MODERN FILTER DESIGN

Active RC and Switched Capacitor

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Preface

The book is intended as a text for senior undergraduates and/or first-year graduate students to be covered in a one-semester course. The book is also intended for self-teaching and reference for practicing engineers.

Chapter 1 covers basic properties and classifications of systems together with filter transmission and approximations. Frequency- and time-domain, continuous-time and discrete-time signal transmissions are considered. The latter is important in the analysis and design of switched capacitor networks discussed later in the book. Chapter 2 deals entirely with op amps. In this book the op amp is considered (for obvious reasons) as the main active element, and other elements, such as gyrators and frequency-dependent negative resistors, are op amp-derived circuits. The discussion of the op amp includes both bipoler and MOS integrators. Chapter 3 gives various definitions of sensitivity as used in the literature. Statistical sensitivity measures, which are becoming more popular and are found to be useful in practical design, as they give good correlation with Monte Carlo analysis, are treated in detail. Chapter 4 discusses continuous-time second-order active sections which are predominantly used as blocks in active filter design. The biquadratic sections considered include single- and multiple-op amp, active-*RC*, active-*R*, and active-*C* realizations. Chapter 5 provides a complete treatment of high-order filter design. Multiple-loop feedback design methods are considered in some detail, as these configurations provide the best sensitivity performance in a properly designed high-order filter. The merits and disadvantages of various design methods are considered so that the designer is made aware of the compromises that must be made in a practical design. Chapter 6 is concerned with the analysis and design of switched capacitor filters. This type of filters is rapidly becoming very popular, as they are compatible with

MOS large-scale integration. Switched capacitor recursive filters are recent developments that fully utilize the advantage provided by MOS LSI, and hence hold future promise in filter design.

Three appendices are included, namely, selected topics in passive-network properties; tables of classical filter functions; and, op amp terminologies and selected data sheets. These appendices are intentionally kept brief to avoid duplication of material found in several books.

Each chapter is complemented by references and problems. The references used are books and published papers. Only available and pertinent references are given. No attempt has been made to cite the original papers or to be complete. The problems are used to provide the reader with exercise material in order to gain a better understanding of the text and to extend the design methods to cover practical situations.

Although active filters have reached maturity, the need for better performance, lower sensitivity, and reduced cost has motivated several recent advances in the state of the art. These advances include computer-aided design (CAD) methods for coupled or multiple-loop feedback topologies used to realize low-sensitivity high-order filters, active-R and active-C networks for monolithic integrated-circuit, high-frequency filters, and switched capacitor networks to realize precision, monolithic integrated-circuit, audio-frequency filters. Thus, one of the purposes of this book is to provide discussions in a tutorial manner, for students as well as practicing engineers, of these and other recent advances which heretofore have appeared only in technical articles scattered in many journals. To the best of our knowledge, some of this material, especially that dealing with switched capacitor networks, appears for the first time in a textbook in a detailed manner. Although we discuss recent advances, we do, of course, include the bread-and-butter active-filter realization methods that have stood the test of time. The book is not, then, just another state-of-the-art text, but one that provides a comprehensive tutorial presentation of active-filter technology from a practical standpoint as well as from the state-of-the-art point of view.

The authors wish to acknowledge the contributions of many authors whose results appear in this text but whose names may or may not appear. We would like especially to thank Professors Adel Sedra and Rolf Schauman, who reviewed the complete manuscript and offered many helpful suggestions and thoughtful advice. Dr. Pradeep Padukone and Dr. Ali Gonuleren also helped in proofreading the manuscript. Some of the original research results of the authors appearing in this book were supported by National Science Foundation grants, for which we wish to express our gratitude.

Some of the content of this book, in particular Chapter 6, received inspiration from my (K.R.L.) work in the Signal Processing and Integrated Circuit Design Department at Bell Laboratories. Significant parts of Chapter 6, which covers the area of switched capacitor networks, are based on collaborative work with Dr. Paul Fleischer. We are indebted to K.R.L.'s former department head, Mr. Carl Simone, and present department head, Dr. Dan Stanzione, for providing their encouragement and for providing the stimulating evironment that influenced the presentation of many of the concepts covered in this book.

A personal note of gratitude goes to Mr. Joe Friend for his encouragement, his careful reading of the various versions of the manuscript, and his helpful suggestions throughout the writing of this book.

Last but not least, we express our gratitude and love to our families for their understanding and patience throughout the development of this manuscript.

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chapter one

Filter Transmission and Related Topics

1.1 INTRODUCTION

An *electrical wave filter* can be defined as an interconnected network of electrical components, such as resistors, capacitors, inductors, and transistors, which operates on or processes applied electrical signals. The applied electrical signal is referred to as the *input signal* or *excitation*. The product of the processing performed by the network on the excitation is referred to as the *output signal* or *response*. The excitation and response will differ according to the processing or filtering performed by the network. As we shall see shortly, there are several means by which we can represent and specify this filtering operation. In this book we are concerned with the analysis, specification, design, and realization of electrical filters for the frequency range $f \leq 500$ kHz, with emphasis on audio-frequency ($f \leq 20$ kHz) applications. It is in this frequency range where the greatest benefits are derived from the use of active components in electrical filter networks.

Filtering in the general sense is a selection process. The output of a filter is then a selected subset of the input. In electrical filtering the object is to perform frequency-selective transmission. We know from elementary signal theory that any periodic wave of period $2\pi/\omega_R$ can be represented by its Fourier series expansion

$$s(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos k\omega_R t + \sum_{k=1}^{\infty} b_k \sin k\omega_R t \tag{1-1}$$

where the coefficients a_k and b_k define the harmonic content of the wave $s(t)$. The frequency-independent term a_0 is referred to as the *dc component*. When a

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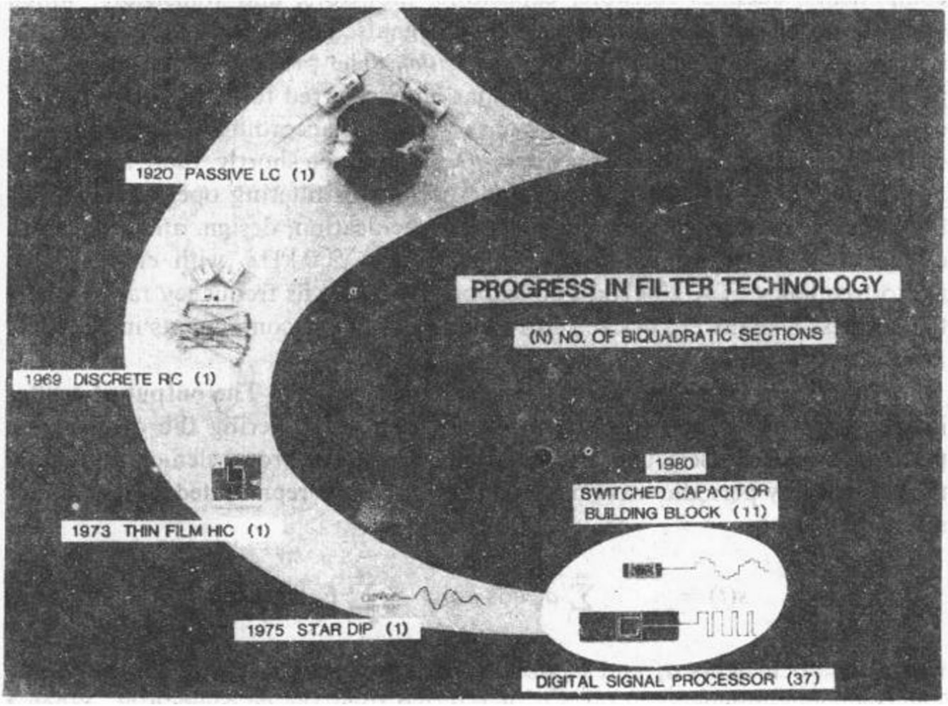
harmonically rich $s(t)$ is applied to a filter, the filtering process will serve to alter the magnitude of the coefficients a_k and b_k; some coefficients may be greatly attenuated, thus defining a stop band. The selective enhancing, attenuating, and removal of the harmonic components of $s(t)$ is what electrical filtering is all about. In a linear filter, the filtering operation cannot yield an output that has more harmonic components than the input. Thus, given an input and a desired output, it is the task of the filter designer to determine:

1. The filtering operation required.
2. The network to implement the filtering operation.
3. The network component values that realize the specific filtering operation.

The choice of network implementation is almost always determined by economic considerations and technology. Because of advances in technology, what was considered the best economical filter implementation 10 years ago, perhaps only one year ago, may not be the most economical implementation today. In Fig. 1-1 is shown the evolution of filter technology in the Bell System, for

Fig. 1-1 Bell System progress in filter technology, depicting the evolution of voice-frequency ($f < 4$ kHz) filters from 1920 to 1980. Shown on the crescent is the hardware required to realize a second-order section with (a) passive *LC*, (b) discrete component active-*RC*, (c) and (d) hybrid thin-film active-*RC* technologies. Shown in the oval are examples of monolithic filters: (e) an analog switched-capacitor building block capable of realizing up to 11 second-order sections and (f) a digital signal processor capable of realizing up to 37 second-order sections with an 8-kHz sampling rate.



voice-frequency ($f < 4$ kHz) applications over a nearly 60-year span. What is shown in each stage of Fig. 1-1 is the hardware to implement a second-order filtering function. Although the evolution depicted is specific to the Bell System, it is to a good approximation representative of the evolution industry wide. From 1920 to the latter 1960s the majority of voice-frequency filters were realized as discrete *RLC* networks. However, it was recognized in the 1950s that size and eventual cost reductions could potentially be achieved by replacing the large costly inductors with active networks. That is, a network comprised of resistors, capacitors, and transistors could be made to resonate like a tuned *RLC* network. These active networks, referred to as *active-RC networks*, remained essentially research curiosities until the mid-1960s, when good-quality active components such as operational amplifiers became inexpensive and readily available. Although size has not been significantly reduced, filtering and amplification were achieved simultaneously. In the early 1970s the economic potential envisioned for active-*RC* filters began to be realized with batch-processed thin-film hybrid integrated circuits (HICs). The HIC shown in Fig. 1-1, composed of two thin-film capacitors, nine thin-film resistors, and one silicon integrated-circuit (SIC) operational amplifier, is 1.05×1.00 in. This circuit represented about a factor-of-2 cost reduction over the equivalent passive-*RLC* realization. By 1975, thin-film technology had advanced such that the 1.05×1.00 in. HIC could be reduced in size to fit into a small 16-pin dual-in-line package (DIP). Today, using switched-capacitor and digital filter techniques, very high order filters can be realized as microminiature silicon integrated-circuit chips. As examples we show in Fig. 1-1 two packaged integrated circuit chips; namely, the digital signal processor (DSP) and the switched capacitor building block (SCBB). The DSP implements up to 37 second order sections of digital filtering where as the SCBB implements up to 11 second order sections of analog filtering. A detailed description of the SCBB is reserved for Chapter 6. The reader interested in pursuing the area of digital filters is referred to references [B9, B10, and B11].

As the evolution depicted in Fig. 1-1 has demonstrated, the use of active networks has enabled engineers to utilize the advances in integrated-circuit technology to implement low-cost, microminiature voice-frequency filters. Although active filters can be used throughout the voice-frequency range ($f < 4$ kHz), voice-frequency applications account for tens of millions of the active filters that are produced yearly throughout the world. Aside from their obvious size and weight advantages over equivalent passive-*RLC* implementations shown in Fig. 1-1, active filters provide the following additional advantages:

1. Increased circuit reliability because all processing steps can be automated.
2. In large quantities the cost of integrated-circuit active filters are much lower than equivalent passive filters.
3. Improvement in performance because high-quality components can be readily manufactured.
4. A reduction in parasitics because of smaller size.

5. Active filters and digital circuitry can be integrated onto the same silicon chip. This advantage has been fully realized with active switched-capacitor filters.

In addition to these advantages, which stem from their physical implementation, there are other advantages which are circuit-theoretic in nature, namely:

1. The design and tuning processes are simpler than those for passive filters.
2. Active filters can realize a wider class of filtering functions than passive filters. Some properties of passive networks are reviewed in Appendix A.
3. Active filters can provide gain; in contrast, passive filters often exhibit a significant loss.

With these advantages there are some drawbacks to active network implementations. They are:

1. Active components have a finite bandwidth, which limits most active filters to audio-frequency applications. In Chapter 4 we see that by properly using this intrinsic band limiting, this limitation can be significantly reduced. In contrast, passive filters do not have such an upper-frequency limitation and they can be used up to approximately 500 MHz.
2. Passive filters are less affected by component drifts in manufacture or drifts due to environmental changes. This phenomenon, referred to as *sensitivity*, will be seen to be an important criterion for comparing similar filter realizations. Much of the disadvantage experienced by active-network implementations stems from the relatively large number of components, required to realize an active filter, as compared to equivalent passive realizations. In Chapter 5 we show techniques for minimizing this disadvantage.
3. Active filters require power supplies, whereas passive filters do not. In this era of energy conservation it is particularly important that active devices be used in a most efficient manner. For this reason the reader will be encouraged to design active filters with the minimum number of active devices consistent with precision performance.

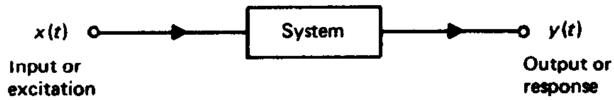
It can be said that in voice and data communication systems, the economic and performance advantages of active filters far outweigh these disadvantages. Testimony to this fact is the general acceptance and usage of active filters throughout the telecommunications industry.

It is assumed that the reader has a working knowledge of network theory and has had some exposure to linear system theory. In this chapter we review some of the fundamental concepts and tools needed to characterize a continuous linear filter. We also take the opportunity to introduce the reader to the fundamental concepts of sampled-data systems and the z-transform as these concepts are used in the analysis, specification, and design of sampled-data switched capacitor filters.

1.2 CLASSIFICATION OF SYSTEMS

A schematic "black-box" representation of a system is shown in Fig. 1-2. On the left side of the box we symbolically show the input to the system or the excitation. As the arrow indicates, the input represents information (e.g., an electrical signal) injected into the system. The output or response, shown symbolically on the right side of the box, represents the systems reaction to its excitation.

Fig. 1-2 Schematic representation of a system.



This output may be observable to the outside (e.g., voltmeter, oscilloscope, etc.), or it may serve as the input to yet another system. Generally, the input $x(t)$ and output $y(t)$ are functions of time and are called *signals*. The black box may, for example, represent a linear or a nonlinear network.

In order to describe a system, let us define the various classes of systems.

1.2.1 Linear and Nonlinear Systems

A system is *linear* if it satisfies the principle of superposition. Mathematically, a system is linear if and only if

$$f(\alpha x_1 + \beta x_2) = \alpha f(x_1) + \beta f(x_2) \tag{1-2}$$

where α and β are arbitrary constants and x_1 and x_2 are input signals. For example, if an input $x_1(t)$ yields an output $y_1(t)$, which we may write symbolically as $x_1(t) \rightarrow y_1(t)$, and if $x_2(t) \rightarrow y_2(t)$, then from Eq. (1-2) we write

$$\alpha x_1(t) + \beta x_2(t) \longrightarrow \alpha y_1(t) + \beta y_2(t) \tag{1-3}$$

To be more germane to the material in this text, it is stated without proof that any system governed by linear differential or difference equations is linear. Only linear systems are considered in this text.

1.2.2 Continuous-Time, Discrete-Time, and Sampled-Data Systems

A system is said to be a *continuous-time* or a *continuous analog system* if input x and output y are capable of changing at any instant of time. We make the fact of continuous change evident by writing x and y as functions of the continuous-time variable t; that is,

$$x = x(t) \qquad \text{and} \qquad y = y(t) \tag{1-4}$$

In discrete-time and sampled-data systems, the input and output signals change at only discrete instants of time. The values of the signals between instants are of no interest in discrete-time systems. In sampled-data systems, which are actually analog systems, the signals are usually held constant between (sampling) instants. More will be said later about the difference between discrete-time and sampled-data systems. However, we can represent sampled-data input and output signals as functions of discrete variables kT:

$$x = x(kT) \qquad \text{and} \qquad y = y(kT) \tag{1-5}$$

where k is an integer and T is the time duration between samples. The signal formats for each of these system types is shown in Fig. 1-3. Note that the amplitude of the output may be larger, smaller, or equal to the input, depending on the type and design of the filter.

An important mathematical distinction between continuous-time and sampled-data (also discrete-time) systems is the fact that continuous-time systems are characterized by differential equations, whereas discrete-time systems are characterized by difference equations. We will explore this difference in greater detail later.

1.2.3 Time-Invariant and Time-Varying Systems

A system is said to be *time-invariant* if the shape of the response to an input applied at any instant of time depends only on the shape of the input and not on the time of application. In mathematical terms, a system is linear and time-invariant if

$$x(t + \tau) \longrightarrow y(t + \tau) \tag{1-6}$$

for all $x(t)$ and all $\tau > 0$. Similarly, for a discrete-time system, if we have

$$x(kT) \longrightarrow y(kT) \tag{1-7}$$

then the system is time-invariant if and only if

$$x[(k - n)T] \longrightarrow y[(k - n)T] \tag{1-8}$$

for any $x(kT)$ and n.

For a causal system, the response cannot precede the excitation; that is, if the excitation is applied at some time t_0 or mT, then the response is zero for $t < t_0$ or $k < m$.

The equilibrium conditions describing a lumped-element linear time-invariant system are characterized by linear differential or difference equations with constant coefficients—in contrast with distributed systems, which are

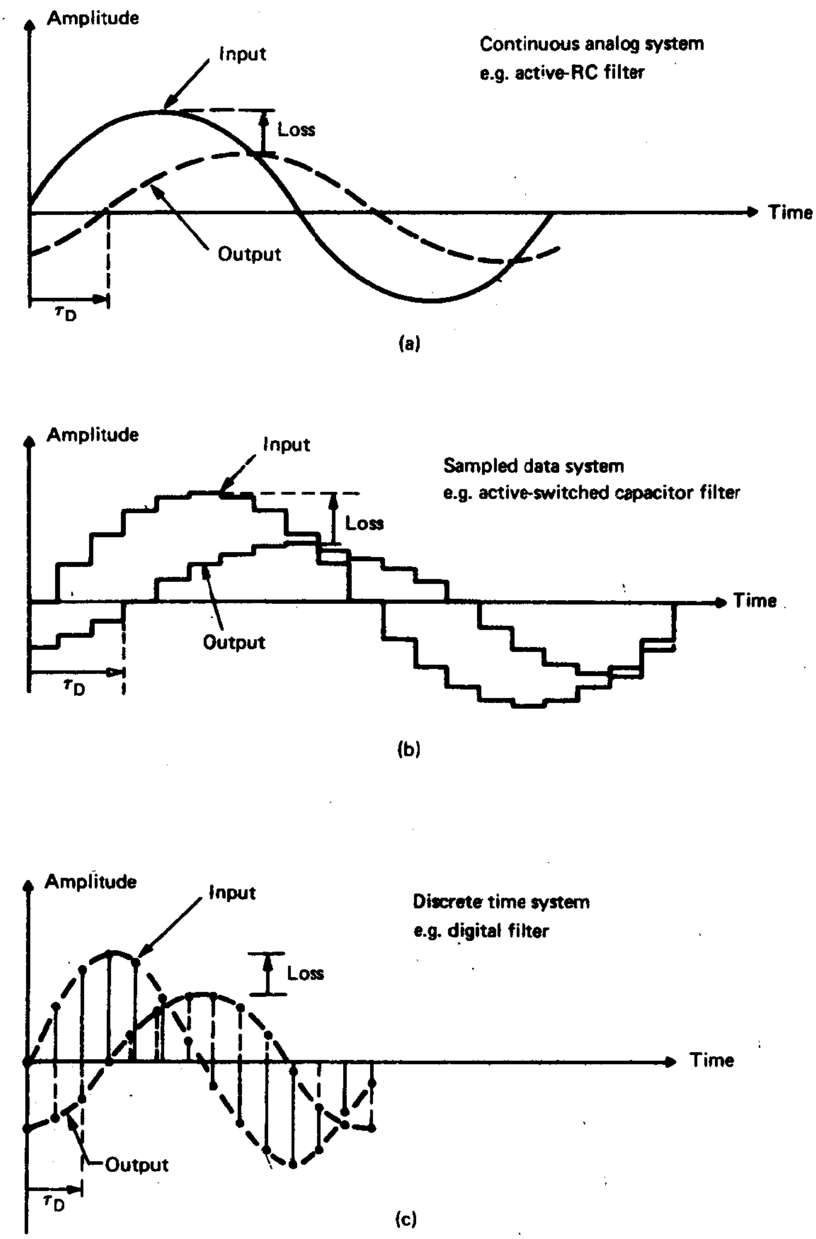


Fig. 1-3 Signal formats for continuous analog, sampled-data analog, and discrete-time systems.

described by partial differential equations. By a *lumped-element system* we mean a system comprised of lumped passive elements and active devices. Lumped sampled-data systems will also contain samplers or switches. Most of the networks considered in this text are time-invariant. However in Chapter 6 we will see that switched-capacitor sampled-data networks are, in general, time-varying systems. Fortunately, after some mathematical manipulation,