

ADVANCED COMMUNICATIONS AND MULTIMEDIA SECURITY

Edited by
Borka Jerman-Blažič
Tomaž Klobučar



IFIP



KLUWER
ACADEMIC
PUBLISHERS

ADVANCED COMMUNICATIONS AND MULTIMEDIA SECURITY

*IFIP TC6 / TC11 Sixth Joint Working Conference on
Communications and Multimedia Security
September 26–27, 2002, Portorož, Slovenia*

Edited by

Borka Jerman-Blažič

Tomaž Klobučar

Institut "Jožef Stefan"

Slovenia



KLUWER ACADEMIC PUBLISHERS
BOSTON / DORDRECHT / LONDON

Distributors for North, Central and South America:

Kluwer Academic Publishers

101 Philip Drive

Assinippi Park

Norwell, Massachusetts 02061 USA

Telephone (781) 871-6600

Fax (781) 681-9045

E-Mail <kluwer@wkap.com>

Distributors for all other countries:

Kluwer Academic Publishers Group

Post Office Box 322

3300 AH Dordrecht, THE NETHERLANDS

Telephone 31 786 576 000

Fax 31 786 576 474

E-Mail <services@wkap.nl>



Electronic Services <<http://www.wkap.nl>>

Library of Congress Cataloging-in-Publication Data

A C.I.P. Catalogue record for this book is available from the Library of Congress.

Advanced Communications and Multimedia Security

Edited by Borka Jerman-Blažič and Tomaž Klobučar

1-4020-7206-6

Copyright © 2002 by International Federation for Information Processing.

All rights reserved. No part of this work may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, microfilming, recording, or otherwise, without written permission from the Publisher (Kluwer Academic Publishers, 101 Philip Drive, Assinippi Park, Norwell, Massachusetts 02061), with the exception of any material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work.

Printed on acid-free paper.

Printed in Great Britain by IBT Global, London

IFIP - The International Federation for Information Processing

IFIP was founded in 1960 under the auspices of UNESCO, following the First World Computer Congress held in Paris the previous year. An umbrella organization for societies working in information processing, IFIP's aim is two-fold: to support information processing within its member countries and to encourage technology transfer to developing nations. As its mission statement clearly states,

IFIP's mission is to be the leading, truly international, apolitical organization which encourages and assists in the development, exploitation and application of information technology for the benefit of all people.

IFIP is a non-profitmaking organization, run almost solely by 2500 volunteers. It operates through a number of technical committees, which organize events and publications. IFIP's events range from an international congress to local seminars, but the most important are:

- The IFIP World Computer Congress, held every second year;
- open conferences;
- working conferences.

The flagship event is the IFIP World Computer Congress, at which both invited and contributed papers are presented. Contributed papers are rigorously refereed and the rejection rate is high.

As with the Congress, participation in the open conferences is open to all and papers may be invited or submitted. Again, submitted papers are stringently refereed.

The working conferences are structured differently. They are usually run by a working group and attendance is small and by invitation only. Their purpose is to create an atmosphere conducive to innovation and development. Refereeing is less rigorous and papers are subjected to extensive group discussion.

Publications arising from IFIP events vary. The papers presented at the IFIP World Computer Congress and at open conferences are published as conference proceedings, while the results of the working conferences are often published as collections of selected and edited papers.

Any national society whose primary activity is in information may apply to become a full member of IFIP, although full membership is restricted to one society per country. Full members are entitled to vote at the annual General Assembly, National societies preferring a less committed involvement may apply for associate or corresponding membership. Associate members enjoy the same benefits as full members, but without voting rights. Corresponding members are not represented in IFIP bodies. Affiliated membership is open to non-national societies, and individual and honorary membership schemes are also offered.

Preface

Security, trust and confidence can certainly be considered as the most important parts of the Information society. Being protected when working, learning, shopping or doing any kind of e-commerce is of great value to citizens, students, business people, employees and employers. Commercial companies and their clients want to do business over Internet in a secure way, business managers when having meetings by videoconferencing tools require the exchanged information to be protected, publishing industry is concerned with the protection of copyright, hospital patients have a right to privacy etc. There is no area in the Information society that can proliferate without extensive use of services that provide satisfactory protection and privacy of data or personality.

In order to gather and present the latest development in the area of communications and multimedia security, and identify future security related research challenges, a Communications and Multimedia Security Conference (CMS 2002) was organised in Portorož, Slovenia, on 26th and 27th of September, 2002. CMS 2002 is the sixth IFIP working conference on communications and multimedia security since 1995. State-of-the-art issues as well as practical experiences and new trends in the areas were the topics of interest again, as proven by preceding conferences.

The book “Advanced Communications and Multimedia Security” contains 22 articles that were selected by the conference programme committee for presentation at CMS 2002. The articles address advanced concepts of communications and multimedia security, such as cryptography, applied

cryptography, biometry, communication systems security, multimedia security, digital watermarking, distributed systems security, applications security, and digital signatures. We would like to express our deep appreciation to all authors for their high-quality contributions. Special thanks also go to members of the programme committee:

- Augusto Casaca, INESC, chairman IFIP TC6, Portugal
- David Chadwick, University of Salford, UK
- Bart de Decker, Katholieke Universiteit Leuven, Belgium
- Yves Deswarte, LAAS CNRS, France
- Dieter Gollmann, Microsoft Research, UK
- Ruediger Grimm, TU Ilmenau, Germany
- Patrick Horster, Universitaet Klagenfurt, Austria
- Steve Kent, BBN, USA
- Klaus Keus, BSI, Germany
- Herbert Leitold, IAIK, Austria
- Peter Lipp, IAIK, Austria
- Antonio Liroy, Politecnico di Torino, Italy
- Guenther Pernul, University of Essen, Germany
- Bart Preneel, Katholieke Universiteit Leuven, Belgium
- Fabien A. P. Petitcolas, Microsoft Research, UK
- Wolfgang Schneider, SIT Fraunhofer Gesellschaft, Germany
- Leon Strous, De Nederlandsche Bank, chairman IFIP TC11, Netherlands

Borka Jerman-Blažič and Tomaž Klobučar

Contents

Preface	ix
APPLIED CRYPTOGRAPHY	
ON THE SECURITY OF A STRUCTURAL PROVEN SIGNER ORDERING MULTISIGNATURE SCHEME Chris J. Mitchell, Namhyun Hur	1
RENEWING CRYPTOGRAPHIC TIMESTAMPS Sattam S. Al-Riyami, Chris J. Mitchell	9
IMPLEMENTING ELLIPTIC CURVE CRYPTOGRAPHY Wolfgang Bauer	17
A NEW ASYMMETRIC FINGERPRINTING FRAMEWORK BASED ON SECRET SHARING Yan Wang, Shuwan Lu, Zhenhua Liu	29
AUTHENTICATION OF TRANSIT FLOWS AND K-SIBLINGS ONE-TIME SIGNATURE Mohamed Al-Ibrahim, Josef Pieprzyk	41

COMMUNICATIONS SECURITY

IMPROVING THE FUNCTIONALITY OF SYN COOKIES André Zúquete	57
A MAC-LAYER SECURITY ARCHITECTURE FOR CABLE NETWORKS Tadauchi Masaharu, Ishii Tatsuei, Itoh Susumu	79
TOWARDS AUTHENTICATION USING MOBILE DEVICES E. Weippl, W. Essmayr, F. Gruber, W. Stockner, T. Trenker	91
CORE: A COLLABORATIVE REPUTATION MECHANISM TO ENFORCE NODE COOPERATION IN MOBILE AD HOC NETWORKS Pietro Michiardi, Refik Molva	107
ENABLING ADAPTIVE AND SECURE EXTRANETS Yves Roudier, Olivier Fouache, Pierre Vannel, Refik Molva	123
MULTIPLE LAYER ENCRYPTION FOR MULTICAST GROUPS Alain Pannetrat, Refik Molva	137
DISTRIBUTED SYSTEMS SECURITY	
ACCESS CONTROL, REVERSE ACCESS CONTROL AND REPLICATION CONTROL IN A WORLD WIDE DISTRIBUTED SYSTEM Bogdan C. Popescu, Chandana Gamage, Andrew S. Tanenbaum	155
THE CORAS APPROACH FOR MODEL-BASED RISK MANAGEMENT APPLIED TO E-COMMERCE DOMAIN Dimitris Raptis, Theo Dimitrakos, Bjørn Axel Gran, Ketil Stølen	169
TOWARDS SECURITY ARCHITECTURE FOR FUTURE ACTIVE IP NETWORKS Dušan Gabrijelčič, Arso Savanović, Borka Jerman-Blažič	183

MULTIMEDIA SECURITY**COMBINED FINGERPRINTING ATTACKS AGAINST
DIGITAL AUDIO WATERMARKING: METHODS,
RESULTS AND SOLUTIONS**

Martin Steinebach, Jana Dittmann, Eva Saar 197

SELECTIVE ENCRYPTION OF VISUAL DATA

Champos J. Skreph, Andreas Uhl 213

**BIOMETRIC AUTHENTICATION - SECURITY AND
USABILITY**

Václav Matyáš, Zdeněk Říha 227

APPLICATIONS SECURITY**AUTOMATIC AUTHENTICATION BASED ON THE
AUSTRIAN CITIZEN CARD**

Arno Hollosi, Udo Payer, Reinhard Posch 241

**AN OPEN INTERFACE ENABLING SECURE
E-GOVERNMENT**

Arno Hollosi, Herbert Leitold, Reinhard Posch 255

CADENUS SECURITY CONSIDERATIONS

Gašper Lavrenčič, Borka Jerman-Blažič,
Aleksej Jerman Blažič 267

DIGITAL SIGNATURES**VALIDATION OF LONG-TERM SIGNATURES**

Karl Scheibelhofer 279

**DIGITAL SIGNATURES AND ELECTRONIC
DOCUMENTS: A CAUTIONARY TALE**

K. Kain, S.W. Smith, R. Asokan 293

Index

309

Index

access control	155	long-term signatures	279
active networks.....	183	mobile agent	1
active packets	183	mobility.....	91
advanced electronic		modelling.....	169
signatures.....	279	multicast security.....	137
asymmetric fingerprinting.....	29	multisignature	1
authentication	91, 123, 227	network security	41
biometrics	227	OCSP	279
certificate status checking	279	one-time signatures.....	41
citizen card	241, 255	open interfaces.....	255
classification.....	227	partial or soft encryption.....	213
coalition attacks.....	197	peer-entity authentication	241
copyright protection	29	PKI.....	9, 293
CRL.....	279	protocol failure	9
cryptanalysis.....	1	replicated objects	155
customer authentication.....	197	revocation checking	279
digital certificates	155	risk analysis	169
digital signature	1, 9	secret sharing	29
digital signatures	279, 293	security	1, 9, 91, 155, 169, 227
distributed systems	155	security architecture.....	183
DPV.....	279	security management.....	183
ECC.....	17	selective image encryption	213
e-commerce	293	simultaneous connection	
e-Commerce	169	initiation.....	57
e-government.....	293	smart card	241
electronic signatures.....	255	smart cards.....	123
encryption.....	137	source authentication.....	41
evaluation	227	SPKI	123
extranets	123	SYN cookies	57
fingerprinting algorithms.....	197	SYN flooding attacks.....	57
firewalls.....	123	TCP options.....	57
Handle System.....	123	timestamping	9
identity card.....	255	TSA	9
identity token.....	241	TSS	9
Java.....	17	watermarking.....	197
k-sibling hashing	41		

ON THE SECURITY OF A STRUCTURAL PROVEN SIGNER ORDERING MULTISIGNATURE SCHEME*

Chris J. Mitchell and Namhyun Hur
Mobile VCE Research Group, Information Security Group
Royal Holloway, University of London
Egham, Surrey TW20 0EX, UK
C.Mitchell@rhul.ac.uk, Namhyun.Hur@rhul.ac.uk

Abstract Certain undesirable features are identified in the 'Structural proven signer ordering' multisignature scheme of Kotzanikolaou, Burmester and Chrissikopoulos. This scheme is a modification of a previous multisignature scheme due to Mitomi and Miyaji.

Keywords: mobile agent, digital signature, multisignature, cryptanalysis, security

1. INTRODUCTION

The notion of a multisignature scheme was introduced nearly 20 years ago [Itakura and Nakamura, 1983], and a number of schemes have been proposed since that time. The fundamental idea of a multisignature scheme is that it enables a number of users to collectively create a digital signature on a document (using their own private keys). Typically, all users will sign the same document, and either the order in which they sign will be fixed or, if it is not fixed, then the verifier will not be able to determine in which order the various users signed the document.

For further details on such multisignature techniques, and also on the ElGamal signature scheme on which the cryptosystems described in this paper are based, see, for example, [Menezes et al., 1997].

*The work reported in this paper has formed part of the Software Based Systems work area of the Core 2 Research Programme of the Virtual Centre of Excellence in Mobile & Personal Communications, Mobile VCE, www.mobilevce.com, whose funding support, including that of EPSRC, is gratefully acknowledged. More detailed technical reports on this research are available to Industrial Members of Mobile VCE.

1.1 Mitomi-Miyaji multisignatures

Recent papers [Mitomi and Miyaji, 2000, Mitomi and Miyaji, 2001] extend the notion of a multisignature. They provide a model for a multisignature scheme that allows three key properties:

- *message flexibility*, i.e., each party can sign a different document,
- *order flexibility*, i.e., the order in which the various parties create their contribution to the multisignature is not fixed, and
- *order verifiability*, i.e., the order in which the various parties created their contribution to the multisignature can be verified by the verifier of the multisignature.

Mitomi and Miyaji also propose two different multisignature schemes fitting this model, one discrete logarithm based and the other RSA based.

1.2 Multisignatures for mobile agents

In [Kotzanikolaou et al., 2001], the application of Mitomi-Miyaji multisignatures to a mobile agent environment is considered. Specifically, mobile agents (essentially autonomous pieces of code) may visit a number of host platforms, and may wish to collectively sign a message, e.g. to commit to a transaction on behalf of the original sponsor of the agents. Each agent will be equipped with its own (multi)signature private key.

The reason to employ such a model is that single agents may not be trusted to complete a transaction on behalf of a remote sponsor, since their operation may be interfered with by the platform on which they run. In general, there are a number of ways in which the threat posed by a small number of malicious platforms can be reduced. One such approach is to send multiple copies of a transaction agent to a number of platforms, and require that a certain number of copies of the agent (running on different platforms) all consent before the transaction is completed. Each copy of the agent is equipped with a distinct signature key pair (thus preventing an agent on one platform masquerading as an agent executing on a different platform). Of course such an approach requires some co-ordination amongst the various platforms involved, but this is not an issue we consider further here.

A variant of the above approach motivates the particular application of multisignatures we consider here. The model discussed in [Kotzanikolaou et al., 2001] involves a series of agents: U_1, U_2, \dots, U_n each contributing to a multisignature in turn. Each agent U_i adds its own message string m_i to the evolving multisignature, and thus user U_i actually contributes to a multisignature on a sequence of messages m_1, m_2, \dots, m_i .

We suppose that the recipient of the multisignature will only accept it if a minimum number of distinct agents have contributed to the signature, and that all the agent messages m_i are 'consistent' in some application-specific way.

In this context, [Kotzanikolaou et al., 2001] identify a potential problem with use of Mitomi-Miyaji multisignatures. Specifically, a malicious user can delete one or more of the most recent agent contributions from a multisignature (Kotzanikolaou et al. call this an *exclude* attack). Kotzanikolaou et al. propose two different ways of addressing this problem.

- The first approach, described in Section 3.4 of [Kotzanikolaou et al., 2001], is called a 'simple solution'. It requires signing agent U_j to include in message m_j the identity of U_{j+1} , the agent which U_j selects to be the next entity to contribute to the multisignature. This clearly prevents a malicious party from 'winding back' a multisignature. No changes to the Mitomi-Miyaji schemes are required.
- The second method, described in Section 3.5 of [Kotzanikolaou et al., 2001], is called 'structural proven signer ordering'. This solution actually involves a minor modification to the discrete logarithm based Mitomi-Miyaji scheme. The multisignature computation performed by U_j is modified to include the value of the public key of the next party to the multisignature, namely U_{j+1} . This is designed to achieve the same objective as the simple solution.

Unfortunately, as we describe below, it is precisely this small modification that enables the manipulation of multisignatures in certain special circumstances. The main conclusion of this paper is therefore that the 'simple solution' is probably preferable.

Specifically, in the remainder of this paper we describe two undesirable features of the structural proven signer modification to the Mitomi-Miyaji discrete logarithm based multisignature scheme.

1.3 Notation and assumptions

We use the notation of [Kotzanikolaou et al., 2001]. Specifically, we suppose that a multisignature is being computed by a series of signers U_1, U_2, \dots, U_j . The part multisignature output by user U_j consists of two sequences of values, namely the messages m_1, m_2, \dots, m_j (where m_i is chosen by U_i , $1 \leq i \leq j$), and the multisignature components s_1, s_2, \dots, s_j (where s_i is computed by U_i , $1 \leq i \leq j$), together with the single value r_j .

As in the scheme described in Section 3.5 of [Kotzanikolaou et al., 2001], we suppose that p and g are universally agreed domain parameters, where p is a large prime satisfying $p = 2q + 1$, q is also prime, and g ($1 < g < p$) has multiplicative order q modulo p .

2. A (PARTIAL) MESSAGE MANIPULATION ATTACK

Suppose a malicious user has succeeded in obtaining iq as its public key, for some integer i . Of course, in general, the malicious user will not know the private key for this public key, i.e. the malicious user will not know a value x for which $g^x \bmod p = iq$. However, this does not prevent at least a partial attack, as we now describe.

2.1 The partial attack

Suppose that a multisignature is being constructed (using the method in Section 3.5 of [Kotzanikolaou et al., 2001]) by a series of signers U_1, U_2, \dots, U_j , and that the next signer (U_{j+1}) is the malicious user; hence U_{j+1} has $y_{j+1} = iq$ as its public key. For convenience we also suppose that $j > 1$, although the attack will work in almost exactly the same way if $j = 1$.

Using the notation of [Kotzanikolaou et al., 2001], U_j will compute

$$\begin{aligned} R_j &= g^{k_j} \bmod p, \\ r_j &= (h(m_j || \text{ID}_j) \cdot r_{j-1})^{-1} \cdot R_j \bmod q, \text{ and} \\ s_j &= (x_j r_j + y_{j+1}) \cdot k_j^{-1} \bmod q \end{aligned}$$

where x_j is the private key of U_j , h is a hash-function, and y_{j+1} is the public key of user U_{j+1} . Hence, since we know that $y_{j+1} \bmod q = 0$, we have

$$s_j = x_j r_j k_j^{-1} \bmod q.$$

User U_j then sends r_j , s_j and m_j to U_{j+1} (together with various other values not of relevance here).

User U_{j+1} can now change the message m_j which user U_j signed. Specifically, suppose user U_{j+1} wishes to make it look as though user U_j signed message $m'_j \neq m_j$. User U_{j+1} first computes $h(m'_j || \text{ID}_j)$ and then computes

$$r'_j = r_j \cdot h(m_j || \text{ID}_j) \cdot (h(m'_j || \text{ID}_j))^{-1} \bmod q.$$

This requires no special knowledge. However, the fact that $y_{j+1} \bmod q = 0$ enables U_{j+1} to compute the 'matching' value s'_j using

$$s'_j = s_j r_j^{-1} r'_j \bmod q = x_j r'_j k_j^{-1} \bmod q = (x_j r'_j + y_{j+1}) \cdot k_j^{-1} \bmod q.$$

These new values r'_j and s'_j can now be used to replace r_j and s_j in the (partial) multisignature, at the same time that m'_j replaces m_j .

2.2 Completing the attack

Whether or not the process described above is a serious attack depends on whether or not U_{j+1} is in a position to complete the modified multisignature. This depends on whether U_{j+1} possesses the private key x_{j+1} corresponding to the public key $y_{j+1} = iq$. In general this appears to be difficult to arrange.

However, there is one specific case where it is possible for a malicious user to calculate the private key corresponding to a public key congruent to zero modulo q . Suppose, as is often described, the domain parameters p and g are selected as follows.

- 1 p is chosen so that $q = (p - 1)/2$ is prime, and thus precisely $q - 1$ of the $p - 1$ non-zero elements modulo p , i.e. approximately 50%, will be primitive (see, for example, Section 4.6.1 of [Menezes et al., 1997]).
- 2 A primitive element modulo p is chosen; call this value e .
- 3 g is set equal to e^2 , guaranteeing that g has order q .

Suppose moreover that $e = 2$. This is not unlikely to be the case; heuristically we expect 2 to be primitive roughly half the time, since roughly half the non-zero elements are primitive, and 2 is typically the first value chosen in a search for a primitive element. In such a case we have $g = 2^2 \bmod p = 4$.

Next observe that $2^q \bmod p = p - 1 = 2q$, and hence $2^{q-1} \bmod p = q$. Thus, $g^{(q-1)/2} \bmod p = 2^{q-1} \bmod p = q$. That is, the private key corresponding to the public key q is simply $(q - 1)/2$. Hence, in this special case, if the malicious user chooses his/her public key to be q , then he/she will know his/her own private key, and hence would be able to complete the forged partial multisignature. This represents a serious compromise of the security of the scheme.

Of course, if this particular special case is avoided then the partial signature cannot be completed and the 'partial attack' is simply a (probably unexploitable) questionable property of the scheme.

Finally note that there is one other way in which the above situation can arise. Suppose that, after selecting p (and hence q), g is found by successively examining values 2, 3, 4, and so on, until an element of order q is found. This is a reasonable approach, since small values of g have implementation advantages. Suppose also that 2 and 3 are primitive

(and hence are not suitable) — as previously, using heuristic arguments we expect this to be true roughly 25% of the time. Then 4 will have order q and will be selected — exactly the same situation now arises.

3. A DESTINATION MANIPULATION ATTACK

We show how three different users can conspire to manipulate a contribution to a multisignature made by an honest user.

Suppose that a multisignature is being constructed (using the method in Section 3.5 of [Kotzanikolaou et al., 2001]) by a series of signers U_1, U_2, \dots, U_j , where $j > 2$.

Then, using the notation of [Kotzanikolaou et al., 2001], U_{j-1} will compute

$$\begin{aligned} R_{j-1} &= g^{k_{j-1}} \bmod p, \\ r_{j-1} &= (h(m_{j-1} || ID_{j-1}) \cdot r_{j-2})^{-1} \cdot R_{j-1} \bmod q, \text{ and} \\ s_{j-1} &= (x_{j-1} r_{j-1} + y_j) \cdot k_{j-1}^{-1} \bmod q \end{aligned}$$

where x_{j-1} is the private key of U_{j-1} , h is a hash-function, and y_j is the public key of user U_j . User U_{j-1} then sends r_{j-1} , s_{j-1} and m_{j-1} to U_j (together with various other values not of relevance here).

Similarly, U_j will compute

$$\begin{aligned} R_j &= g^{k_j} \bmod p, \\ r_j &= (h(m_j || ID_j) \cdot r_{j-1})^{-1} \cdot R_j \bmod q, \text{ and} \\ s_j &= (x_j r_j + y_{j+1}) \cdot k_j^{-1} \bmod q \end{aligned}$$

where x_j is the private key of U_j and y_{j+1} is the public key of user U_{j+1} . User U_j then sends r_j , s_j and m_j to U_{j+1} (together with various other values not of relevance here).

We now show how a collaboration of three users, namely U_{j-1} , U_{j+1} and a third user which we denote by U'_{j+1} , can modify the multisignature contribution of user U_j to make it look as though the next user specified by U_j was U'_{j+1} and not U_{j+1} . The modifications required are as follows.

First, when computing the original values of R_{j-1} , r_{j-1} and s_{j-1} , user U_{j-1} must choose k_{j-1} equal to x'_{j+1} , where x'_{j+1} is the private key of user U'_{j+1} (we also denote the private key of user U_{j+1} by x_{j+1}). Hence

$$R_{j-1} = g^{k_{j-1}} \bmod p = g^{x'_{j+1}} \bmod p = y'_{j+1}.$$

Second, the values R_{j-1} , r_{j-1} and s_{j-1} are replaced with new values R'_{j-1} , r'_{j-1} and s'_{j-1} computed using a new 'random value' k'_{j-1} , where $k'_{j-1} = x_{j+1}$, the private key of user U_{j+1} .

The replacement values are now computed as follows:

$$\begin{aligned}
 R'_{j-1} &= g^{k'_{j-1}} \bmod p = g^{x_{j+1}} \bmod p = y_{j+1}, \\
 r'_{j-1} &= r_{j-1} \cdot (y'_{j+1} \bmod q)^{-1} \cdot (y_{j+1} \bmod q) \bmod q \\
 &= r_{j-1} \cdot (R_{j-1} \bmod q)^{-1} \cdot (R'_{j-1} \bmod q) \bmod q \\
 &= (h(m_{j-1} || ID_{j-1}) \cdot r_{j-2})^{-1} \cdot R'_{j-1} \bmod q, \\
 s'_{j-1} &= (x_{j-1} r'_{j-1} + y_j) \cdot (k'_{j-1})^{-1} \bmod q \\
 &= (x_{j-1} r'_{j-1} + y_j) \cdot (x_{j+1})^{-1} \bmod q.
 \end{aligned}$$

(Note that computing these replacement values is simple since U_{j-1} is a member of the conspiracy).

Replacement values are also computed for R_j , r_j and s_j as follows, this time *without* the co-operation of user U_j :

$$\begin{aligned}
 R'_j &= R_j, \quad (k_j \text{ is thus as before}), \\
 r'_j &= r_j \cdot (r'_{j-1})^{-1} \cdot r_{j-1} \bmod q \\
 &= (h(m_j || ID_j) \cdot r'_{j-1})^{-1} \cdot R'_j \bmod q, \\
 s'_j &= s_j \cdot r'_j \cdot (r_j)^{-1} \bmod q.
 \end{aligned}$$

It remains to show that s'_j has the required properties. Observe that

$$\begin{aligned}
 s'_j &= s_j \cdot r'_j \cdot (r_j)^{-1} \bmod q, \\
 &= (x_j r_j + y_{j+1}) \cdot k_j^{-1} \cdot r'_j \cdot (r_j)^{-1} \bmod q, \quad (\text{by definition of } s_j), \\
 &= (x_j r'_j + y_{j+1} \cdot r'_j \cdot (r_j)^{-1}) \cdot k_j^{-1} \bmod q, \\
 &= (x_j r'_j + y_{j+1} \cdot r_{j-1} \cdot (r'_{j-1})^{-1}) \cdot k_j^{-1} \bmod q, \quad (\text{by definition of } r'_j), \\
 &= (x_j r'_j + y'_{j+1}) \cdot k_j^{-1} \bmod q \quad (\text{by definition of } r'_{j-1}).
 \end{aligned}$$

This completes the demonstration, since it is clear that s'_j identifies U'_{j+1} as the next participant in the multisignature instead of U_{j+1} .

4. ANALYSIS

Observe that, in most circumstances, the (partial) forgery described in Section 2 cannot be completed to a full multisignature. Hence its impact is very limited. Moreover, if users are required to prove possession of their private key before their public key is certified (or otherwise distributed), as is now deemed 'good practice', then in most cases the partial attack is prevented. However, the existence of such a partial attack (which can be extended to a full attack in certain special cases) is nevertheless of concern.