HANDBOOK OF HEAT TRANSFER

EDITED BY

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Section 1

Introduction

Part A

Basic Concepts of Heat Transfer

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1. Types of Heat Transfer Mechanisms

Heat is defined as energy transferred by virtue of a temperature difference or gradient and is vectorial in the sense that it flows from regions of higher temperature to regions of lower temperature. The basic modes of heat transfer are *conduction* and *radiation*.

Conduction is the transfer of heat from one part of the body at a higher temperature to another part of the same body at a lower temperature, or from one body at a higher temperature to another body at a lower temperature in physical contact with it. The conduction process takes place at the molecular level and involves the transfer of energy from the more energetic molecules to those with a lower energy level. This can be easily visualized within gases where we note that the average kinetic energy of molecules in the higher-temperature regions is greater than those in the lower temperature regions. The more energetic molecules, being in constant and random motion, periodically collide with molecules of a lower energy level and exchange energy and momentum. In this manner there is a continuous transport of energy from the high temperature regions to those of lower temperature. In liquids the molecules are more closely spaced than in gases, but the molecular energy exchange process is qualitatively similar to that in gases. In solids which are non-conductors of electricity (dielectrics), heat is conducted by lattice waves caused by atomic motion. In solids which are good conductors of electricity this lattice vibration mechanism is only a small contribution to the energy transfer process, with the principal contribution being that due to the motion of free electrons which move about in the same way as molecules in a gas.

At the macroscopic level we state that the heat flux is proportional to the temperature gradient with the proportionality factor being identified as the thermal conductivity k:

$$\frac{q}{A} = -k\left(\frac{\partial T}{\partial y}\right) \tag{1}$$

This relationship is used for the conduction process in solids, liquids, and gases. From the foregoing, as one might expect, the magnitude of the thermal conductivity of electrically conducting solids is higher than for dielectrics and solids in general have higher conductivity than liquids.

In treating conduction problems it is often convenient to introduce another property which is related to the thermal conductivity, namely the thermal diffusivity α

$$\alpha = \frac{k}{\rho c} \tag{2}$$

Here ρ is the density and c is the specific heat.

Radiation, or more correctly thermal radiation, is electromagnetic radiation emitted by a body by virtue of its temperature. Thus thermal radiation is of the same nature as visible light, X-rays, and radio waves, the difference between them being in their wave lengths. The eye is sensitive to electromagnetic radiation in the region from 35 to 75 microns; this is identified as the visible regions of the spectrum. Radio waves have a wave length of 10^4 microns and above, X-rays have wave lengths of 0.01 to 1, while thermal radiation occurs in rays from 0.1 to 100 microns. All heated solids and liquids as well as some gases emit thermal radiation. On the macroscopic level, the calculation of thermal radiation is based on the Stefan-Boltzmann law which relates the energy flux emitted by an ideal radiator to the fourth power of the absolute temperature

$$e_b = \sigma T^4 \tag{3}$$

Here σ is the Stefa Boltzmann constant. Engineering surfaces in general do not perform as ideal radiat s and for real surfaces the above law is modified to read

$$e = \epsilon \sigma T^4 \tag{4}$$

The term ϵ is called e emissivity of the surface with a value between 0 and 1.

Convection, somet nes identified as a separate mode of heat transfer, relates to the transfer of heat fron a bounding surface to a fluid in motion, or to the heat transfer across a flow plane ithin the interior of the flowing fluid. If the fluid motion is induced by a pump, lower, fan, or some similar device, the process is called forced convection. If the fluid motion occurs as a result of the density differences produced by the heat transfer itself, the process is called free or natural convection. Detailed inspection of the heat transfer process in these cases reveals that the basic heat transfer mechanisms are conduction and radiation, both of which are generally influenced by the fluid motion. In convective processes involving heat transfer to or from a boundary surface exposed to a low velocity fluid stream, it is convenient to introduce a heat transfer coefficient h defined by Eq. (5), which is known as Newton's law of cooling

$$\frac{q}{A} = h(T_f' - T_s) \tag{5}$$

Here T_s is the surface temperature and T_f is a characteristic fluid temperature.

For surfaces in unbounded convection, such as plates, tubes, bodies of revolution, etc., immersed in a large body of fluid, it is customary to define h in Eq. (5) with T_f being the temperature of the fluid far away from the surface often identified as $T_{f\infty}$. For bounded convection, such as fluids flowing in tubes, channels, across tubes in bundles, etc., T_f is usually taken as the enthalpy-mixed-mean temperature, customarily identified as T_m.

The heat transfer coefficient so defined may include both radiation and conductive contributions. If the radiation contribution is negligible, then the total transfer is due to conduction. In this case we may note

$$h = \frac{q/A}{T_f - T_s} = \frac{-k(\partial T/\partial y)_s}{T_f - T_s} \tag{6}$$

The heat transfer coefficient is then recognizable as the gradient of dimensionless temperature at the surface. It is sensitive to the geometry, to the physical properties of the fluid, and to the fluid velocity.

For convective processes involving high velocity gas flows (high subsonic or supersonic), a more meaningful and useful definition of the heat transfer coefficient is given by

$$\frac{q}{A} = h(T_r - T_s) \tag{5a}$$

Here T_r, commonly called the adiabatic wall temperature or the recovery temperature, is the equilibrium temperature the surface would attain in the absence of any heat transfer to or from the surface and in the absence of radiation exchange between the surroundings and the surface. In general the adiabatic wall temperature is dependent on the fluid properties and the properties of the bounding wall. Generally, the adiabatic wall temperature is reported in terms of a dimensionless recovery factor r defined as

$$T_r = T_f + r\frac{V^2}{2C_p}$$

The value of r for gases normally lies between 0.8 and 1.0. It can be seen that for low-velocity flows the recovery temperature is equal to the free stream temperature T_f. In this case, Eq. (5a) reduces to Eq. (5). From this point of view, Eq. (5a) can be taken as the generalized definition of the heat transfer coefficient.

In some physical situations, it is possible to determine analytically the details of the flow field and the temperature distribution and thereby to evaluate the heat transfer coefficient h. In these cases which are amenable to analysis, if the heat transfer process involves both radiation and conduction it is convenient to determine the magnitude of each mode and to define the heat transfer coefficient in terms of the conductive contribution alone as given by Eq. (6). Needless to say, in determining the total heat transfer the radiation contribution must be added. Unfortunately, in many engineering problems the convective heat transfer cannot be determined analytically but must be evaluated by experiment. In such cases, if radiation is an important mechanism it is usually impossible to separate the conductive and radiative modes and the heat transfer coefficient may not be interpreted as the dimensionless temperature gradient; rather it is defined by Eq. (5). The chapters on forced and free convection will concentrate on the determination of the heat transfer coefficient.

2. Rate Equations

Equation (1) for heat conduction relates the heat flux to the temperature gradient. It is an example of the type of phenomenological equation which is used for the prediction of other rate processes. Of particular interest here are the rate equations for transfer of momentum and mass.

a. Momentum Transfer. Consider a fluid confined between two parallel plates separated by a distance S with the bottom plate at rest and the top one moving with a velocity V. Under steady-state conditions the velocity distribution will be linear as shown in Figure 1. To sustain the motion, a force F must be exerted on the top plate

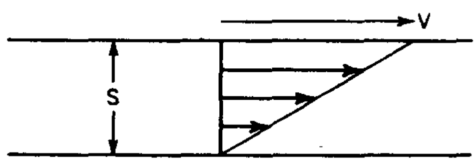

Fig. 1.

and an equal and opposite force is transmitted to the plate at rest. The ratio of this force to the area of the plate F/A is called the shearing stress τ and for many fluids of engineering importance its magnitude is directly proportional to the velocity V and inversely proportional to the plate spacing S

$$\tau = \mu \frac{V}{S} \tag{7}$$

The proportionality factor μ is called the dynamic viscosity and is a property of the fluid. Fluids such as water and air which conform to this relationship are called Newtonian fluids. There are other more complex fluids called non-Newtonian which follow more complex laws and these will be treated in later chapters. Extending the basic Newtonian law to a more general fluid motion we may write

$$\tau = \mu \left(\frac{dV_x}{dy}\right) \tag{8}$$

This shearing stress τ may be interpreted in terms of the transport of momentum ρV_x. The faster moving molecules transfer momentum to their slower moving neighbors and consequently there is a net transfer of x momentum in the y direction. This may be seen from the Newtonian relationship restated as follows

$$\tau = \left(\frac{\mu}{\rho}\right)\frac{d}{dy}(\rho V_x) \tag{9}$$

The ratio (μ/ρ) has been given its own identity, and is called the kinematic viscosity,

$$\nu = \frac{\mu}{\rho} \tag{10}$$

The dimensions of kinematic viscosity are L^2/T which in the English system of units becomes ft^2/hr. For liquids, both the dynamic viscosity μ and the kinematic viscosity ν are primarily dependent on temperature and are relatively insensitive to pressure except in the neighborhood of the critical point. For gases the dynamic viscosity is also temperature dependent, showing little sensitivity to pressure whereas the kinematic viscosity is strongly dependent on both temperature and pressure, being inversely proportional to pressure. Generally, the dynamic viscosity is higher for liquids than for gases, while the kinematic viscosity of gases tends to be higher than for liquids. As an example, at atmosphere pressure and 70°F the dynamic viscosity of water is approximately 50 times higher than that of air, while the kinematic viscosity of air is approximately 10 times that of water.

b. Mass Transfer. Consider a stagnant pure dry gas positioned between two parallel surfaces separated by a distance S. The lower surface is wetted, which can be accomplished by using a porous wick for this surface or by maintaining a thin layer of liquid on a solid surface. The upper surface is so selected that it is capable of absorbing any vapor which may be transferred from the lower surface. The partial density of the vapor immediately above the wetted surface is maintained at C_s, while the partial density at the ideal absorbing surface is found to be negligible. If these conditions are maintained for a sufficiently long period, a steady state will ensue in which it will be found that the partial density profile is a linear one. Under these conditions, the amount of vapor transported from the lower surface to the upper one is found to be directly proportional to the value of C_s and inversely proportional to the plate spacing

$$\frac{W}{A} = D\frac{C_s}{S} \tag{11}$$

Here W is the mass of vapor transported and A is the surface area. The proportionality factor D is called the mass diffusivity or the ordinary coefficient of diffusion, having units of L^2/T or ft^2/hr in the English system. If this relationship is extended to more complex systems we have

$$\frac{W}{A} = -D\frac{\partial C}{\partial y} \tag{12}$$

This relation is called Fick's Law of Diffusion.

3. Basic Fluid Mechanics

In dealing with convection problems it is important to have an understanding of the behavior of fluids in motion over external surfaces or through enclosed channels. For liquids or gases flowing over external surfaces under continuum conditions, it will be found that the relative velocity between the surface and the fluid goes to zero at the surface. Moving away from the surface, the velocity increases rapidly toward the free-stream value, effectively reaching the free-stream value at a distance S not far from the surface. The thin region where the velocity is varying is called the boundary layer, a term suggested by Prandtl who first recognized this basic phenomenon. Since the shearing stress is proportional to the product of the viscosity and the velocity gradient, it is clear that substantial shearing stresses will occur only in the boundary layer where a velocity gradient exists, whereas outside the boundary the shearing stress will be vanishingly small. Accordingly, it may be stated that the effects of viscosity are confined to the boundary layer, whereas outside the boundary layer the flow may be considered to be inviscid. Thus in analyzing the flow field over external surfaces, the inviscid flow equations may be used to predict the free-stream flow field. The resulting velocity distribution may then be used in conjunction with the boundary layer equations which include the influence of viscosity for the prediction of the flow field in the immediate vicinity of the wall. In this manner the drag on external surfaces can be determined.

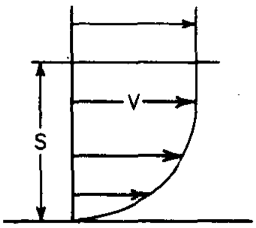

Fig. 2.

In the case of heat transfer (or mass transfer) to or from external surfaces placed in a flow field, it will be found that there is a thermal boundary layer (or concentration boundary layer) analogous to the velocity boundary layer, within which the influence of thermal conductivity (or diffusivity) is confined. Outside this region the flow is essentially nonconducting and nondiffusing.

In the case of a fluid flow through an enclosed channel, a boundary layer begins at the channel entrance. In this entrance region there is an inviscid core flow and a

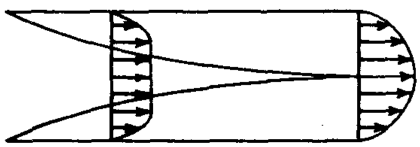

Fig. 3.

viscous boundary layer. Some distance downstream the boundary layers grow together and the velocity is at a maximum in the central region of the duct, decreasing to a value of zero at the bounding surfaces.

a. Laminar and Turbulent Flows. Osborne Reynolds in 1883 reported that there are two basically different types of fluid motion which he identified as laminar flow and turbulent flow. For example, in the case of flow over a flat plate geometry the boundary layer motion near the leading edge of the plate is smooth or streamlined. Locally within the boundary layer the velocity is constant and invariant with time.

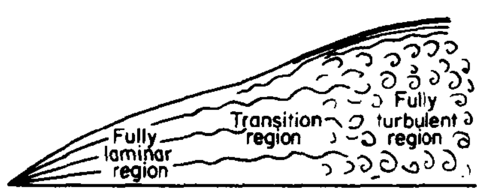

Fig. 4.

In this region momentum and energy transfer occur by a diffusion process as described by the Newtonian shearing stress law and by the Fourier conduction relationship. This is the region of laminar flow. If the plate is long enough or the velocity sufficiently high and we proceed far downstream, the nature of the flow is markedly changed. At any point in the boundary layer the velocity varies with time about some mean value as shown in Fig. 5. The exchange of momentum and energy is now no longer controlled by diffusional processes. Rather macroscopic eddies randomly move from one fluid

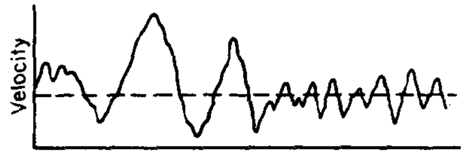

Fig. 5.

layer to another, and in the process momentum and energy are transferred. The analysis of transport processes in turbulent flows is inherently more difficult than in the laminar cases and, in general, the treatment is semi-empirical in nature.

The flow does not change abruptly from laminar to turbulent motion, but rather there is an intermediate region connecting the well-defined laminar and the well-defined turbulent motion. This is the transition region. It has been found that the laminar boundary layer begins to experience transition where the dimensionless quality $(u_e x/\nu)$, called the critical Reynolds number for flow over external surfaces, is of the order of 500,000, but this is dependent on the level of turbulence in the free stream.

For flow in circular tubes, it has been found that the flow is generally laminar if the Reynolds number $\bar{u}\, d/\nu$, where $\bar{u}$ is mean velocity, d is pipe diameter, and ν is kinematic viscosity, is lower than 2,300. If this Reynolds number is greater than 10,000, the flow is considered to be fully turbulent. In the 2,300 to 10,000 region, the flow is described as transition flow. It is possible to shift these Reynolds values by minimizing the disturbances in the inlet flow, but for general engineering applications the numbers cited are representative.

b. Flow Separation. In region of adverse pressure gradient such as encountered in flow over curved bodies, the boundary layer, in effect, separates from the surface. At this location the shear stress goes to zero and beyond this point there is a reversal of flow in the vicinity of the wall as shown in Fig. 6. In this separated region, the boundary layer equations are no longer valid and the analysis of the flow is generally very difficult.

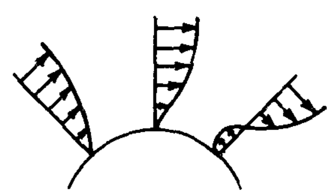

Fig. 6.

4. Units

Generally, the English system of engineering units is used throughout the handbook, although in a few chapters the authors have selected other systems. To assist the handbook user, a conversion table (Table 2) is given to aid rapid calculation in any system of units. Furthermore, when possible, engineering results are presented in a dimensionless fashion, independent of the unit system, and it is a relatively straightforward matter to proceed from the dimensionless number to the desired dimensioned quantity. A listing of dimensionless groups frequently encountered in heat transfer is given in Table 1.

5. General Equations

The following are some of the general equations encountered in heat transfer. They have been collected in this section for use as a ready reference.

a. Continuity Equations.

Vector form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

or

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{V} = 0$$

Incompressible

$$\nabla \cdot \mathbf{V} = 0$$

TABLE 1. Dimensionless Groups

Group*	Symbol	Name
$\Delta p/\rho V^2$	Eu	Euler number
$\alpha t/r_0^2$	Fo	Fourier number
$(L/d)(k/Vd\rho c_p)$	Gz [= (L/d)/RePr]	Graetz number
$g\beta(\Delta T)L^3\rho^2/\mu^2$	Gr	Grashof number
λ/L	Kn	Knudsen number
α/D	Le	Lewis number
V/V_{sound}	Ma	Mach number
hL/k, hd/k	Nu	Nusselt number
$Vd\rho c_p/k$	Pe (= RePr)	Peclet number
$c_p\mu/k$	Pr	Prandtl number
$g\beta(\Delta T)L^3\rho^2 c_p/\mu k$	Ra (= GrPr)	Rayleigh number
$\rho VD/u$, $\rho VL/\mu$	Re	Reynolds number
$\mu/\rho D$	Sc	Schmidt number
$h_p d/D$	Sh	Sherwood number
$h/c_p G$	St (= Nu/RePr)	Stanton number
$V_\infty^2/C_p(\Delta T)_0$	E	Eckert number
V^2/gL	Fr	Froude number
$f_r d/V$	St	Strouhal number
$\rho V^2 L/\sigma$	We	Weber number

*f_r = frequency of oscillation
σ = surface tension.

Cartesian: $$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

Cylindrical: $$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_r\frac{\partial}{\partial r} + \frac{v_\theta}{r}\frac{\partial}{\partial \theta} + v_z\frac{\partial}{\partial z}$$

Spherical: $$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_r\frac{\partial}{\partial r} + \frac{v_\theta}{r}\frac{\partial}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial}{\partial \phi}$$

Cartesian: u, v, and w are the velocities in the x, y, and z directions respectively.

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

Incompressible

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Cylindrical: v_r, v_θ, and v_z are the velocities in the r, θ, and z directions respectively.

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Incompressible

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

Spherical: v_r, v_θ, and v_ϕ are the velocities in the r, θ, and ϕ directions respectively

$$\frac{\partial\rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\rho v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(\rho v_\phi) = 0$$

Incompressible

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} = 0$$

b. Momentum Equations (Navier-Stokes).

P = pressure
F = body force per unit volume
μ = viscosity
λ = second coefficient of viscosity ($\lambda = -\frac{2}{3}\mu$, monatomic gas)
$\zeta = \lambda + \frac{2}{3}\mu$ (= zero for monatomic gas)

Vector form

$$\rho\frac{D\mathbf{V}}{Dt} = \rho\left[\frac{\partial\mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V}\right] = \rho\left[\frac{\partial\mathbf{V}}{\partial t} + \nabla\left(\frac{V^2}{2}\right) - \mathbf{V}\times(\nabla\times\mathbf{V})\right]$$

$$= -\nabla P + \mathbf{F} - \nabla\times[\mu(\nabla\times\mathbf{V})] + \nabla\left[\left(\zeta + \frac{4}{3}\mu\right)\nabla\cdot\mathbf{V}\right]$$

or in terms of λ

$$\rho\left[\frac{\partial\mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V}\right] = \rho\left[\frac{\partial\mathbf{V}}{\partial t} + \nabla\left(\frac{V^2}{2}\right) - \mathbf{V}\times(\nabla\times\mathbf{V})\right]$$

$$= -\nabla P + \mathbf{F} - \nabla\times[\mu(\nabla\times\mathbf{V})] + \nabla[(\lambda + 2\mu)\nabla\cdot\mathbf{V}]$$

ρ, μ constant

$$\rho\frac{D\mathbf{V}}{Dt} = \rho\left[\frac{\partial\mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V}\right] = -\nabla P + \mathbf{F} + \mu\nabla^2\mathbf{V}$$

Cartesian

$$\nabla\cdot\mathbf{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

$$\rho\frac{Du}{Dt} = F_x - \frac{\partial P}{\partial x} + \frac{\partial}{\partial x}\left[2\mu\frac{\partial u}{\partial x} + \lambda\nabla\cdot\mathbf{V}\right] + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)\right]$$

$$\rho\frac{Dv}{Dt} = F_y - \frac{\partial P}{\partial y} + \frac{\partial}{\partial y}\left[2\mu\frac{\partial v}{\partial y} + \lambda\nabla\cdot\mathbf{V}\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)\right] + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right]$$

$$\rho\frac{Dw}{Dt} = F_z - \frac{\partial P}{\partial z} + \frac{\partial}{\partial z}\left[2\mu\frac{\partial w}{\partial z} + \lambda\nabla\cdot\mathbf{V}\right] + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)\right] + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)\right]$$

ρ, μ constant

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial P}{\partial x} + F_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial P}{\partial y} + F_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial P}{\partial z} + F_z + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

Cylindrical

$$\nabla\cdot\mathbf{V} = \frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = \frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

$$\rho\left[\frac{Dv_r}{Dt} - \frac{v_\theta^2}{r}\right] = F_r - \frac{\partial P}{\partial r} + \frac{\partial}{\partial r}\left[2\mu\frac{\partial v_r}{\partial r} + \lambda\nabla\cdot\mathbf{V}\right]$$

$$+ \frac{1}{r}\frac{\partial}{\partial \theta}\left[\mu\left(\frac{1}{r}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)\right] + \frac{2\mu}{r}\left(\frac{\partial v_r}{\partial r} - \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} - \frac{v_r}{r}\right)$$

$$\rho\left[\frac{Dv_\theta}{Dt} + \frac{v_r v_\theta}{r}\right] = F_\theta - \frac{1}{r}\frac{\partial P}{\partial r} + \frac{1}{r}\frac{\partial}{\partial \theta}\left[\frac{2\mu}{r}\frac{\partial v_\theta}{\partial \theta} + \lambda\nabla\cdot\mathbf{V}\right]$$

$$+ \frac{\partial}{\partial z}\left[\mu\left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right)\right] + \frac{\partial}{\partial r}\left[\mu\left(\frac{1}{r}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right)\right] + \frac{2\mu}{r}\left[\frac{1}{r}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right]$$

$$\rho\frac{Dv_z}{Dt} = F_z - \frac{\partial P}{\partial z} + \frac{\partial}{\partial z}\left[2\mu\frac{\partial v_z}{\partial z} + \lambda\nabla\cdot\mathbf{V}\right] + \frac{1}{r}\frac{\partial}{\partial r}\left[\mu r\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)\right]$$

$$+ \frac{1}{r}\frac{\partial}{\partial \theta}\left[\mu\left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right)\right]$$

ρ, μ constant

$$\rho\left[\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right] = F_r - \frac{\partial P}{\partial r}$$

$$+ \mu\left[\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r}\frac{\partial v_r}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{v_r}{r} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]$$

$$\rho\left[\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right] = F_\theta - \frac{1}{r}\frac{\partial P}{\partial \theta}$$

$$+ \mu\left[\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial v_\theta}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2}\right]$$

$$\rho\left[\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right] = F_z - \frac{\partial P}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right]$$

Spherical

$$\rho\left[\frac{Dv_r}{Dt}-\frac{v_\theta^2+v_\phi^2}{r}\right]=F_r-\frac{\partial P}{\partial r}+\frac{\partial}{\partial r}\left[2\mu\frac{\partial v_r}{\partial r}+\lambda\nabla\cdot\mathbf{V}\right]$$

$$+\frac{1}{r}\frac{\partial}{\partial\theta}\left[\mu\left\{r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{1}{r}\frac{\partial v_r}{\partial\theta}\right\}\right]+\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[\mu\left\{\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi}+r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right\}\right]$$

$$+\frac{\mu}{r}\left[4\frac{\partial v_r}{\partial r}-\frac{2}{r}\frac{\partial v_\theta}{\partial\theta}-\frac{4v_r}{r}-\frac{2}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}-\frac{2v_\theta\cot\theta}{r}+r\cot\theta\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{\cot\theta}{r}\frac{\partial v_r}{\partial\theta}\right]$$

$$\rho\left[\frac{Dv_\theta}{Dt}+\frac{v_r v_\theta}{r}-\frac{v_\phi^2\cot\theta}{r}\right]=F_\theta-\frac{1}{r}\frac{\partial P}{\partial\theta}+\frac{1}{r}\frac{\partial}{\partial\theta}\left[\frac{2\mu}{r}\left(\frac{\partial v_\theta}{\partial\theta}+v_r\right)+\lambda\nabla\cdot\mathbf{V}\right]$$

$$+\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[\mu\left\{\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right)+\frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right\}\right]+\frac{\partial}{\partial r}\left[\mu\left\{r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{1}{r}\frac{\partial v_r}{\partial\theta}\right\}\right]$$

$$+\frac{\mu}{r}\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial\theta}-\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}-\frac{v_\theta\cot\theta}{r}\right)\cdot\cot\theta+3\left\{r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{1}{r}\frac{\partial v_r}{\partial\theta}\right\}\right]$$

$$\rho\left[\frac{Dv_\phi}{Dt}+\frac{v_\phi v_r}{r}+\frac{v_\theta v_\phi\cot\theta}{r}\right]=F_\phi-\frac{1}{r\sin\theta}\frac{\partial P}{\partial\phi}$$

$$+\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[\frac{2\mu}{r}\left(\frac{1}{\sin\theta}\frac{\partial v_\phi}{\partial\phi}+v_r+v_\theta\cot\theta\right)+\lambda\nabla\cdot\mathbf{V}\right]$$

$$+\frac{\partial}{\partial r}\left[\mu\left\{\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi}+r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right\}\right]+\frac{1}{r}\frac{\partial}{\partial\theta}\left[\mu\left\{\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right)+\frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right\}\right]$$

$$+\frac{\mu}{r}\left[3\left\{\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi}+r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right\}+2\cot\theta\left\{\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right)+\frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right\}\right]$$

ρ, μ constant

$$\rho\left[\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial\phi}-\frac{v_\theta^2+v_\phi^2}{r}\right]$$

$$=F_r-\frac{\partial P}{\partial r}+\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_r}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v_r}{\partial\theta}\right)\right.$$

$$\left.+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial\phi^2}-\frac{2v_r}{r^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}-\frac{2v_\theta\cot\theta}{r^2}-\frac{2}{r^2\sin\theta}\frac{\partial v_\phi}{\partial\phi}\right]$$

$$\rho\left[\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi{}^2\cot\theta}{r}\right]$$

$$= F_\theta - \frac{1}{r}\frac{\partial P}{\partial \theta} + \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v_\theta}{\partial \theta}\right)\right.$$

$$\left. + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial \phi^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2\sin^2\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\phi}{\partial \phi}\right]$$

$$\rho\left[\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r}{r} + \frac{v_\theta v_\phi\cot\theta}{r}\right]$$

$$= F_\phi - \frac{1}{r\sin\theta}\frac{\partial P}{\partial \phi} + \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v_\phi}{\partial \theta}\right)\right.$$

$$\left. + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial \phi^2} - \frac{v_\phi}{r^2\sin^2\theta} + \frac{2}{r^2\sin^2\theta}\frac{\partial v_r}{\partial \phi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\theta}{\partial \phi}\right]$$

c. Energy Equations.

e = internal energy (per unit mass).
P = pressure.
Q = internal heat generation.
$\mathbf{q}_r$ = radiation heat flux vector.
T = temperature.
k = thermal conductivity.
ρ = mass density
Φ = mechanical or viscous dissipation function.
i = enthalpy (per unit mass).

Vector Form

$$\frac{\partial Q}{\partial t} + \Phi + \nabla\cdot(k\nabla T) - \nabla\cdot\mathbf{q}_r = \rho\frac{De}{Dt} + P\nabla\cdot\mathbf{V} = \rho\left[\frac{De}{Dt} + P\frac{D}{Dt}\left(\frac{1}{\rho}\right)\right]$$

$$\rho\frac{Di}{Dt} = \frac{DP}{Dt} + \frac{\partial Q}{\partial t} + \Phi + \nabla\cdot(k\nabla T) - \nabla\cdot\mathbf{q}_r$$

ρ, k constant

$$\frac{\partial Q}{\partial t} + \Phi + \kappa\nabla^2 T - \nabla\cdot\mathbf{q}_r = \rho\frac{De}{Dt}$$

$$\rho\frac{Di}{Dt} = \frac{DP}{Dt} + \frac{\partial Q}{\partial t} + \Phi + k\nabla^2 T - \nabla\cdot\mathbf{q}_r$$

For perfect gases

$$\frac{Di}{Dt} = c_p\frac{DT}{Dt}$$