Approximation Methods for Electronic Filter Design

With Applications to Passive, Active, and Digital Networks

By
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Preface

This book was written to help fill a void that exists in the literature of filter theory. There are many electrical-engineering books that can be used to help synthesize passive, active, or digital filters; however, there is no satisfactory book that treats the approximation problem from the viewpoint of an electrical engineer—and the approximation problem must be solved before one can synthesize a filter. This book is concerned with finding approximations for attenuation and delay functions; the approximations are ratios of polynomials, and the polynomials are expressed in terms of the frequency variable s.

There are many approximation books written by mathematicians, but these are not directly applicable to filter design. Although some filter-synthesis books devote a chapter to the approximation problem, this is not nearly enough for a comprehensive treatment. Thus, electrical engineers have had to read many articles to become familiar with the approximation problem. This book presents various solutions to the problem in an organized and logical manner.

The first few chapters are concerned with what might be termed the classical approximation theory: they discuss Butterworth, Tschebycheff, inverse Tschebycheff, and elliptic filters. These chapters apply to low-pass filters; Chapter 6 presents transformations that can be used for other types of filters.

Chapter 7 discusses a special transformation. It introduces filter design in terms of the transformed variable and serves as a bridge between classical approximation theory and modern approximation theory. Modern approximation theory makes use of the transformed variable because it provides computational accuracy and simplifies many derivations. The transformed variable is used in Chapters 8 through 12 to investigate filters that have arbitrary stopbands and either equiripple or maximally flat passbands.

After treating approximations for magnitude response, the book discusses delay and transient response. Chapters 14 and 15 contain formulas that can be used to calculate the delay and transient response for any of the filters mentioned in previous chapters.

The remaining chapters discuss methods for synthesizing the transfer functions obtained in the first part of the book. Chapter 16 treats passive network synthesis, Chapter 17 active filter synthesis, and Chapter 18 digital filter synthesis. This material emphasizes the fact that the transfer functions obtained in the book are not just mathematical abstractions; networks can be constructed to perform these filtering functions.

Modern approximation theorists utilize computers to help solve their problems; therefore it seemed essential to include a substantial number of programs in the book. The programs were written in Telcomp II because that language contains statements that are easy to understand. It is not expected that the reader will ever program in Telcomp, but he should be able to write similar programs in other languages. An appendix discusses Telcomp II in enough detail so that the programs in this book can be comprehended.

Problems have been included at the ends of most chapters for two reasons: First, they can help test the reader's comprehension of the material and thus serve as a learning aid. Second, many of the problems are used to establish results that appear in the text. This approach keeps lengthy proofs from cluttering up the presentations. Answers to selected problems are given at the end of the text.

The working environment at Bell Telephone Laboratories helped make this book possible. There were numerous stimulating discussions with colleagues, and most of the material in the book was taught in a course to fellow employees. Special thanks are due them for their valuable comments and suggestions, as well as to the many people who helped prepare the manuscript.

Finally, to my wife and children, who found out firsthand how much time is required to write a book of this nature, my appreciation for your patience and understanding.

Richard W. Daniels

Symbols

A	attenuation, loss
A_i	attenuation to the right of FS_i
A_{max}	maximum passband loss
A_{min}	minimum stopband loss
A_{min_i}	attenuation of ith arc at frequency F_{min_i}
B_n	nth-order Butterworth polynomial
cn	elliptic cosine function
C_H	constant multiplier of H
dn	elliptic difference function
D	delay
D_{min_i}	minimum difference (excess attenuation) of the ith arc
$D(Z)$	polynomial that determines the loss minimums of arcs
$D(\omega)$	delay
e	numerator of H, as in $H(s) = e(s)/q(s)$
e_e	even part of $e(s)$
e_o	odd part of $e(s)$
Ev	even part of
$E(Z)$	transformed version of $e(s)$
$E^*(Z)$	transformed version of $e(-s)$
f	frequency
f	numerator of K, as in $K(s) = f(s)/q(s)$
f_e	even part of $f(s)$

f_i	location of *i*th attenuation pole
f_o	odd part of $f(s)$
F	frequency
FA	lower passband edge
FB	upper passband edge
FH	upper stopband edge
FL	lower stopband edge
$F_{\min_i}$	frequency at which *i*th arc is closest to the requirement
FS_i	*i*th attenuation step
$F(Z)$	transformed version of $f(s)$
H	transfer function (ratio of input to output)
$i(t)$	impulse response
IA	image attenuation
IL	insertion loss
IP	integer part of
j	$\sqrt{-1}$
k	modulus of the elliptic integral
k'	complementary modulus of the elliptic integral
K	characteristic function, complete elliptic integral
K'	complementary complete elliptic integral
$K_{\min}$	minimum stopband value of $K(s)$
L	minimum stopband value of $\|R_n(x,L)\|$
$L(Z)$	loss function
m	a parameter defined by m = NZ + NIN + 2N
n	indicates degree of a filter, polynomial etc.
N	number of attenuation poles (excluding those at zero and infinity)
NA	number of poles in lower stopband
NB	number of poles in upper stopband
NIN	number of attenuation poles at infinite frequency
NZ	number of attenuation poles at zero frequency
P	parametric multiplier
P_0	power dissipated in load when coupling network is replaced by a short circuit
P_1	power delivered to coupling network
P_2	power dissipated in load
P_m	maximum available power
P_r	reflected power
q	denominator of $H(s)$, as in $H(s) = e(s)/q(s)$
Q	quality of a root
Q_p	quality of a pole
Q_z	quality of a zero
$Q(Z)$	transformed version of $q(s)$
R_c	relative change in ripple
R_n	*n*th-order Tschebycheff rational function
s	complex frequency $s = \sigma + j\omega$
sn	elliptic sine function
S	normalized complex frequency

SA	number of attenuation steps in lower stopband
SB	number of attenuation steps in upper stopband
T	sampling time
T	transfer function (ratio of output to input)
T_1	reflection function
T_n	nth-order Tschebycheff polynomial
u	elliptic integral of the first kind
V_0	load voltage when coupling network is replaced by a short circuit
V_1	voltage at input of coupling network
V_2	load voltage
x_L	the first value of x at which $R_n(x,L) = L$
Y	admittance
z	variable of the Z transformation ($z = e^{sT}$)
Z	impedance
Z_1	input impedance of coupling network
Z_{fi}	transformed version of ω_{fi}
Z_i	transformed version of f_i
ZF_i	ith zero of $H(s)$
ZQ_i	quality of the ith zero of $H(s)$
α	attenuation function, parametric constant
β	phase function
β_i	angle of a typical term of $L(Z)$
$\delta(t)$	unit impulse function
ϵ	a constant uniquely determined by A_{max}
σ	real part of complex frequency s
ϕ	amplitude of the elliptic integral
ω	imaginary part of complex frequency s
ω_A	lower passband edge
ω_B	upper passband edge
ω_H	upper stopband edge
ω_i	ith attenuation pole
ω_L	lower stopband edge
ω_0	frequency of maximum flatness, critical frequency
ω_s	sampling frequency
Ω	imaginary part of complex frequency S

Contents

CHAPTER ONE

Introduction

1.1 APPROXIMATION THEORY

The mathematical discipline known as *approximation theory* is very general and contains many useful theorems. The books *Theory of Approximation*[1,*] and *Approximation Theory*[2] may be examined by those interested in the niceties of this area of mathematics.

This book is not an attempt to treat approximation theory in general. Instead, the approximation problem is investigated from the viewpoint of an electrical engineer interested in designing filters. Thus, while the treatment is not elementary, neither does it become bogged down with mathematical details. For example, in discussing how to measure approximation errors, we will not become involved in a lengthy discussion of norms. We will, instead, use our time and energy to develop the approximation theory that is the basis for modern network-synthesis computer programs.

Network synthesis is used to find networks that will perform a desired task. For example, in an AM communication system it is usually necessary to synthesize networks that attenuate certain unwanted frequencies.

* Superscript numbers indicate references listed at the end of the chapter.

The approximation problem of network synthesis refers to the determination of a system function that, when synthesized, will perform the desired task. There are usually many different approximating functions that could be used to solve a specific approximation problem; which one is "best" will depend on many factors, such as the complexity of the resulting network. Thus this book does not present a unique solution to the approximation problem; instead it offers tools that can be used by the engineer interested in filter design.

1.2 FILTER JARGON

This book is written from the viewpoint of a filter designer, and the material is described in terms commonly used by filter designers. Since some of the jargon might not be familiar to readers from other disciplines, this section provides a brief introduction to the terminology.

Most of the functions encountered in this book are expressed in terms of the complex frequency $s = \sigma + j\omega$. This is the variable commonly encountered in Laplace transformation theory; given a function $v(t)$ (for example, a voltage that is a function of time), its Laplace transformation is defined as

$$V(s) = \int_0^{\infty} v(t)e^{st}\,dt = \mathcal{L}[v(t)] \tag{1.1}$$

A lowercase letter is usually used to denote a time-domain function, and an uppercase letter is employed for its Laplace transformation. For example, for currents one writes

$$I(s) = \mathcal{L}[i(t)] \tag{1.2}$$

An impedance is defined to be a voltage-to-current ratio and is always expressed in terms of the complex variable s. It is usually denoted by $Z(s)$; that is,

$$Z(s) = \frac{V(s)}{I(s)} \tag{1.3}$$

The reciprocal of an impedance is defined to be an admittance:

$$Y(s) = \frac{I(s)}{V(s)} \tag{1.4}$$

All lumped network response functions—impedances, admittances, and dimensionless ratios—are rational functions of the complex variable s.[6] This is the major reason for introducing the complex frequency s; it allows us to consider only rational functions and to employ our previous knowledge about such functions. For example, a transfer function which is a ratio of voltages may be written as

$$\frac{V_{out}}{V_{in}} = \frac{a_0 + a_1 s + a_2 s^2 + \cdots + a_m s^m}{b_0 + b_1 s + b_2 s^2 + \cdots + b_n s^n} = T(s) \tag{1.5}$$

For this transfer function to represent a stable system, the denominator of $T(s)$ must be Hurwitz; that is, the poles must be in the left half-plane.

If a transfer function is written in terms of the complex variable s, then it is very easy to investigate the steady-state response of the network. Consider a linear system that has an input which is a sinusoid of frequency ω:

$$v_{in}(t) = \sin \omega t \tag{1.6}$$

The output will be a sinusoid of the same frequency:

$$v_{out}(t) = C \sin (\omega t + \phi) \tag{1.7}$$

The amplitude C and phase ϕ arise because, in general, the output of a network will not have the same amplitude or phase as the input. The ratio of the output amplitude to the input amplitude can be found by simply evaluating the transfer function $T(s)$ at $s = j\omega$. Because $s = j\omega$ corresponds to a sinusoidal frequency, the letter ω is often said to represent *real* frequencies while the letter s is said to represent *complex* frequencies.

If the amplitude of a sinusoid at the output of a network is smaller than the amplitude at the input, then the signal is said to have been *attenuated* (i.e., it has encountered loss). The attenuation is usually expressed in terms of decibels (dB) as

$$A(\omega) \triangleq 20 \log \left| \frac{V_{in}(j\omega)}{V_{out}(j\omega)} \right| \tag{1.8}$$

By our definition of the transfer function $T(s)$, this can also be written as

$$A(\omega) = -20 \log |T(j\omega)| \tag{1.9}$$

In filter theory it is common practice to consider transfer functions that are ratios of input to output; i.e., we work with

$$H(s) = \frac{1}{T(s)} = \frac{b_0 + b_1 s + b_2 s^2 + \cdots + b_n s^n}{a_0 + a_1 s + a_2 s^2 + \cdots + a_m s^m} \tag{1.10}$$

$$= \frac{e(s)}{q(s)} \tag{1.11}$$

It follows that the attenuation can be expressed as

$$A(\omega) = 20 \log |H(j\omega)| \tag{1.12}$$