Stochastic models, estimation, and control VOLUME 3

PETER S. MAYBECK

Stochastic models, estimation, and control

VOLUME 3

PETER S. MAYBECK

DEPARTMENT OF ELECTRICAL ENGINEERING AIR FORCE INSTITUTE OF TECHNOLOGY WRIGHT-PATTERSON AIR FORCE BASE



ACADEMIC PRESS

A Subsidiary of Harcourt Brace Jovanovich, Publishers

New York London

Paris San Diego San Francisco São Paulo Sydney Tokyo Toronto

100000

COPYRIGHT © 1982, BY ACADEMIC PRESS, INC

ALL RIGHTS RESERVED.

NO PART OF THIS PUBLICATION MAY BE REPRODUCED OR TRANSMITTED IN ANY FORM OR BY ANY MEANS, ELECTRONIC OR MECHANICAL, INCLUDING PHOTOCOPY, RECORDING, OR ANY INFORMATION STORAGE AND RETRIEVAL SYSTEM, WITHOUT PERMISSION IN WRITING FROM THE PUBLISHER.

ACADEMIC PRESS, INC.
111 Fifth Avenue, New York, New York 10003

United Kingdom Edition published by ACADEMIC PRESS, INC. (LONDON) LTD. 24/28 Oval Road, London NW1 7DX

Library of Congress Cataloging in Publication Data

Maybeck, Peter S. Stochastic models, estimation, and control.

(Mathematics in science and engineering)
Includes bibliographies and indexes.
1. System analysis. 2. Control theory. 3. Estimation theory. I. Title. II. Series.
QA402.M37 519.2 78-8836
ISBN 0-12-480703-8 (v.3) AACR2

PRINTED IN THE UNITED STATES OF AMERICA

82 83 84 85 9 8 7 6 5 4 3 2 1

Preface

As was true of Volumes 1 and 2, the purpose of this book is twofold. First, it attempts to develop a thorough understanding of the fundamental concepts incorporated in stochastic processes, estimation, and control. Second, and of equal importance, it provides experience and insights into applying the theory to realistic practical problems. Basically, it investigates the theory and derives from it the tools required to reach the ultimate objective of systematically generating effective designs for estimators and stochastic controllers for operational implementation.

Perhaps most importantly, the entire text follows the basic principles of Volumes 1 and 2 and concentrates on presenting material in the most lucid. best motivated, and most easily grasped manner. It is oriented toward an engineer or an engineering student, and it is intended both to be a textbook from which a reader can learn about estimation and stochastic control and to provide a good reference source for those who are deeply immersed in these areas. As a result, considerable effort is expended to provide graphical representations, physical interpretations and justifications, geometrical insights, and practical implications of important concepts, as well as precise and mathematically rigorous development of ideas. With an eye to practicality and eventual implementation of algorithms in a digital computer, emphasis is maintained on the case of continuous-time dynamic systems with sampled-data measurements available; nevertheless, corresponding results for discrete-time dynamics or for continuous-time measurements are also presented. These algorithms are developed in detail to the point where the various design trade-offs and performance evaluations involved in achieving an efficient, practical configuration can be understood. Many examples and problems are used throughout the text to aid comprehension of important concepts. Furthermore, there is an extensive set of references in each chapter to allow pursuit of ideas in the open literature once an understanding of both theoretical concepts and practical implementation issues has been established through the text.

x PREFACE

This volume builds upon the foundations set in Volumes 1 and 2. Chapter 13 introduces the basic concepts of stochastic control and dynamic programming as the fundamental means of synthesizing optimal stochastic control laws. Subsequently, Chapter 14 concentrates attention on the important LQG synthesis of controllers, based upon linear system models, quadratic cost criteria for defining optimality, and Gaussian noise models. This chapter and much of Chapter 13 can be understood solely on the basis of modeling and estimation concepts from Volume 1. It covers the important topics of stability and robustness, and synthesis and realistic performance analysis of digital (and analog) controllers, including many practically useful controller forms above and beyond the basic LQG regulator. Finally, Chapter 15 develops practical nonlinear controllers, exploiting not only the linear control insights from the preceding two chapters and Volume 1, but also the nonlinear stochastic system modeling and both adaptive and nonlinear filtering of Volume 2.

Thus, these three volumes form a self-contained and integrated source for studying stochastic models, estimation, and control. In fact, they are an outgrowth of a three-quarter sequence of graduate courses taught at the Air Force Institute of Technology; and thus the text and problems have received thorough class testing. Students had previously taken a basic course in applied probability theory, and many had also taken a first control theory course, linear algebra, and linear system theory, but the required aspects of these disciplines have also been developed in Volume 1. The reader is assumed to have been exposed to advanced calculus, differential equations, and some vector and matrix analysis on an engineering level. Any more advanced mathematical concepts are developed within the text itself, requiring only a willingness on the part of the reader to deal with new means of conceiving a problem and its solution. Although the mathematics becomes relatively sophisticated at times, efforts are made to motivate the need for, and to stress the underlying basis of, this sophistication.

The author wishes to express his gratitude to the many students who have contributed significantly to the writing of this book through their feedback to me—in the form of suggestions, questions, encouragement, and their own personal growth. I regard it as one of God's many blessings that I have had the privilege to interact with these individuals and to contribute to their growth. The stimulation of technical discussions and association with Professors Michael Athans, John Deyst, Nils Sandell, Wallace Vander Velde, William Widnall, and Alan Willsky of the Massachusetts Institute of Technology, Professor David Kleinman of the University of Connecticut, and Professors Jurgen Gobien, James Negro, J. B. Peterson, and Stanley Robinson of the Air Force Institute of Technology has also had a profound effect on this work. I deeply appreciate the continual support provided by Dr. Robert Fontana, Chairman of the Department of Electrical Engineering at AFIT.

PREFACE xi

and the painstaking care with which many of my associates have reviewed the manuscript. Finally, I wish to thank my wife, Beverly, and my children, Kristen and Keryn, without whose constant love and support this effort could not have been fruitful.

Notation

Vectors, Matrices

Scalars are denoted by upper or lower case letters in italic type.

Vectors are denoted by lower case letters in boldface type, as the vector \mathbf{x} made up of components x_i .

Matrices are denoted by upper case letters in boldface type, as the matrix A made up of elements A_{ij} (ith row, jth column).

Random Vectors (Stochastic Processes), Realizations (Samples), and Dummy Variables

Random vectors are set in boldface sans serif type, as $\mathbf{x}(\cdot)$ or frequently just as \mathbf{x} made up of scalar components \mathbf{x}_i ; $\mathbf{x}(\cdot)$ is a mapping from the sample space Ω into real Euclidean *n*-space R^n : for each $\omega_k \in \Omega$, $\mathbf{x}(\omega_k) \in R^n$.

Realizations of the random vector are set in boldface roman type, as \mathbf{x} : $\mathbf{x}(\omega_k) = \mathbf{x}$.

Dummy variables (for arguments of density or distribution functions, integrations, etc.) are denoted by the equivalent Greek letter, such as ξ being associated with \mathbf{x} : e.g., the density function $f_{\mathbf{x}}(\xi)$. The correspondences are (\mathbf{x}, ξ) , (\mathbf{y}, \mathbf{p}) , (\mathbf{z}, ξ) , $(\mathbf{Z}, \mathcal{L})$.

Stochastic processes are set in boldface sans serif type, just as random vectors are. The *n*-vector stochastic process $\mathbf{x}(\cdot,\cdot)$ is a mapping from the product space $T \times \Omega$ into R^n , where T is some time set of interest: for each $t_j \in T$ and $\omega_k \in \Omega$, $\mathbf{x}(t_j,\omega_k) \in R^n$. Moreover, for each $t_j \in T$, $\mathbf{x}(t_j,\cdot)$ is a random vector, and for each $\omega_k \in \Omega$, $\mathbf{x}(\cdot,\omega_k)$ can be thought of as a particular time function and is called a sample out of the process. In analogy with random vector realizations, such samples are set in boldface roman type: $\mathbf{x}(\cdot,\omega_k) = \mathbf{x}(\cdot)$ and $\mathbf{x}(t_j,\omega_k) = \mathbf{x}(t_j)$. Often the second argument of a stochastic process is suppressed: $\mathbf{x}(t,\cdot)$ is often written as $\mathbf{x}(t)$, and this stochastic process evaluated at time t is to be distinguished from a process sample $\mathbf{x}(t)$ at that same time.

xiii

xiv NOTATION

Subscripts

a: augmented I: ideal

continuous-time; m: model (command generator model) nominal; or noise disturbance or controller

d: discrete-time: r: reference variable or desired steady state SS:

truth model error t: f) final time: initial time 0:

or filter (shaping filter)

Superscripts

transpose (matrix) was religious as well as wright inverse and any enthant z at: complex conjugate transpose; i w#: coseudoinversed by a detail. or optimal : estimate remandance de la région 2. Tal.: aninverse (matrix) allowed and appending Time Fourier transform; and anit L: left inverse territorial or steady state solution of the

e engle pour description for the value of a market. A

 $x_i^{i} \leq a_i^{i}$ for all i to the second state a_i^{i}

Conditional telefolds also against the segment of the segment

PENNSY AND THE

Matrix and Vector Relationships and was the second and a second medianise

registers for course were A > 0: A is positive definite.

For the page to recover the first form the first the present and there is the continue for the second state of

 $A \ge 0$: A is positive semidefinite.

 $x \le a$: componentwise, $x_1 \le a_1, x_2 \le a_2, \dots$, and $x_n \le a_n$.

Commonly Used

sgn(·)

Abbreviations and Symbols of the observations of the engine of the control of the reta posed salikeliji nji dane in diot di 1900 ni darki pa seli yelikotoria, sen kini, saki

E{-} expectation w.p.1 with probability of one 19 M. M. 15 M. $E\{\cdot|\cdot\}$ conditional expectation determinant of exponential norm of $exp(\cdot)$ Himit of the grant have some lim. matrix square root of (see Volume 1) Li.m. limit in mean (square) natural log element of mean square m.s. subset of maximum il minimar a la liggis max. set of; such as minimum seed and down and h min. $\{x \in X : x \le a\}$, i.e., the set R^{h} Euclidean n-space of $x \in X$ such that

signum (sign of) as a three tenset one

trace with to both it was to need accer-

NOTATION

List of symbo are defined or	ols and pages where r first used	they 15		\$\$: !) **; 34 25
	163	1 " 5 4	C	167	+ · · ·
\mathbf{A}_{ij}	153 153 153 153		G_{cy}	167	
a	247	A Line	Ger	6	
.⊗/	105	- 7,	G_d	6	1 - F3
В	6; 88		$\mathbf{G}_{\mathrm{d}}\big[\mathbf{x}(t_i),t_i\big]$	16, 188	
B _{ey}	167	: 53	g	10, 100	
\mathbf{B}_{cz}	167	, 1/85	G	148	
$\mathbf{B}_{\mathbf{d}}$	6, 76	.16/15	\mathscr{G}_{L}	6; 184	4:
B	74; 76	1 85	H	6; 184	()
B	105	9.4 1 .	h	5	44
C	69, 231	-5		45	10 2
c	223	4, 43	I	76	
С	13; 26		\mathbf{I}_i	45	
C*	13; 15; 47; 48		I	105	(110
\mathbf{D}_{y}	81; 89; 231	. ¥ . 7×	J	10	
\mathbf{D}_z	81	(4 E)	$J_{\rm c}$	44.57 4 13	
d	131; 132		${J}_{r}$	75	ķ
dx	5	the Ar	${ar J}_{ m r}$	[™] 76	₹:
dβ	5		K	16; 19; 185	Ų.
ďβ _m	5	e gir	<u> </u>	100	à
E	137	100	K	101	į ₂ !
$\mathbf{E}_{\mathbf{x}}$	151	्र की	K _c	15; 70; 73;	186
$\mathbf{E}_{\mathbf{v}}$	151	33	$\bar{\mathbf{K}}_{\mathbf{c}}$	71; 95; 188	17
$E\{\cdot\}$	9	15.	$\mathbf{K}_{\mathbf{s}\mathbf{s}}$	79	z:
$E\{\cdot \cdot\}$	13	3.5	\mathbf{K}_{x}°	ं ^स 144; 145	1 7 2 1
- (,	50; 100; 115	A.	$\mathbf{K}_{v}^{}$	144	H _y
$\exp\{\cdot\}$	24	,	K _ž	144; 145	;
F	6; 88	72	$\mathscr{K}_{\mathbf{i}}$	14 8	
f	5	11.4	Ж̂р	148	<u>.</u>
$f_{\mathbf{x}}$	31	1000	L,	75	5-
$f_{\mathbf{x}}^{\mathbf{x}}(\boldsymbol{\xi},t \boldsymbol{\rho},t')$	15	1 1 4	L L	26: 40	j.
	54	$(V^{*}(\lambda), \varphi)$	$\overline{\overline{L}}$	46; 47	,0%
$f_{\mathbf{x},\mathbf{y}}$	15: 24		\overline{L}'	48	
$egin{array}{c} f_{x i y} \ \mathbf{G} \end{array}$	6	.**	$L_{ m f}$	21	91
	5 18		$\overline{L}_{\mathbf{f}}$	46; 47	
$G[\mathbf{x}(t),t]$	104		\overline{L}_{t}'	48	T
$\mathbf{G}_{\mathbf{L}}$	96		$\mathbf{M}^{L_{\mathrm{f}}}$	90	
G _c	•		M _D	92°	
G * Č *	16; 69; 73; 186			93	
$ar{\mathbf{G}}_{c}^{*}$	71; 78; 95; 242		$\mathbf{M}_{ extsf{DTI}}$	93	
G_{cx}	167		\mathbf{M}_{TI}	93	

xvi NOTATION

$\mathbf{m}_{\mathbf{x}}$	174	u	14
\mathbf{m}_{xa}	174	u	2
\mathbf{m}_{ya}	171	u _I	153
m	5	\mathbf{u}_k	253
\mathbf{n}_{d}	, 127	\mathbf{u}_{0}	123; 230
n	5	u ^{ĂCE}	241
$\mathbf{P} = \mathbf{P}(t/t)$	185	\mathbf{u}^{CL}	226
$\mathbf{P}(t_i^-)$	19	u ^{DO}	227
$\mathbf{P}(t_i^+)$	15; 19	$\mathbf{u}^{\mathbf{F}}$	226
₽	100; 188	\mathbf{u}^{MF}	226
$ar{\mathbf{P}}^-$	100; 243	\mathbf{u}^{OL}	225
\mathbf{P}_{xa}	174	u*	13; 16; 17; 47; 48
$\mathbf{P}_{xx}(t)$	174	U	26
\mathbf{P}_{ya}	171	u_0	230
\mathbf{P}_{o}	10	V	85
$\mathbf{p}(t_i)$	40; 204	v	5
p	2	$\mathbf{v_c}$	5; 185
$p_k(t_i)$	40; 254	$\hat{\mathbf{W}}$	90; 100
Q	5	$\mathbf{W}_{_{\mathbf{D}}}$	91
Q	75; 76	$\mathbf{W}_{\mathtt{DTI}}^{-}$	92
\mathbf{Q}_{d}	6; 9; 76	$\mathbf{W}_{ ext{TI}}$	92
q .	133	\mathbf{W}_{uu}	74
\mathbf{q}_{k}	171	\mathbf{W}_{xu}	74
$\mathbf{q}_{\mathbf{o}}$	137	\mathbf{W}_{xx}^{m}	74
q_k	171	w	74
R	5; 10	$\mathbf{w_d}$	6
$\mathbf{R_c}$	5; 184	X	10; 76
R"	13	$ar{\mathbf{X}}$	76
r	254	$\mathbf{X_f}$	10
r	2	X	3
S	48	$\hat{\mathbf{x}}(t)$	185
S S	73; 76	$\hat{\mathbf{x}}(t_i^-)$	18
S	76	$\hat{\mathbf{x}}(t_i^+)$	15; 19
S_i	47	$\mathbf{\hat{x}}(t/t_{i-1})$	243
$\mathbf{S}_{\mathbf{y}}$	115	$\tilde{\mathbf{x}}$	101
S	52	$\mathbf{x}_{\mathbf{I}}$	153
S	5	\mathbf{X}_{n}	83
T	xiii	\mathbf{x}_0	123; 230
t_{N+1}	10	$\widehat{\mathbf{x}}_{k}$	254
t_{f}	186	$\mathbf{\hat{x}}_{0}$	52
t_i	6	$\mathbf{x}^{\mathbf{d}}$	99
<u>U</u>	10; 76	x_0	230
Ū	76	Y	69

$\mathbf{Y}_{\mathbf{f}}$	69	$\delta {f y}_{ m c}$	124; 159
y _a	94; 170	ε	148
y _c	2; 69; 81; 89; 223	λ	109
$\mathbf{y_d}$	2	$\hat{\lambda}_{\max}$	109
$\mathbf{\hat{y}_c}(t_i^-)$	133	ξ	147; 164
$\hat{\mathbf{y}}_{\mathbf{c}}(t_i^+)$	133	П	123
Z	12; 45	$oldsymbol{\Sigma}$	57
\mathbf{Z}_{i}	12; 45	σ	171
$\mathbf{z}(t)$	184	σ^2	171
$\mathbf{z}(t_i)$	5; 6; 81	σ_{max}	106
\mathbf{Z}_{i}	12; 18	σ_{\max}^*	107; 111
\mathbf{z}_{o}	126; 238	σ_{\min}	106
Z	97	Φ	6; 76
β	5	$\Phi_{\rm c}$	167
$\beta_{\rm m}$	5; 184	ϕ	6; 40
ΔG	104	$\overset{\cdot}{\phi}_{\mathtt{x}}$	64
Δu	133	Ψ_{xx}	171
$\Delta\mathscr{G}$	105	$\Psi_{-}(\omega)$	182
δχ	88; 159	Ψ_{ya}	171; 17 4
δu	88; 159	$\Omega^{'}$	xiii; 15
δz	126	ω	12; 107
$\delta \mathbf{q}$	136	w.	107

Contents

Preface	
Notation	1

ix xiii

VOLUME 1

Chapter 1 Intro	duction
-----------------	---------

Chapter 2 Deterministic system models

Chapter 3 Probability theory and static models

Chapter 4 Stochastic processes and linear dynamic system models

Chapter 5 Optimal filtering with linear system models

Chapter 6 Design and performance analysis of Kalman filters

Chapter 7 Square root filtering

VOLUME 2

Chapter 8 Optimal smoothing

Chapter 9 Compensation of linear model inadequacies

Chapter 10 Parameter uncertainties and adaptive estimation

Chapter 11 Nonlinear stochastic system models

Chapter 12 Nonlinear estimation

VOLUME 3

Chapter 13	Dynamic programming and stochastic control	
13.1	Introduction	- 1
13.2	Basic Problem Formulation	2
13.3	Introduction to Concepts: Overview of Simple LQG Problem	. i 9
13.4	The Backward Kolmogorov Equation	20
13.5	Optimal Stochastic Control with Perfect Knowledge of the State	24
13.6	Optimal Stochastic Control with Noise-Corrupted Measurements	45
13.7	Summary	58
	References	60
	Problems	6
Chapter 14	-	• •
	performance analysis	
14.1	Introduction	68
14.2	The LQG Stochastic Regulator	69
14.3	Stability	82
14.4	Stability of LQG Regulators	91
14.5	Stability Robustness of LQG Regulators	402
14.6	The LQG Synthesis of Trackers	114
14.7	Nonzero Setpoint Controllers	1762
14.8 14.9	Rejection of Time-Correlated Disturbances The LQG Synthesis of PI Controllers	126
14.10	Command Generator Tracking	132
14.11	Performance Evaluation of Linear Sampled-Data Controllers	151 166
14.12	Systematic Design Procedure	175
14.43	The LQG Controller for Continuous-Time Measurements	184
14.14	Summary	190
	References	193
	Problems	202
Chapter 15	Nonlinear stochastic controllers	
15.1	Introduction	223
15.2	Basic Problem Formulation and Controller Characteristics	223
15.3	Linear Perturbation Control Laws for Nonlinear Systems: Direct	
	Application of LQG Synthesis	250
15.4	Assumed Certainty Equivalence Design	241
15.5	Closed-Loop Law Approximations and Dual Effect	245
15.6 15.7	Stochastic Adaptive Control	247
15.7	Design Philosophy	256
13.8	Summary and Perspective References	257
*	Problems	260 266
•	- COMMING	
Index		271

Abord add yo hadinggala ad maa tamanna i ama tama haranna agta a realis na l appropriate scalar terraina la poste en decide de conserva de sua Politica de la colonida en contra podrocy a set of me hand metablished for the principle of hatches come entermount in emili experimonante sinancia della metale di se la come della come benezia e sundamen iraja zardalnea hallerirans ada an i esa di Evdiresar, ana berezia u gundunar sega zanomini in ini. Barezea en gidote tem iglalitup izioenti en tity (lunger ini. inter luscustida). Barezea en gidote tem iglalitup izioenti en tity (lunger ini. inter luscustida). a gainfinialla in 1945, ifanonyo a gainemi af en ele e CHARTER. no zig wares to sa) stanton Dynamic programming telegrates and soldier and standard former of any experience and stochastic control A their country and adomed foremap without dien ten agreement replace with some time. trove traence vitrafavi a kortaliko esira astesalari resora etertirata ceno ano eterti testadorado estro estretes de amegacido de la distracción de la tractición Bedeen. De order in ediscuse cartoir reports of the sound sistem bulliprion. vice to frequent arise the areas courses a consequence of the arrangement of the areas villes, con pairin, bas and dissibilities of the street from the control of the street in the street 13.1 INTRODUCTION Tap Bahkarap méli da 2000 dan bendulih dan Clab

Up to this point, we have considered estimation of uncertain quantities on the basis of both sampled-data (or continuous) measurements from a dynamic system and mathematical models describing that system's characteristics. Now we wish to consider exerting appropriate control over such a system, so as to make it behave in a manner that we desire. This chapter will formulate the optimal stochastic control problem, provide the theoretical foundation for its solution, and discuss the potential structure for such solutions. The two subsequent chapters will then investigate the practical design of stochastic controllers for problems adequately described by the "LQG" assumptions (Linear system model, Quadratic cost criterion for optimality, and Gaussian noise inputs; see Chapter 14) and for problems requiring nonlinear models (see Chapter 15).

Within this chapter, Sections 13.2 and 13.3 are meant to be a basic overview of the optimal stochastic control problem, providing insights into important concepts before they are developed in detail later. Stochastic dynamic programming is the fundamental tool for solving this problem, and this algorithm is developed in the remainder of the chapter. It will turn out to be inherently a backward recursion in time, and the backward Kolmogorov equation useful for reverse-time propagations is presented in Section 13.4. Dynamic programming itself is then used to generate the optimal stochastic control function, first in Section 13.5 assuming that perfect knowledge of the entire state is available from the system at each sample time, and then in Section 13.6 under the typically more realistic assumption that only incomplete, noise-corrupted measurements are available.

13.2 BASIC PROBLEM FORMULATION

Fundamentally, control problems of interest can be described by the block diagram in Fig. 13.1. There is some dynamic system of interest, whose behavior is to be affected by r applied control inputs $\mathbf{u}(t)$ in such a way that p specified controlled variables $y_c(t)$ exhibit desirable characteristics. These characteristics are prescribed, in part, as the controlled variables $y_c(t)$ matching a desired p-dimensional reference signal $y_d(t)$ as closely, quickly, and stably as possible, either over time intervals (as in tracking a dynamic $y_d(t)$ or maintaining a piecewise constant setpoint y_d) or at specific time instants (as at some given terminal time in a problem). Stability of the controlled system is an essential requirement, after which additional performance measures can be considered. However, the system responds not only to the control inputs, but also dynamics disturbances $\mathbf{n}(t)$ from the environment, over which we typically cannot exert any control. These usually cause the system to behave in an other-than-desired fashion. In order to observe certain aspects of the actual system behavior, sensor devices are constructed to output measurements $\mathbf{z}(t)$, which unfortunately may not correspond directly to the controlled variables, and which generally are not perfect due to measurement corruptions $\mathbf{n}_{m}(t)$. These typically incomplete and imperfect measurements are then provided as inputs to a controller, to assist in its generation of appropriate controls for the dynamic system.

EXAMPLE 13.1 For instance, the dynamic system might be the internal environment of a building, with controlled variables of temperature and humidity at various locations to be maintained at desired values through the control inputs to a furnace, air conditioner, and flow control dampers in individual ducts. Environmental disturbances would include heat transfer with external and internal sources and sinks, such as atmospheric conditions and human beings. Direct measurements of temperature and humidity at all locations may not be available, and additional measurements such as duct flow rates might be generated, and all such indications are subject to sensor dynamics, biases, imprecision, readout quantization, and other errors.

The case of matching the controlled variables to desired values at a single time might be illustrated by an orbital rendezvous between two space vehicles. On the other hand, continuously

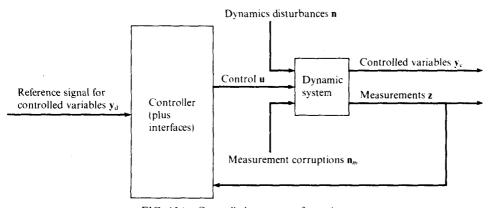


FIG. 13.1 Controlled system configuration.

matching $y_c(t)$ to a dynamic $y_d(t)$ is displayed in the problem of tracking airborne targets with communication antennas, cameras, or weapon systems.

Let us consider the "dynamic system" block in Fig. 13.1 in more detail. The first task in generating a controller for such a system is to develop a mathematical model that adequately represents the important aspects of the actual system behavior. Figure 13.2 presents a decomposition of the "dynamic system" block assuming that an adequate model can be generated in the form of a stochastic state model, with associated controlled variable and measurement output relations expressed in terms of that state. As shown, the controller outputs, $\bf u$, command actuators to impact the continuous-time state dynamics, which are also driven by disturbances, $\bf n$. The sensors generate measurements that are functions of that state, corruptions, $\bf n_m$, and possibly of the controls as well. Similarly, the controlled variables are some function of the system state; the dashed line around "controlled variable output function" denotes the fact that it does not necessarily correspond to any physical characteristic or hardware, but simply a functional relationship.

A number of different types of measurements might be available. In the trivial case, nothing is measured, so that the controller in Fig. 13.1 would be an open-loop controller. However, in most cases of interest, there are enough disturbances, parameter variations, modeling inadequacies, and other uncertainties associated with the dynamic system that it is highly desirable to feed back observed values of some quantities relating to actual system response. One might conceive of a system structure in which all states could be measured perfectly; although this is not typically possible in practice, such a structure will be of use to gain insights into the properties of more physically realistic feedback control systems. Most typically, there are fewer measuring devices than states, and each of these sensors produces a signal as a nonlinear or linear function of the states, corruptions, and in some cases, controls. Physical sensors are often analog devices, so that continuous-time measurements from continuous-time systems are physically realistic. However, in most of the applications of interest to us, the controller itself will be implemented as software in a digital computer, so we will concentrate on continuous-time state descriptions with sampled-data measurements, compatible as inputs to the computer. We will also consider continuous-time measurements, and discrete-time measurements from systems described naturally only in discrete time, but the sampled-data case will be emphasized. On the other hand, it will be of importance to consider the controlled variables not only at the sampling instants, but throughout the entire interval of time of interest: reasonable behavior at these discrete times is not sufficient to preclude highly oscillatory, highly undesirable performance between sample times.

The noises \mathbf{n} and \mathbf{n}_{m} in Fig. 13.2 correspond to disturbances, corruptions, and sources of error that can be physically observed. Since physical continuous-time noises cannot truly be white, shaping filter models driven by fictitious

को गार स्थित है। जा महिला पूर्ण प्रकार के जा हमारे जा कमा के केदर में हुमें जा हमार को असम्बन्धी जाने हैं, आहे स्थान जाने के असमार समाम केदर के समाम केदर के समाम केदर के स्थान केदर के समाम केदर केदर केदर केदर केदर केदर के

Ziri ani tatim ta ze ta Measurements z Courrelled variable output function Sciisors System state x, Continuous-tune state dynamics Measurement corruptions Actuators

에 가장 하는 사람들이 되었다. 그 그 사람들이 가장 그를 보고 있다. 그 사용하는 사람들이 가용하는 사용하는 사람들이 되었다. 사람들이 가장 그를 보고 있다. 주는 사람들은 기용하는 기용하는 기업이 좋아하는 기업이 되는 사람들이 있다.