmidt), general properties of maximal surfaces in closed and open space times (D. Brill), an analysis of the field equations and singularity properties of Bianchi universes (A. Taub), a review of the advances towards the proof of the vacuum black hole uniqueness theorem (B. Carter) and finally, in a somewhat different line, a conceivable role for torsion in general relativity (F. Hehl).

More on these topics* is presented in the communications section: modification of black holes horizons due to external fields (R. Hajicek), background field formulation of general relativity (A. Quale), electromagnetic fields in stationary geometries (B. Lukács and Z. Perjes), radiation reaction in a freely falling charge

* Not contained in the proceedings are the reports presented at the meeting by E. Lifshitz, H. Sato, E. Schücking, A. Tomimatsu and Abdus Salam, since the material presented, though highly relevant to the work of the sessions concerned, had already appeared elsewhere in print.

D. Wilkins) and finally, contributions on possible effects of torsion and spin on general relativity (C. Lopez, E. Mielke, N. Hari Dass and V. Radhakrishnan, and A. Prasanna).

In the afternoon of the third day of the meeting, a round table was held on the advances in perturbation techniques in a given stationary background metric. The following reports of this round table are contained in these proceedings: electromagnetic perturbations of a rotating black hole in the Chandrasekhar formalism (S. Detweiler), electromagnetic perturbations in Reissner-Nordstrøm spacetimes in the Newman-Penrose formalism (M. Johnston), perturbations in any analytic background using the Hadamard's elementary solution (M. Peterson and R. Ruffini), forced perturbations of a Reissner-Nordstrøm geometry in the Dirac-Arnowitt-Deser-Misner formalism (J. Weinstein).

A scan of these contributions will convince the reader that in all the topics covered, very conspicuous progress has been made in recent years. One may safely forecast that future work on singularities and their properties and on the complexification of spacetime will lead to a new understanding of entropy in cosmological models and to a simpler and deeper explanation of spinning relativistic systems and spacetime structures. In a different direction, work on the uniqueness and perturbation techniques of black holes are giving a tool of unprecedented power, by providing a complete set of eigen functions, designed to describe and predict physical processes occurring in the field of gravitationally collapsed stars. These advances could lead to the direct identification and observation of some of the novel non-linear effects of general relativity in the limit of very strong gravitational fields. Finally, the work on torsion and on the possible modifications of general relativity in microphysics is likely to promote a revival of the classical Riemann programme presented as far back as 1857 on the possibility that "the metric relations of space in the infinitely small do not conform to the hypotheses of geometry". This message could very likely play a major role in future developments of physics and general relativity, especially with regards to the quantization problems.

The last three days of the meeting were devoted to topics which have already become, or are expected to become, of paramount importance for the understanding of experimental observations in physics and astrophysics. The basic theory of relativistic magnetohydrodynamics (A. Lichnerowicz) and the extensive theoretical and numerical work on magnetohydrodynamics in the fields of black holes (J. Wilson and communications by R. Hanni and W. Kundt) have by now become the basic tools for building up models able to explain the observed x-ray fluxes from close binary systems (R. Ruffini). Similarly, the coupling between electromagnetic and gravitational radiation (Y. Choquet-Bruhat) and the analysis of processes of vacuum polarization in strong gravitational fields (T. Damour, N. Deruelle and A. Starobinsky and communications by L. Davis and by W. Zaumen) are likely to be subjects of direct experimental verification within the forthcoming years (R. Ruffini). One of the topics of great interest in the meeting was the analysis of black holes thermodynamics along the lines suggested by Christodoulou, Bekenstein and Hawking. In particular the Hawking process of black holes evaporation was discussed from varying approaches, some of which are contained in these proceedings (G. Gibbons, T. Damour, P. Davies, U. Gerlach, H. Rumpf and W. Unruh). It may be thought unlikely that the Hawking process can actually be observed in the real physical world for some time to come. The analysis of the process, even viewed as a Gedanken experiment, however, is surely likely to lead to a new, deep and unifying view of thermodynamics and relativistic field theories in a curved background.

In a final session K. Nordtvedt reviewed the theoretical significance of present day gravity experiments, F. Everitt gave a critical historical review of experiments on gravitation leading to an analysis of the most sophisticated experiments currently being performed in the solar system, while B. Partridge gave an experimentalist's review of the recent status of observational cosmology.

If one may summarize the feelings of those attending the meeting, one had the overwhelming impression that general relativity has moved from being an extremely elegant mathematical theory, all the way to having acquired the maturity of a modern field theory at the very centre of physics and astrophysics. On the one hand, nearly the totality of the recent discoveries in astrophysics seem to have found their basic explanations in the framework of general relativity. On the other, the processes of vacuum polarization in highly curved spacetimes give an occasion to formulate (in a fully relativistic language) some of the basic notion of quantum field theories (the vacuum, the concept and definition of positive and negative frequencies, the particle number operator, functional integral methods and the like). General relativity offers an important example of a complex non-linear field theory, presenting a challenge and demanding a consistent formulation as a quantized field theory, while at the same time arrogating to itself the deepest notions on the very basic structure of space and time.

It is our hope that these impressions, so sharply conveyed during the meeting, are reflected in these proceedings. Clearly, what we have not been able to reproduce is the lively and beautiful presentation of the material by the lecturers themselves. If one may select one set of lectures for mention, perhaps the most outstanding were the lectures on cosmology by E. Lifschitz: all the participants, we are sure, will keep in their memory the three superb lectures he delivered, and even more, the passionate discussions following each lecture. This was a splendid example of strong interaction between speaker and audience, leading to flashes of unsuspected insights.

We would like now to thank the people who have made this meeting possible. Besides the sponsors of the International Centre for Theoretical Physics, these include the Consorzio per l'Incremento degli Studi e delle Ricerche degli Instituti di Fisica dell'Università di Trieste, the University of Trieste and the Alfred P. Sloan Foundation.

We would also like to thank the speakers and participants who came from all parts of the world. Most of the success of the meeting was due to the efficacy of the chairmen of the sessions - B. Bertotti, K. Bleuler, S. Chandrasekhar, J. Ehlers, W. Fairbank, H. Sato and F. Zerilli - who kept an excellent equilibrium between the presentations, interventions and the general discussions, and gave, with their scientific knowledge, due perspective to important new contributions.

Our deep gratitude goes to the local organizers of the meeting in Trieste; first of all Gallieno Denardo and then to the entire staff of the International Centre for Theoretical Physics, who worked devotedly and tirelessly for its success.

Finally, the publication of these proceedings would never have been possible without massive help from T. Damour and the generous and careful secretarial help and typing work of Mrs. D. Battle and Mrs. C. Kappes.

Trieste, 18 November 1976

Remo Ruffini
Abdus Salam

List of participants

P.C. Aichelburg - Inst. für Theoretische Physik, Wien, Austria.

S. Ames - Los Alamos Scientific Laboratory, Los Alamos, U.S.A.

C. Aragone - Dep. de Fisica, Universidad S. Bolivar, Caracas, Venezuela.

A. Barnes - School of Mathematics, Dublin, Ireland.

R. Beig - Inst. für Theoretische Physik, Wien, Austria.

B. Bertotti - Istituto di Fisica Teorica, Università, Pavia, Italy.

N.T. Bishop - Dep. of Mathematics, University, Southampton, England.

K. Bleuler - Inst. für Theoretische Kernphysik, Bonn, W. Germany.

S. Bonanos - N.R.C. "Demokritos", Attiki, Greece.

D. Brill - University of Maryland, College Park, U.S.A./W. Germany.

M.R. Brown - Dep. of Astrophysics, Oxford, England.

H.A. Buchdahl - Dep. of Theoretical Physics, Aust. Nat. University, Canberra, Australia.

C. Callias - Dep. of Physics, Princeton University, U.S.A./Greece.

M. Calvani - Istituto di Astronomia, Padova, Italy.

P. Campbell - Dep. of Mathematics, University of Lancaster, Bailrigg, England.

B. Carter - D.A.F. Observatoire de Paris, Meudon, Paris, France/England.

R. Catenacci - Istituto di Fisica Teorica, Università, Pavia, Italy.

A. Chamorro - Dep. de Fisica, Universidad de Bilbao, Spain.

S. Chandrasekhar - The Enrico Fermi Institute, University of Chicago, Illinois, U.S.A.

J. Chela-Flores - Inst. Venezolano Investigaciones Cientificas, Caracas, Venezuela.

Y. Choquet-Bruhat - Dep. de Mécanique, Université de Paris, France.

D. Christodoulou - International Centre for Theoret. Physics, Trieste, Italy/Greece.

F. Cooperstock - Dep. of Physics, University of Victoria, Canada.

T. Damour - Dep. of Physics, Princeton University, U.S.A./France.

P. Davies - Dep. of Mathematics, King's College, London, England.

L.R. Davis - Dep. of Physics, Princeton University, U.S.A.

F. De Felice - Istituto di Fisica "G. Galilei", Padova, Italy.

J.C. Dell - Physics Dep., University of Maryland, College Park, U.S.A.

L. De Luca - Istituto di Macchine, Università, Milano, Italy.

G. Denardo - Istituto di Fisica Teorica, Università, Trieste, Italy.

W. Deppert - Philosophisches Seminar der Universität Kiel, W. Germany.

N. Deruelle - Ecole Normale Superieure de Jeunes Filles, Paris, France.

S. Detweiler - Dep. of Physics, University of Maryland, College Park, U.S.A.

W. Dietz - Physikalisches Institut der Universität, Würzburg, W. Germany.

C.T.J. Dodson - Dep. of Mathematics, University of Lancaster, Bailrigg, England.

P. Dolan - Dep. of Mathematics, Imperial College, London, England.

J.S. Dowker - Dep. of Theoretical Physics, University of Manchester, England.

C. Duval - Dep. de Physique, Université de Marseille, France.

R. Ebert - Physikalisches Institut der Universität, Würzburg, W. Germany.
J. Ehlers - Max-Planck Institut für Physik, Munich, W. Germany.
J. Elhadad - U.E.R. Mathématiques, Université de Provence, Marseille, France/Marocco.
H.G. Ellis - Dep. of Mathematics, University of Colorado, Boulder, U.S.A.
C.W.F. Everitt - W.W. Hansen Laboratories, Stanford University, Cal., U.S.A./England.
W.M. Fairbank - Physics Dep., Stanford University, Cal., U.S.A.
E. Fliche - U.E.R. Mathématiques, Université de Provence, Marseille, France.
H. Friedrich - Max-Planck Institut für Physik, Munich, W. Germany.
W. Garczynski - Institute of Theoretical Physics, University of Wrocław, Poland.
J. Geheniau - Université Libre de Bruxelles, Belgium.
U. Gerlach - Dep. of Mathematics, Ohio State University, Columbus, U.S.A.
G.W. Gibbons - D.A.M.T.P., University of Cambridge, England.
L. G'rardello - Istituto di Fisica, Università di Milano, Italy.
A. Goddard - Mathematical Institute, Oxford, England.
R. Goldoni - Scuola Normale Superiore, Pisa, Italy.
W. Graf - Fachbereich Physik der Universität, Konstanz, W. Germany.
R. Grassini - Istituto di Matematica "Renato Caccioppoli", Napoli, Italy.
F. Grilli - Via Montanari 29, Mirandola, Modena, Italy.
P. Hajicek - Institute for Theoret. Phys., Univ. of Berne, Switzerland/Czechoslovakia.
R. Hakim - Section d'Astrophysique, Observatoire de Paris, Meudon, France.
L. Halpern - Dep. of Physics, Florida State University, Tallahassee, U.S.A./Austria.
R. Hanni - Physics Dep., Stanford University, Cal., U.S.A.
N.D. Hari Dass - Max-Planck Institut, Munich, W. Germany.
F.W. Hehl - Institut für Theoretische Physik, Clausthal-Zellerfeld, W. Germany.
J. Hoek - Instituut voor Theoretische Fysica, Amsterdam, The Netherlands.
A. Inomata - Dep. of Physics, University of New York, Albany, U.S.A./Japan.
M.D. Johnston - Massachusetts Institute of Technology, Cambridge, U.S.A.
C.-G. Källman - NORDITA, Copenhagen, Denmark/Finland.
I. Khan - Dep. of Physics, University of Khartoum, Sudan/Pakistan.
F.S. Klotz - Dep. of Applied Mathematics, Dublin, Ireland/U.S.A.
E. Krotscheck - Institut für Theoretische Physik, Hamburg, W. Germany/Austria.
W. Kundt - Institut für Theoretische Physik, Hamburg, W. Germany.
J. Ławrynowicz - Polish Academy of Sciences, Łodź Branch, Poland.
C.W. Lee - Dep. of Mathematics, University of Lancaster, Bailrigg, England.
A. Lichnerowicz - College de France, Paris, France.
E.M. Lifshitz - Institute for Physical Problems, Moscow, U.S.S.R.
C.A. Lopez - Dep. de Fisica, Universidad de Chile, Santiago, Chile.
R. Mansouri - Institut für Theoretische Physik der Universität Wien, Austria/Iran.
M. Martellini - Istituto di Scienze Fisiche, Università di Milano, Italy.
M.A. Mashkour - Dep. of Physics, University of Mosul, Iraq.
P. Mathieu - Université Catholique de Louvain, Belgium.

E.W. Mielke - Institut für Reine und Angewandte Kernphysik, Universität Kiel, W. Germany.

H. Müller Zum Hagen - Hochschule der Bundeswehr Hamburg, W. Germany.

A. Nduka - Dep. of Physics, University of Ife, Nigeria.

E.T. Newman - Dep. of Physics, University of Pittsburgh, U.S.A.

K. Nordtvedt - Dep. of Physics, Montana State University, Bozeman, U.S.A.

E. Nowotny - Fachbereich Physik, Universität Konstanz, W. Germany.

M.A. Obeid - College of Engineering, Riyadh, Saudi Arabia.

F. Occhionero - Consiglio Nazionale delle Ricerche, Roma, Italy.

H.C. Ohanian - Rensselaer Polytechnic Institute, Troy, New York, U.S.A.

J. O'Hanlon - Dep. of Applied Mathematics, University of Dublin, Ireland.

H. Okamura - Dep. of Physics, Kogakuin University, Tokyo, Japan.

D. Papadopoulos - Astronomy Dep., University of Thessaloniki, Greece.

L. Papaloucas - Mathematics Institute, University of Athens, Greece.

G. Papini - Dep. of Physics, University of Regina, Canada.

R.B. Partridge - Astronomy Dep., Haverford College, U.S.A.

R. Penrose - Mathematical Institute, Oxford, England.

Z. Perjes - Central Research Institute for Physics, Budapest, Hungary.

M.J. Perry - University of Cambridge, England.

S. Persides - Astronomy Dep., University of Thessaloniki, Greece.

M. Peterson - Physics Dep., Amherst College, U.S.A.

G. Platania - Osservatorio Astronomico di Capodimonte, Napoli, Italy.

L. Platania - Dep. of Mathematics, Università di Cosenza, Italy.

A.R. Prasanna - Institut für Theoretische Kernphysik, Universität Bonn, W. Germany/India.

A. Qadir - Mathematics Dep., Islamabad University, Pakistan.

A. Quale - Institute of Physics, Oslo University, Norway.

H. Quintana - Dep. of Astronomy, University of Chile, Santiago, Chile.

T. Regge - Institute for Advanced Study, Princeton, U.S.A./Italy.

L. Rodrigues - Faculté des Sciences Mirande, Dijon, France/Brazil.

W. Roos - Institut für Theoretische Physik, Universität Hamburg, W. Germany.

M. Rosenbaum - Universidad Nacional Autonoma de Mexico, Mexico.

F. Rosso - Mathematical Institute, Università di Napoli, Italy.

R. Ruffini - Dep. of Physics, Princeton, U.S.A./Italy.

H. Rumpf - Institut für Theoretische Physik der Universität Wien, Austria.

M.P. Ryan - Universidad Nacional Autonoma de Mexico, Mexico/U.S.A.

F. Sacchetti - Comitato Nazionale per l'Energia Nucleare, Roma, Italy.

J.L. Safko - Dep. of Physics, University of South Carolina, Columbia, U.S.A.

Abdus Salam - International Centre for Theoretical Physics, Trieste, Italy/Pakistan.

N.G. Sanchez - Institute of Astronomy, Buenos Aires, Argentina.

H. Sato - Res. Institute for Fundamental Physics, Yukawa Hall, Kyoto, Japan.

B. Schmidt - Max-Planck Institut für Physik, Munich, W. Germany.

E.L. Schucking - Dep. of Physics, New York University, U.S.A./W. Germany.
H.-J. Seifert - Hochschule der Bundeswehr Hamburg, W. Germany.
A. Selloni - Istituto di Fisica "G. Marconi", Roma, Italy.
M. Soffel - Johann W. Goether University, Frankfurt/Main, W. Germany.
J.M. Souriau - Centre de Physique Théorique, Marseille, France.
P. Spindel - Faculté des Sciences, Université de l'Etat à Mons, Belgium.
E. Streeruwitz - Institut für Theoretische Physik der Universität Wien, Austria.
G. Strini - Istituto di Fisica, Università di Milano, Italy.
W. Szczyrba - Polish Academy of Sciences, Warsaw, Poland.
J. Tafel - Uniwersytet Warszawski, Instytut Fizyki Teoretycznej, Warsaw, Poland.
A.H. Taub - Mathematics Dep., University of California, Berkeley, U.S.A.
C. Teitelboim - Dep. of Physics, Princeton University, U.S.A./Chile.
L.R. Thomlinson - King's College, Cambridge, England.
H. Tilgner - Mathematische Physik, Freie Universität Berlin, W. Germany.
A. Tomimatsu - Res. Institute for Theoretical Physics, Takehara, Hiroshima, Japan.
D.A. Tranah - Dep. of Mathematics, University of Southampton, England.
A. Treves - Università di Milano, Italy.
D. Trevese - Osservatorio Astronomico di Roma, Italy.
M.J. Trinkala - Physics Dep., State University of New York, Albany, U.S.A.
W.G. Unruh - McMaster University, Hamilton, Ontario, Canada/U.S.A.
P. Urban - Institut für Theoretische Physik der Universität Graz, Austria.
H. Urbantke - Institüt fur Theoretische Physik der Universität Wien, Austria.
F. Vagnetti - Laboratorio Astrofisica Spaziale, Frascati, Italy.
E. Van Der Spuy - Atomic Energy Board, Pretoria, South Africa.
P. Van Nieuwenhuizen - Brandeis University, Waltham, Mass., U.S.A./The Netherlands.
L. Verhoustraete - Institut de Physique Théorique, Louvain, Belgium.
J. Weinstein - Dep. of Physics, Princeton University, U.S.A.
D.C. Wilkins - University of California, Santa Barbara, Cal., U.S.A.
R.M. Williams - Girton College, Cambridge, England.
J.R. Wilson - Lawrence Livermore Laboratory, Livermore, Cal., U.S.A.
W.T. Zaumen - Lockheed Palo Alto Res. Laboratory, Cal., U.S.A.
F. Zerilli - Dep. of Physics, University of Washington, Seattle, U.S.A.
D. Zivanovic - Institute of Physics, Beograd, Yugoslavia.
A. Zupancic - Istituto di Fisica Teorica, Università di Trieste, Italy.

CONTENTS

AN INTRODUCTION TO COVARIANT QUANTIZATION
OF GRAVITATION*

P. van Nieuwenhuizen

Institute for Theoretical Physics
State University of New York at
Stony Brook, L.I., New York 11794

CONTENTS

1. INTRODUCTION

Gravitation is the gauge field theory describing massless self-interacting spin 2^+ bosons, just as Yang-Mills fields describe massless self-interacting spin 1^- bosons and quantumelectrodynamics massless non-self-interacting spin 1^- bosons. During the past five years important developments have been made in the general theory of covariant quantization of gauge fields, the main results obtained being of course the renormalization of field theories which unify the weak and electromagnetic interactions. Also for gravitation, however, our advancement in understanding quantized gauge theories has had significant consequences. In this review we will discuss some of the applications of these new ideas about covariant quantization to gravitation as a theory of particle physics.†

Covariant quantization of a gauge theory consists of deriving Feynman rules for the propagators and vertices of this gauge theory. Once the propagators and vertices are given, Feynman diagrams can be constructed in terms of which the S-matrix can be expressed. One may then investigate whether the theory is renormalizable, i.e. whether one can eliminate the unavoidable infinities of loop diagrams in such a way that the theory gets some predictive power. Covariant quantization is not the only way to quantize gauge field theories; one can also use canonical quantization (in which case one determines the p's and the q's of the theory). When one wants to determine S-matrix elements, covariant quantization is by far the simpler scheme. For Yang-Mills theory (and electrodynamics) both quantization schemes have been shown to be equivalent [1] but for gravitation

*Supported in part by the NSF Grant #MPS-74-13208 A01.

†In particle physics a distinction is sometimes made between relativistic particle theory and relativistic field theory. In this review we always talk about relativistic field theory.

a similar proof is still absent in the literature. Most physicists believe that this is only a matter of time.

According to the particle physics approach, gravitons are treated on exactly the same basis as other particles such as photons and electrons. In particular, particles (including gravitons) are always in flat Minkowski space and move as if they followed their geodesics in curved spacetime because of the dynamics of multiple graviton exchange.† This particle physics approach is entirely equivalent to the usual geometrical approach. Pure relativists often become somewhat uneasy at this point because of the following two aspects entirely peculiar to gravitation:

1. In canonical quantization one must decide before quantization which points are spacelike separated and which are timelike separated, in order to define the basic commutation relations. However, it is only after quantization that the fully quantized metric field can tell us this spacetime structure. It follows that the concept of space- or time-like separation has to be preserved under quantization, and it is not clear whether this is the case. (One might wonder whether the causal structure of space-time need be the same in covariant quantization as in canonical quantization.)
2. Suppose one wanted to quantize the fluctuations (for example of a scalar field, or even of the gravitational field itself) about a given curved classical background instead of about flat Minkowski space-time. In order to write the field operators corresponding to these fluctuations in second-quantized form, one needs positive and negative frequency (annihilation and creation) solutions. In non-stationary spaces it is not clear whether one can define such solutions. (It may help to think of non-stationary space-time as giving rise to a time-dependent Hamiltonian.)

The strategy of particle physics has been to ignore these two problems for the time being, in the hope that they will ultimately be resolved in the final theory. Consequently we will not discuss them here any further.

Historically, there have been many (and often conflicting) opinions on whether and how to quantize gravitation. Heisenberg [2] already pointed at that feature which we know nowadays to be the chief problem of quantum gravity: the dimension of an inverse mass of the gravitational coupling constant κ. He

---

†As suggested by this picture, only asymptotically flat spaces are considered. For any finite number of gravitons the corresponding amplitudes are singularity free. It is only by summing the whole perturbation series that one can expect to produce such singularities as for example are present in the Schwarzschild solution. Most of the results we will discuss are to first order in quantum effects only (one-loop diagrams). Consequently we expect that these results are only adequate approximations to solutions which are singularity free.

predicted that this would force gravitation to be nonrenormalizable. Pauli, Salam and others [3,4] on the other hand hoped that gravitation, when coupled to matter, would quench all infinities, including its own. Wheeler [5] suggested that one should quantize the geometry of spacetime itself rather than the fields in it ("no fixed arena"). Penrose [6] goes even further; if I interpret his work correctly, he wants to quantize the points themselves. (Since in his approach points are intersections of two quantized twistors, they are no longer sharply defined but subject to quantum oscillations.) Some people believe that gravitation is something like van der Waals forces: present where matter is, but not propagating in free radiation modes (which might then explain the limited success in detecting gravitational radiation). Others, like Zumino [7], have speculated that general relativity is a low-energy phenomenological reflection of an unknown deeper-lying renormalizable theory. Weinberg [8] considers the possibility that the graviton is a bound state, with the extra positive-metric zero helicity part of the gravitational field serving to cancel a negative-metric Goldstone boson generated by spontaneous symmetry breakdown of scale invariance. (The generation of a scalar boson, the dilation, is known in the literature [9], but a tensor boson has not yet been produced.)

This review was written for pure relativists who are not familiar with particle physics techniques. Most results are therefore stated without derivations and the physical aspects are stressed. Time and space being finite, such interesting topics as nonpolynomial Lagrangians [4,10,11] (Delbourgo, Isham, Salam and Strathdee and others), f-gravity [12], super-symmetry in gravitation (Arnowitt, Nath and Zumino) and particle creation in curved spacetime (Hawking, DeWitt, Boulware, and others) have been omitted with reluctance. This seemed to us necessary in order to exhibit in a clear way the fundamental problems. In particular all results will be presented in the language of normal field theory, although many results were first obtained in the background [13-23] field formalism of DeWitt. Although this elegant formalism greatly reduces the horrendous algebra of quantum gravity, most physicists are still more familiar with normal field theory.

The following conventions are used in this review: $\delta_{\mu\nu} = (+1,+1,+1,+1)$ and $p^1 \cdot p^2 = \vec{p}^1 \cdot \vec{p}^2 + p^1_4 p^2_4$. The scalar curvature R is related to the Ricci tensor $R_{\mu\nu}$ by $R = g^{\mu\nu} R_{\mu\nu}$ with $R_{\mu\nu} = R^{\alpha}{}_{\mu\nu\alpha}$ and the Riemann-Christoffel tensor is defined by

$$R^{\alpha}{}_{\mu\nu\beta} = (\partial_{\nu}\Gamma^{\alpha}_{\mu\beta} + \Gamma^{\alpha}_{\nu\lambda}\,\Gamma^{\lambda}_{\mu\beta}) - (\,\nu \leftrightarrow \beta\,)$$

The Christoffel symbols are defined by

$$\Gamma^{\sigma}_{\alpha\beta} = \tfrac{1}{2} g^{\sigma\rho}\,[\,\partial_{\alpha}\, g_{\beta\rho} + \partial_{\beta} g_{\alpha\rho} - \partial_{\rho}\, g_{\alpha\beta}]\ .$$

The Einstein field equations read $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R = -\frac{\kappa^2}{4}\, T_{\mu\nu}$ where $T_{\mu\nu}$ is the