APPLIED CALCULUS

Third Edition

Taylor
Gilligan

EDITION

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APPLIED CALCULUS

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This book is dedicated
in memory of my mentor and friend, James Bender C. D. T.
and to
my dad, Andrew J. Gilligan L. G. G.

PREFACE

This book represents the third generation of a text that was written expressly for students majoring in business, management science, economics, life science, social science, and psychology. As such, it recognizes both the particular needs of that student audience and the unique set of constraints placed upon instructors teaching the calculus in a three- or four-hour, one-semester course.

Our main purpose in writing *Applied Calculus* was to make the calculus *meaningful* to students who would be going directly into applied disciplines. To achieve our goal, we have presented differential and integral calculus within an applied, problem-solving setting. If students leave this course with a strong sense of the usefulness of calculus as a tool for solving real-world problems, then we as educators have succeeded in meeting the most fundamental goal of the course.

We are proud of the fact that, as the manuscript for this book was reviewed by our colleagues across the country, the most often-repeated comment was about its *writing style.* From using the text in class, we know that students *do* read it and don't just use it as a collection of homework exercises. To help its "coefficient of readability" we have incorporated the following features:

- New concepts and techniques are presented directly and concisely. Prose is enhanced by numerous step-by-step examples. In our applied calculus classes, each of us has found the technique of teaching by carefully selecting examples to be highly effective.
- We have included enough theory to substantiate the mathematical development of concepts for the purposes of this course and this student audience. The instructor has the choice of deciding how much theory to include, since proofs and verifications generally appear at the end of their sections. (See, for example, the treatment of the Product and Quotient Rules in Section 3.2.)
- The design and format of the book have been especially crafted with its audience in mind. Important rules, theorems, and properties are highlighted and boxed for easy reference. Graphics and diagrams are used liberally throughout the text to help students visualize ideas.
- We have found that students need to master vocabulary in order to read any mathematics text. Consequently, new vocabulary terms are listed at the end of each chapter for easy reference in vocabulary/formula lists.
- Exercise sets are carefully constructed. There are four types of exercises: Set A, Set B, Writing and Critical Thinking, and Using Technology in Calculus.

Set A exercises are designed with groups of exercises keyed to specific examples in that section. The example serves as a direct problem-solving model for the student.

Set B includes application problems and the more challenging exercises.

Writing and Critical Thinking exercises are optional and are designed to encourage students to think about concepts and express their reasoning in writing.

Using Technology in Calculus exercises are optional, designed for students with access to computer programs or graphics calculators. They extend the concepts using the power of the technology.

ADDITIONAL FEATURES

- Over 200 exercises have been added to the third edition, most of which are applied to Business, Social Science, or Life Science.
- Since individual instructors' interest in using technology ranges from none at all to extensive, we have built in a great deal of flexibility:

  There are seven Alternate sections, indicated clearly in the table of contents, where the concepts of calculus are approached using the advantages of a graphics calculator. The actual graphics screens from a TI-81 calculator are displayed; however, the sections are not calculator-specific—any graphics display calculator can be used.

  There are 32 Using Technology in Calculus exercises, which offer challenging looks at the concepts of calculus, using either computer software or graphics calculators.*

  As in the second edition, there are also standard calculator exercises and a few spreadsheet exercises.

- Chapter reviews include a vocabulary/formula list, review exercises (keyed to specific sections), and a chapter test. Each chapter test has two parts: Part One is a multiple-choice test, and Part Two is a more standard test composed of questions that require complete answers.
- A second color has been used to highlight key elements of the presentation and to "point" to the kinds of terms and expressions an instructor would point to in a classroom demonstration.

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* We have chosen to concentrate on three technology components: *DERIVE*®, a computer algebra system, *CONVERGE*®, a general-purpose mathematics software package, and the TI-81 Graphics Calculator®. The authors would like to thank the following three companies for their cooperation and enthusiasm:

DERIVE is a registered trademark of Soft Warehouse, Inc. (3660 Waialae Ave., Suite 304, Honolulu, HI 96816; (808) 734-5801).

CONVERGE is a registered trademark of JEMware (The Kawaiahao Plaza Executive Center, 567 South King St., Suite 178, Honolulu, HI 96813; (808) 947-1853).

TI-81 Graphics Calculator is a trademark of Texas Instruments, Inc. (7800 Banner Drive, Dallas, TX 75265; (214) 917-1539).

SUPPLEMENTS

- The *Instructor's Manual* contains the answers to even-numbered exercises, suggestions on how to use the Writing and Critical Thinking exercises, answers and suggestions for the Writing and Critical Thinking exercises, and answers to the Using Technology in Calculus exercises.
- The *Instructor's Test Booklet* offers two forms of tests for each chapter.
- An *Electronic Test Bank*, using Brooks/Cole's *EXP-Test*®, contains a wide variety of questions (ranked easy, medium, or difficult) and can be used to create many different forms of tests of equivalent difficulty. *EXP-Test* is available free to adopters of Brooks/Cole texts.
- A *Student Solutions Manual* is available for sale in your college bookstore. It contains the completely worked out solutions to all odd-numbered exercises.
- A manual, *Calculus and the DERIVE® Program: Experiments with the Computer, Second Edition* by L. Gilligan and J. Marquardt is available as a supplement for instructors or students interested in using the computer algebra system *DERIVE*® as part of the optional technology component of the text.

OTHER THIRD EDITION CHANGES

- Chapter 5, "Exponential and Logarithmic Functions," has been reorganized to incorporate the applications of these functions into the first two sections.
- Derivatives of exponential and logarithmic functions now appear in two separate sections in Chapter 5.
- Several sections were rewritten in response to suggestions from reviewers, most notably Section 2.6 on the derivative and Section 7.3 on numerical integration techniques.

ACKNOWLEDGMENTS

This book has benefited from the contributions of many student users—from early generations of students who used the original manuscript to users of the first and second editions of this work. Also, an army of extremely talented professionals reviewed the manuscript and offered many valuable suggestions. They are Bela Bajnok, University of Houston–Downtown; Darrell Clevidence, Carl Sandburg College; Lillie Crowley, Lexington Community College; George Duchossois, University of South Dakota; Vuryl Klassen, California State University at Fullerton; Phillip McGill, Illinois Central College; R. L. Richardson, Appalachian State University; Henry Smith, Metairie, Louisiana; and Lenore Vest, Lower Columbia College. We would also like to thank Paula Heighton, our editor on this project, and the talented crew at Brooks/Cole for their hard work.

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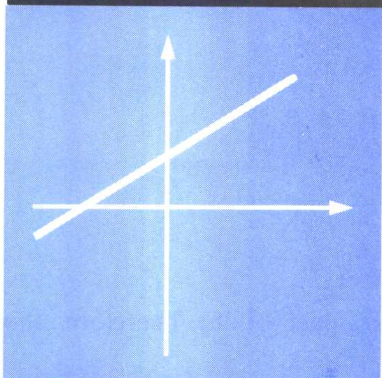

ALGEBRA REVIEW

1.1 SIMPLIFYING EXPRESSIONS

There are many situations in solving mathematical problems in which it is important to perform some kind of *simplification* process. One of the most basic types of simplification is called **combining like terms. Terms** are parts of an expression joined by addition (or subtraction) and **like terms** are terms that differ from each other *only* by the numerical coefficients. To *combine* like terms, add (or subtract) their coefficients. The following examples illustrate this process.

Example 1 Simplify each of the following by combining like terms:

a. $2x + 3 + 5x - 2 - y + 1$
b. $7x^2 - 4y + x - 2x^2 + 3x + y$

Solution **a.** Rearranging the terms, we have

$$\underbrace{2x + 5x}_{7x} \underbrace{- y}_{-y} + \underbrace{3 - 2 + 1}_{2}$$

$$7x \quad - y + \quad 2$$

(Notice that the sign preceding a term is part of it.) Therefore, the simplified form is

$$7x - y + 2$$

b. $$\underbrace{7x^2 - 2x^2}_{5x^2} + \underbrace{x + 3x}_{4x} - \underbrace{4y + y}_{3y}$$

$$5x^2 \quad + \quad 4x \quad - \quad 3y$$

The result is $5x^2 + 4x - 3y$. ■

The next example involves expressions containing *grouping* symbols of parentheses and brackets. The procedure is to start on the inside and work your way out.

Example 2 Simplify the following:

a. $(2x + 3) - (4x - 1)$ **b.** $2[5x - 3(x + 2)]$

Solution **a.** When removing the parentheses, change the sign of the terms in the second "piece," since the operation is subtraction.

$(2x + 3) - (4x - 1)$

$2x + 3 - 4x + 1$ Combine like terms

or $-2x + 4$

b. Start inside with the parentheses.

$2[5x - 3(x + 2)]$

$2[5x - 3x - 6]$ Use the distributive property

$2[2x - 6]$ Combine like terms

$4x - 12$ Distributive property ■

In addition to *simplifying* an algebraic expression, we will also, under certain circumstances, want to *find the value of* an expression. To *evaluate* $3x - 4y$ when $x = 2$ and $y = 7$, for example, means to substitute 2 for x and 7 for y in $3x - 4y$.

$$\begin{array}{c} 3x - 4y \\ \downarrow \quad \downarrow \\ 3 \cdot 2 - 4 \cdot 7 \\ 6 - 28 \\ -22 \end{array}$$

Multiplication is performed before subtraction

Example 3 Evaluate the following expression for $x = -2$:

$$4x^3 - 2x + 3$$

Solution We need to replace x with -2 everywhere x appears in the expression.

$$4(-2)^3 - 2(-2) + 3$$
$$4(-8) + 4 + 3$$
$$-32 + 7$$
$$-25$$

■

Before we simplify any expressions that involve multiplying variables, we review some definitions and properties for exponents and radicals in Table 1.1.

Exponents and Radicals

Property	Example
1. $x^m \cdot x^n = x^{m+n}$	$x^3 \cdot x^5 = x^8$
2. $\frac{x^m}{x^n} = x^{m-n},\ x \neq 0$	$\frac{x^7}{x^4} = x^3$
3. $(x^m)^n = x^{mn}$	$(x^3)^2 = x^6$
4. $(xy)^n = x^n y^n$	$(xy)^4 = x^4 y^4$
5. $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n},\ y \neq 0$	$\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$
6. $x^{-n} = \frac{1}{x^n},\ x \neq 0$	$x^{-3} = \frac{1}{x^3}$
7. $x^0 = 1,\ x \neq 0$	$3^0 = 1$
8. $x^{1/n} = \sqrt[n]{x}$	$x^{1/3} = \sqrt[3]{x}$
9. $x^{m/n} = \sqrt[n]{x^m}$ or $(\sqrt[n]{x})^m$	$x^{2/3} = \sqrt[3]{x^2}$ or $(\sqrt[3]{x})^2$

TABLE 1.1

Property 1 is used in conjunction with the distributive property in the next two examples.

Example 4 Simplify each of the following:

a. $2x(3x^2 + 1)$ **b.** $(x^4 + 3)(2x^4 - 4)$

Solution **a.** Using the distributive property, we get

$$2x(3x^2 + 1)$$
$$2x \cdot 3x^2 + 2x \cdot 1$$
or $$6x^3 + 2x$$

b. Using the distributive property twice produces four products, as illustrated:

$$(x^4 + 3)(2x^4 - 4)$$
$$x^4 \cdot 2x^4 + 3 \cdot 2x^4 - 4 \cdot x^4 - 4 \cdot 3$$
$$2x^8 + 6x^4 - 4x^4 - 12 \quad \text{Combine like terms}$$
or $$2x^8 + 2x^4 - 12$$ ■

Example 5 Simplify the following:

$$x(2x - 3) - 4x(x^2 - 1)$$

Solution

$$x(2x - 3) - 4x(x^2 - 1)$$
$$2x^2 - 3x - 4x^3 + 4x$$
or $$-4x^3 + 2x^2 + x$$ ■

The process of multiplying two expressions by applying the distributive property sometimes results in expressions that have a specific form. It will be important, in Section 1.2, to recognize these forms, called **special products,** and we summarize them in Table 1.2.

The presence of a *fractional* exponent means that an expression may be rewritten using a radical symbol, as properties 8 and 9 in Table 1.1 suggest. For example, $8^{1/3}$ is another way of writing $\sqrt[3]{8}$ and $9^{3/2}$ means $(\sqrt{9})^3$. Examples 6 and 7 show how you can convert from fractional exponent form to radical form and vice versa.

Example	Name	Form
$(x+4)(x-4) = x^2 + 4x - 4x - 16$ $= x^2 - 16$	Difference of two squares	$(x+a)(x-a)$ $= x^2 - a^2$
$(x+5)(x-6) = x^2 + 5x - 6x - 30$ $= x^2 - x - 30$	Trinomial (x^2 coefficient is 1)	$(x+a)(x+b)$ $= x^2 + (a+b)x + ab$
$(2x-5)(x-7) = 2x^2 - 5x - 14x + 35$ $= 2x^2 - 19x + 35$	Trinomial (x^2 coefficient $\neq 1$)	$(ax+b)(cx+d)$ $= acx^2 + (ad+bc)x + bd$
$(x-2)(x^2+2x+4) = x^3 + 2x^2 + 4x - 2x^2$ $- 4x - 8 = x^3 - 8$	Difference of two cubes	$(x-a)(x^2+ax+a^2)$ $= x^3 - a^3$
$(x+3)(x^2-3x+9) = x^3 - 3x^2 + 9x + 3x^2$ $- 9x + 27 = x^3 + 27$	Sum of two cubes	$(x+a)(x^2-ax+a^2)$ $= x^3 + a^3$

TABLE 1.2

Example 6 Change to radical form:

a. $2x^{2/3}$ **b.** $x^{-1/2}$ **c.** $(x-1)^{1/3}$ **d.** $x^{-4/3}$

Solution **a.** $2\sqrt[3]{x^2}$ (not $\sqrt[3]{2x^2}$) **b.** $\dfrac{1}{\sqrt{x}}$ **c.** $\sqrt[3]{x-1}$ **d.** $\dfrac{1}{\sqrt[3]{x^4}}$ ■

Example 7 Change to exponential form:

a. $-5\sqrt{x}$ **b.** $\dfrac{x}{\sqrt[4]{2-x}}$ **c.** $\sqrt[5]{x^4}$

Solution **a.** $-5x^{1/2}$ **b.** $x(2-x)^{-1/4}$ **c.** $x^{4/5}$ ■

Example 8 Evaluate each of the following for the given value:

a. $2x^{2/3}$ for $x = -8$

b. $\dfrac{1}{\sqrt[4]{1-x}}$ for $x = 0$

c. $-5x^{-1/2}$ for $x = 9$

Solution **a.** $2(-8)^{2/3} = 2[(-8)^{1/3}]^2 = 2[-2]^2 = 2[4] = 8$

b. $\dfrac{1}{\sqrt[4]{1-0}} = \dfrac{1}{\sqrt[4]{1}} = 1$

c. $-5(9)^{-1/2} = \dfrac{-5}{\sqrt{9}} = \dfrac{-5}{3}$ ■

It is sometimes desirable in a rational expression involving radicals to have no radicals in the denominator or in some cases no radicals in the numerator. The process of removing radicals from the denominator (or numerator) is called **rationalizing** the denominator (or numerator).

Example 9 Rationalize the denominator in the following:

a. $\dfrac{3}{\sqrt{2}}$ **b.** $\dfrac{1}{1-\sqrt{x}}$

Solution **a.** Multiply by $\sqrt{2}$ in both the numerator and denominator.

$$\frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{4}} = \frac{3\sqrt{2}}{2}$$

b. In this case, we need to multiply by $1 + \sqrt{x}$ in the numerator and denominator.

$$\frac{1}{1-\sqrt{x}} \cdot \frac{1+\sqrt{x}}{1+\sqrt{x}} = \frac{1+\sqrt{x}}{1-\sqrt{x}+\sqrt{x}-\sqrt{x^2}}$$

$$= \frac{1+\sqrt{x}}{1-x}$$ ■

Example 10 Rationalize the numerator for $\dfrac{\sqrt{x}+\sqrt{x+3}}{3}$.

Solution Multiply by $\sqrt{x} - \sqrt{x+3}$ in the numerator and denominator.

$$\frac{\sqrt{x}+\sqrt{x+3}}{3} \cdot \frac{\sqrt{x}-\sqrt{x+3}}{\sqrt{x}-\sqrt{x+3}}$$

$$= \frac{\sqrt{x^2}+\sqrt{x}\sqrt{x+3}-\sqrt{x}\sqrt{x+3}-\sqrt{(x+3)^2}}{3(\sqrt{x}-\sqrt{x+3})}$$

$$= \frac{x-(x+3)}{3(\sqrt{x}-\sqrt{x+3})}$$

$$= \frac{-3}{3(\sqrt{x}-\sqrt{x+3})}$$

$$= \frac{-1}{\sqrt{x}-\sqrt{x+3}}$$ ■

We conclude this section by multiplying and simplifying expressions involving fractional exponents.

Example 11 Simplify by multiplying and combining like terms:

$$2x^{1/2}(x^{-1/2} + 1) - x^{-1/2}(x^{1/2} - x)$$

Solution

$$2x^{1/2}(x^{-1/2} + 1) - x^{-1/2}(x^{1/2} - x)$$
$$2x^{1/2} \cdot x^{-1/2} + 2x^{1/2} - x^{-1/2} \cdot x^{1/2} + x^{-1/2} \cdot x^{1}$$
$$2x^{0} + 2x^{1/2} - x^{0} + x^{1/2} \qquad (x^0 = 1)$$

or $$3x^{1/2} + 1$$ ■

1.1 EXERCISES

SET A

In exercises 1 through 14, perform the indicated operations and simplify. (See Examples 1, 2, 4, and 5.)

1. $4x^2 - 2x - x^2 - 5x + 3$

2. $5a - 4b + 2a - 3 + b$

3. $(2x - x^2) - (5 + 3x^2)$

4. $(x^2 + 1) - (x^2 + 2x) - (x + 4)$

5. $4(x - 3) - 2(4x + 1)$

6. $2x(x + 1) - 4(2x + 5)$

7. $-2[5 - 3(x - 2)]$

8. $6[2(1 - x) - 3(2 - x)]$

9. $x^2(2x^2 - x + 4)$

10. $-2x^3(x^2 + x - 1)$

11. $(2x + 3)(x - 7)$

12. $x(x + 3)(x - 4)$

13. $5x - x[2x - (x - 3)]$

14. $2x[4 - 2(x^2 - 1)] + 3x$

In exercises 15 through 22, let $x = -2$ and $y = 3$. Evaluate each expression. (See Example 3.)

15. $2x^3 - 3x + 5$

16. $4y^2 - 5y + 1$

17. $3x^4 - x + 6$

18. $y^3 - y + 7$

19. $5x^2y - 3xy^2$

20. $8x^3y - 5xy^3$

21. $x^2 + y^2$

22. $(x + y)^2$

In exercises 23 through 28, change each expression to a radical form. (See Example 6.)

23. $x^{3/4}$

24. $-x^{4/3}$

25. $4x^{1/2}$

26. $-4x^{-1/2}$

27. $3(x + 2)^{-1/2}$

28. $-x(1 - 2x)^{2/3}$

In exercises 29 through 34, change each radical expression to an exponential form. (See Example 7.)

29. $\sqrt[3]{x}$

30. $x\sqrt{x}$

31. $-2x\sqrt[4]{x}$

32. $\dfrac{1}{\sqrt[3]{x^2 + 1}}$

33. $\dfrac{2x}{\sqrt[3]{x}}$

34. $\dfrac{x - 2}{\sqrt{x + 1}}$

In exercises 35 through 44, evaluate each expression for the given value. (See Example 8.)

35. $x^{1/2}$ for $x = 16$

36. $x^{1/3}$ for $x = 27$

37. $x^{1/3}$ for $x = -27$

38. $x^{2/3}$ for $x = 64$