

**GAS TURBINE  
ENGINEERING HANDBOOK**

**Editor  
JOHN W. SAWYER**

# GAS TURBINE ENGINEERING HANDBOOK

**Editor**

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*Dedicated to*

**R. Tom Sawyer— “Mr. Gas Turbine”**

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## PREFACE

Engineers have long been faced with the need for complete technical data on design, manufacture, test, selection, installation, operation and maintenance of gas turbines. This Gas Turbine Engineering Handbook will, to a large degree, assist in filling those needs. It is, however, appreciated that no single volume can cover in detail the technical know-how of the past as well as that currently being developed.

It is most appropriate to acknowledge the tremendous part that the aircraft engine development has contributed to the many major improvements in this relatively new prime mover. The material in the Handbook is arranged generally in four major categories with a number of supporting chapters in each; these are:

- a. Fundamentals, thermodynamics, and cycle calculations
- b. Design, including aerodynamic and mechanical
- c. Application, progress, status and selection considerations
- d. Maintenance in major areas of use

To provide the readers with more detailed information on the various subjects covered, extensive references have been included with each chapter.

This Handbook was not the work of a single individual. The scope of the material covered was of such magnitude that it was essential to enlist the knowledge and skill of a number of renowned engineers and aerodynamists to assure adequate technical coverage.

The authors are recognized authorities in their respective fields, and the editor has relied on them for the accuracy of the material and data which they have presented. The editor is indebted to many people and organizations for their help in collecting and compiling the information presented in this book. He particularly appreciates the assistance from governments, industries, research organizations, universities, consultants and professional societies. The authors have worked with the editor many hours in reviewing and re-drafting material. Special acknowledgment is due to Mr. R. T. Sawyer, Consultant and Editor of Gas Turbine Magazine and Gas Turbine Catalog, for his suggestions and ideas. The aid of the staff of Gas Turbine Publications, Inc. has been of considerable help in this work. Mr. Wilbur W. Young did an outstanding job in developing the page arrangements for the book. Special thanks are due to Mr. G. Renfrew Brighton, publisher of this book, for his encouragement and agreement to freedom in selecting authors, subject material and space allocations within the book.

The editor expresses his sincere appreciation to his wife Dorothy and daughters Barbara, Nancy and Peggy for their understanding and help during the several years he spent in preparation of this book.

JOHN W. SAWYER  
*Editor*

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# 1 THERMODYNAMICS AND FUNDAMENTALS OF THE GAS TURBINE CYCLE

By CHARLES P. HOWARD

**T**HIS chapter will review the thermodynamic fundamentals which are necessary for an understanding of the basic gas turbine cycle analysis. A more detailed consideration of the thermodynamic analysis as applied to variations in cycles and use of working substances other than air will be presented in Chapter 2.

## NOMENCLATURE

$a$  = Acceleration, ft/sec<sup>2</sup>  
 $a_0, a_1, \dots$  = Constants as indicated in the equations  
 $c_p$  = Specific heat at constant pressure, Btu/lbm - R  
 $c_v$  = Specific heat at constant volume, Btu/lbm - R  
 $E_c$  = Internal chemical energy, Btu  
 $e_c$  = Specific internal chemical energy, Btu/lbm  
 $F$  = Force, lbf  
 $g$  = Local acceleration due to gravity, ft/sec<sup>2</sup>  
 $g_c$  = Universal constant of Newton's second law, 32.1739 lbm-ft/lbf - sec<sup>2</sup>  
 $H$  = Enthalpy, Btu  
 $h$  = Specific enthalpy, Btu/lbm  
 $I_{rev}$  = Irreversibility rate, Btu/sec - R  
 $J$  = Energy conversion constant, 778.16 ft-lbf/Btu  
 $k$  = Ratio of specific heats,  $c_p/c_v$   
 $k_c$  = Proportionality constant of Newton's second law.  
 $M$  = Mass, lbm  
 $\dot{m}$  = Mass flow rate,  $dM/dt$ , lbm/sec  
 $\bar{M}$  = Molal mass, lbm/lb mole  
 $p$  = Pressure, lbf/ft<sup>2</sup>, normally referenced to absolute zero  
 $P_r$  = Pressure ratio as used in the Gas Charts and defined in equation 19  
 $Q$  = Heat energy, Btu  
 $\dot{Q}$  = Heat rate,  $dQ/dt$ , Btu/sec  
 $q$  = Specific heat energy, Btu/lbm  
 $R$  = Universal gas constant, 1545.32 ft-lbf/lb mole - R  
 $R$  = Gas constant for a mole of a particular gas,  $R/\bar{M}$ , ft-lbf/lbm - R  
 $r_p$  = Pressure ratio for compression or expansion  
 $S$  = Entropy, Btu/R  
 $\dot{S}$  = Entropy rate,  $dS/dt$ , Btu/sec - R  
 $s$  = Specific entropy, Btu/lbm - R  
 $T$  = Temperature, R or K as indicated  
 $t$  = Time, seconds  
 $U$  = Internal thermal energy, Btu  
 $u$  = Specific internal thermal energy, Btu/lbm  
 $V$  = Velocity, ft/sec

$v$  = Specific volume, ft<sup>3</sup>/lbm,  $1/\rho$   
 $W$  = Work, ft-lbf  
 $\dot{W}$  = Work rate,  $dW/dt$ , ft-lbf/sec, horsepower  
 $X$  = Variable for calculation as used  
 $y$  = Variable for calculation as used  
 $Z$  = Height for measure of potential energy of position, ft  
 $\alpha$  = The ratio of  $(k - 1)/k$   
 $\Delta$  = Difference  
 $\eta$  = Efficiency, ratio of output to input  
 $\rho$  = Density, lbm/ft<sup>3</sup>  
 $\Sigma$  = Summation

## Subscripts

0 = Stagnation conditions,  
 1, 2, 3, ... = State points  
 $a$  = Actual  
 $A$  = Added  
 $B$  = Brake  
 $C$  = Compression, compressor  
 $R$  = Rejection  
 $S$  = Standard  
 $T$  = Expansion, turbine  
 avg = Average

## MASS, FORCE, AND ENERGY

Mass is a quantity of matter, and the quantity which has been arbitrarily selected as a fundamental unit on the macroscopic scale is the kilogram. However, in the English engineering system of units it has become commonplace to use the pound mass (lbm) as the basic quantity of matter to be considered for the working substance in a thermodynamic system. The pound mass is defined as 1/2.205 part of the kilogram mass. Force then becomes a derived unit and is related to mass by the principle of Newton's second law as

$$F = k_c M a \quad (1)$$

where the proportionality constant,  $k_c$ , depends upon the quantity of matter used for the mass,  $M$ , and the field of acceleration,  $a$ , used for the datum. Since it is most useful, if sometimes confusing, to also work with the pound force (lbf) as a fundamental unit as well as the pound mass, the proportionality constant becomes

$$k_c = 1/32.1739 \text{ lbf-sec}^2/\text{lbm-ft}$$

where the acceleration,  $a = 32.1739 \text{ ft/sec}^2$ , is the selected arbitrary datum. Thus, in the English engineering system of units with both pound mass and pound force as fundamental

units, it is necessary to apply the universal constant

$$g_c = 1/k_c = 32.1739 \text{ lbm-ft/lbf-sec}^2 \quad (2)$$

to obtain a consistent set of units in the various working equations.

Energy is a property of matter. The exchange and transformation of energy, particularly in the forms of heat and work, make up the science of thermodynamics. The units of energy are foot-pounds force (ft-lbf) in the English engineering unit system, but the British Thermal Unit (Btu) has become a standard form for measuring and tabulating this property for a quantity of matter. The British Thermal Unit is defined as equalling 778.16 ft-lbf, and thus giving the conversion constant

$$J = 778.16 \text{ ft-lbf/Btu} \quad (3)$$

The Btu is historically related to the caloric theory of heat as that amount of energy, transferred as heat, necessary to raise 1 lbm of water 1 F at "room" conditions.

### PROPERTIES, CONCEPTS, AND DEFINITIONS

Properties are those quantities which when determined will allow the state of a substance to be completely defined. The immediate properties of interest are those which relate to the thermodynamic state of the substance. Odor, color, and flavor are some non-thermodynamic properties, but might influence the selection of a substance for eventual use. There are two general types of properties: those which are independent of the quantity of mass are called intensive and those which are proportional to the quantity of mass are called extensive. Pressure, temperature, and density are typical thermodynamic intensive properties. Volume, energy, and entropy are typical extensive properties. However, it frequently becomes convenient to intensify an extensive property by dividing by the mass involved, thus obtaining the "specific volume," the "specific energy," and the "specific entropy" as intensive properties.

While it has been stated that "mass" is a quantity of matter and that "energy" is a property of matter, mass and energy are also two of the most important concepts of thermodynamics. Concepts should be related to our physical senses in order to derive physical meaning. The concept of length and time are readily perceived when one travels from one place to another. The concept of temperature is also readily understood by the "hotness" or "coldness," to the touch. The concept of mass, however, must come indirectly through the "push" or "pull" of a force due to an acceleration or deceleration on the body. The concept of mass is, therefore, more than a mere quantity of matter. Likewise, the concept of energy is indirect and must be experienced by the pushing or pulling over a distance (work) or the feeling of becoming warm or cold (heat). The fact that neither heat nor work can be stored further complicates the concept of energy as a property. Energy then is that property of matter which manifests itself either in the form of work or heat.

Work is a mechanism for the transfer of energy and is computed by the displacement of a quantity of matter times the force required to produce the displacement. Heat is another mechanism for the transfer of energy to a quantity of matter and is that energy transferred which cannot be recognized as work. There are several forms of energy which can be divided into classes, and only those thermodynamic

forms related to the gas turbine will be given here:

#### Mechanical Forms

1. Work— $W$ , transient, non-storable and a path function, (ft-lbf)— $w$ , (ft-lbf/lbm)
2. Kinetic energy— $V^2/2g_c$ , storable and a state function, (ft-lbf/lbm)
3. Potential energy of position— $gZ/g_c$ , storable and a state function, (ft-lbf/lbm)
4. Flow work— $p/\rho$ , transient, storable and a state function.

Note that for a closed system (i.e., a system in which no mass crosses the boundaries) there will be no flow work even though it will be possible to compute a value for  $p/\rho$ , (ft-lbf/lbm)

#### Thermal Forms

1. Heat— $Q$ , transient, non-storable and a path function, (Btu)— $q$ , (Btu/lbm)
2. Internal thermal— $U$ , storable and a state function, (Btu)— $u$ , (Btu/lbm)

#### Chemical Form

1. Internal chemical— $E_c$ , storable, and a state function, (Btu)— $e_c$ , (Btu/lbm)

Besides the various energy forms, other quantities which need definition are:

Enthalpy— $h = u + p/\rho J$ , (Btu/lbm)

Entropy— $ds = dq/T$  for a reversible process, (Btu/lbm-R)

Specific heat at constant pressure— $c_p = \left(\frac{\partial h}{\partial T}\right)_p$ , (Btu/lbm-R)

Specific heat at constant volume— $c_v = \left(\frac{\partial u}{\partial T}\right)_v$ , (Btu/lbm-R)

Ratio of specific heats— $k = c_p/c_v$

The temperature scale—the absolute temperature scale in the English engineering system of units is the Rankine scale, R, and it begins at zero; the relative temperature scale, Fahrenheit—F, is so chosen that 1 F = 1 R and absolute zero temperature equals -459.67 F. In the metric system of units the absolute temperature scale is the Kelvin scale, K, and it begins at zero; the relative temperature scale, Celsius—C, is so chosen that 1 C = 1 K and absolute zero temperature equals -273.15 C. The metric and English scales are related as

$$1 \text{ K} = 5/9 \text{ R}$$

The pressure scale—the absolute pressure scale begins at zero and has the standard atmosphere defined as:

$$1 \text{ atm} = 14.696 \text{ lbf/sq. in. abs. (psia)}$$

$$= 2116 \text{ psfa} = 29.92 \text{ in. Hg} = 760 \text{ mm Hg}$$

### FIRST LAW OF THERMODYNAMICS

The principle which states that energy is conserved for any system either open or closed is the first law of thermodynamics. For the usual energy analysis of a system, the principle of conservation of mass is applied along with the conservation of energy. The more general formulation which relates the interchange of energy and mass as put forth by Einstein is excluded in this treatment. As a principle the

first law of thermodynamics (along with the conservation of mass principle) can be mathematically formulated as

$$\Sigma \text{ Energy in} = \Sigma \text{ Energy out} + \Sigma \text{ Energy stored} \quad (4)$$

for the system at a given instant of time or for a specified time interval. Note the requirements that the system must be defined, all energy forms must be recognized, those energy forms crossing the system boundaries and undergoing change within the boundaries must be specified, and finally a time basis must be specified. For the gas turbine the energy forms are those which have previously been listed except the potential energy of position is frequently neglected. For the example of a gas turbine compressor at an instant of time, the control volume (system) is chosen as the boundaries of the compressor where there is no heat transfer to the surroundings (adiabatic walls). It is further useful to choose entry and exit planes for the energy where the values can be represented by single average values (one-dimensional) for each form. Then applying the first law to the energy diagram as shown in Fig. 1

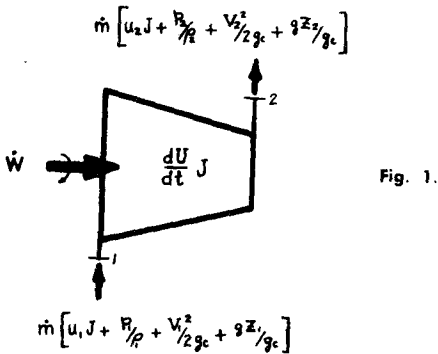


Fig. 1.

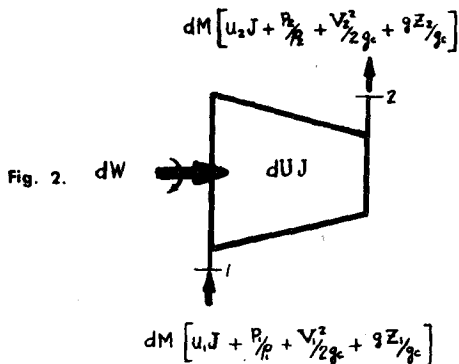


Fig. 2.

$$\begin{aligned} \dot{W} + \dot{m}[u_1J + p_1/\rho_1 + V_1^2/2g_c + gZ_1/g_c] = \\ \dot{m}[u_2J + p_2/\rho_2 + V_2^2/2g_c + gZ_2/g_c] + dU/dt, \end{aligned} \quad (5)$$

ft-lbf/sec

where for the instant of time the dot ( $\dot{W}$ ,  $\dot{m}$ ) quantities imply the "rate." If it is further taken that the energy states at each point remain constant with time (steady state) then  $dU/dt$  would be zero. Again the potential energy of position change,  $g/g_c(Z_2 - Z_1)$ , is usually sufficiently small to be negligible. Finally, if the concept of decelerating the flow to a state of zero velocity without incurring any losses (reversibly), and defining this state as "stagnation," is introduced into equation 5, the work rate of compression (power) becomes

$$\begin{aligned} \dot{W} &= \dot{m}[(u_2J + p_2/\rho_2) - (u_1J + p_1/\rho_1) \\ &\quad + 1/2g_c(V_2^2 - V_1^2)] \\ &= \dot{m}[(h_2 - h_1)J + 1/2g_c(V_2^2 - V_1^2)] \\ &= \dot{m}[(h_2J + V_2^2/2g_c) - (h_1J + V_1^2/2g_c)] \\ &= \dot{m}(h_{02} - h_{01})J, \text{ ft-lbf/sec} \end{aligned} \quad (6)$$

It should be noted that the energy diagram is an integral part of this methodology as the free body diagram is in mechanics, and that the signs of the quantities resolve themselves (i.e., if the sign of the work rate is calculated to be negative in this example, it then means the direction of energy transfer as work is in the opposite direction to that shown in the diagram and work is actually being transferred from the system).

Figure 2 is an energy diagram for the same example set up for the same system on the basis of a time-interval. With the same assumptions of steady state ( $dU = 0$ ), negligible potential energy of position change ( $g/g_c(Z_2 - Z_1) \approx 0$ ) and the use of the stagnation definition, the first law of thermodynamics (conservation of energy principle), give

$$\begin{aligned} dW &= dM[(u_2J + p_2/\rho_2) - (u_1J + p_1/\rho_1) \\ &\quad + 1/2g_c(V_2^2 - V_1^2)] \\ &= dM[(h_2 - h_1)J + 1/2g_c(V_2^2 - V_1^2)] \\ &= dM[(h_2J + V_2^2/2g_c) - (h_1J + V_1^2/2g_c)] \\ &= dM(h_{02} - h_{01})J, \text{ ft-lbf} \end{aligned} \quad (7)$$

## SECOND LAW OF THERMODYNAMICS

The first classical statement of the second law of thermodynamics is attributed to Clausius who said, "It is impossible for a self acting (cyclic) machine unaided by external agency to move heat from one body to another at a higher temperature." The other frequently quoted classical statement is that of Kelvin-Planck: "It is impossible to construct an engine which while operating in a cycle produces no effects except to do work and exchange heat with a single reservoir." While these statements can sometimes be used as a means for overall evaluation of inventors' claims, they are not operationally convenient for general thermodynamic analysis of systems. It is the consequences of these statements which are most useful. One of these consequences is that there is a property of matter called entropy which, for a reversible process involving this matter, can be evaluated as

$$dS = dQ/T, \text{ Btu/R}$$

or for a given quantity of matter (mass) "specific entropy" as

$$ds = dq/T, \text{ Btu/lbm - R}$$

Another consequence was the establishment of the thermodynamic temperature scale which has been experimentally determined on the Kelvin and Rankine scales in use today.

For operational convenience a major consequence can be established for any system either open or closed as

rate of creation of entropy =

$$[\Sigma \dot{S}] \geq \sum_{in} \dot{Q}/T - \sum_{out} \dot{Q}/T, \text{ Btu/sec - R} \quad (8)$$

where "creation" is defined as

$$[\Sigma \text{ outflow} - \Sigma \text{ inflow} + \Sigma \text{ increase of storage}] \quad (9)$$

The "rate" of course applies for an instant of time for the system. Thus for any system, the rate of irreversibility can be determined as

$$\text{Irrev.} = \{\Sigma \dot{S}\} - \sum_{\text{in}} \dot{Q}/T + \sum_{\text{out}} \dot{Q}/T \geq 0, \quad \text{Btu/sec} \cdot \text{R} \quad (10)$$

The equality sign holds for the reversible process and the inequality sign is a measure of the "strength" of the irreversibility of a process. The above formulation is credited to Professor A. L. London of Stanford University. Examples demonstrating its utility will be included in later paragraphs. The above statement, equation 10, contains the means of evaluating entropy changes between specified terminal states by means of a nonflow reversible process as

$$ds = \frac{du}{T} + \frac{p}{TJ} d(1/\rho), \text{ Btu/lbm} \cdot \text{R} \quad (11)$$

or, by using the enthalpy definition,

$$ds = \frac{dh}{T} - \frac{1}{T\rho J} dp, \text{ Btu/lbm} \cdot \text{R} \quad (12)$$

### EQUATION OF STATE

The relationship between properties of matter at some state is the "equation of state." Whether the relationship is presented in tabular, graphical, or mathematical form usually depends on the substance and in many cases all three forms are used. While it is not necessary, most "equations of state" give the relationship of intensive properties. When an extensive property has been "intensified," the quantity of mass is the pound-mass (lbm) for the English engineering system of units. The equation of state for water, for example, is adequately presented in the "Thermodynamic Properties of Steam" by Keenan and Keyes. Likewise, the equations of state for many common gases are tabulated in the Keenan and Kaye "Gas Tables."

Since the majority of gas turbines operate as open cycle using air as the primary working fluid, the Bureau of Ships, U. S. Navy, has prepared a graphical representation for the equation of state of air to assist in gas turbine cycle analysis. Through the courtesy of the U. S. Navy a portion of the properties of air charts from Research Memorandum No. 6-44, NAVSHIPS 250-330-6 of December, 1944, entitled "Gas Turbine Gas Charts," are reproduced in this chapter. These charts as well as the specific heat at constant pressure,  $c_p$ , can be approximated to a high order of accuracy in the temperature range of  $400 \text{ R} \leq T \leq 2500 \text{ R}$  by the following equations supplied by the Westinghouse Electric Corporation, Steam Division, Philadelphia, Pennsylvania:

$$c_p = c_p(T), \text{ Btu/lbm} \cdot \text{R} \quad (13)$$

$$c_p = a_0 + a_1 T + a_2 T^2 + a_3 T^3 + a_4 T^4 + a_5 T^5$$

$$a_0 = 0.24913875$$

$$a_1 = -0.48196782 \times 10^{-4}$$

$$a_2 = 0.68100588 \times 10^{-7}$$

$$a_3 = -0.21358038 \times 10^{-10}$$

$$a_4 = 0.48224330 \times 10^{-15}$$

$$a_5 = 0.47985777 \times 10^{-18}$$

$$h = h(T), \text{ Btu/lbm} \quad (14)$$

$$X = T/100$$

$$h = a_0 + a_1 X + a_2 X^2 + \dots + a_6 X^6$$

$$a_0 = -0.9774677 \times 10^2$$

$$a_1 = 0.25257199 \times 10^2$$

$$a_2 = -0.31167200$$

$$a_3 = 0.30219636 \times 10^{-1}$$

$$a_4 = -0.94272843 \times 10^{-3}$$

$$a_5 = 0.11868992 \times 10^{-4}$$

$$a_6 = -0.32615268 \times 10^{-7}$$

$$Pr = Pr(T) \quad (15)$$

$$X = T/10$$

$$Pr = e^y/10$$

$$y = a_0 + a_1 X + a_2 X^2 + \dots + a_8 X^8$$

$$a_0 = -0.404308 \times 10$$

$$a_1 = 0.2759446$$

$$a_2 = -0.45085958 \times 10^{-2}$$

$$a_3 = 0.53019327 \times 10^{-4}$$

$$a_4 = -0.41468032 \times 10^{-6}$$

$$a_5 = 0.21127191 \times 10^{-8}$$

$$a_6 = -0.67216409 \times 10^{-11}$$

$$a_7 = 0.12101712 \times 10^{-13}$$

$$a_8 = -0.94003173 \times 10^{-17}$$

and the inverse functions for temperature

$$T = T(h), \text{ R} \quad (16)$$

$$X = h/10$$

$$T = a_0 + a_1 X + a_2 X^2 + \dots + a_9 X^9$$

$$a_0 = 0.4000029 \times 10^3$$

$$a_1 = 0.41827332 \times 10^2$$

$$a_2 = -0.64799748 \times 10^{-1}$$

$$a_3 = 0.12160439 \times 10^{-1}$$

$$a_4 = -0.15484329 \times 10^{-2}$$

$$a_5 = 0.86703630 \times 10^{-4}$$

$$a_6 = -0.27042766 \times 10^{-5}$$

$$a_7 = 0.48670442 \times 10^{-7}$$

$$a_8 = -0.47242455 \times 10^{-9}$$

$$a_9 = 0.19149538 \times 10^{-11}$$

$$T = T(Pr), \text{ R} \quad (17)$$

$$X = \ln(10 \cdot Pr)$$

$$T = a_0 + a_1 X + a_2 X^2 + \dots + a_8 X^8$$

$$a_0 = 0.23796617 \times 10^3$$

$$a_1 = 0.37807000 \times 10$$

$$a_2 = 0.49211414 \times 10^2$$

$$a_3 = -0.15095696 \times 10^2$$

$$a_4 = 0.36226837 \times 10$$

$$a_5 = -0.45916847$$

$$a_6 = 0.33772184 \times 10^{-1}$$

$$a_7 = -0.13466334 \times 10^{-2}$$

$$a_8 = 0.23704971 \times 10^{-4}$$

These equations are particularly useful for computer analysis work.

The fact that the equation of state for air can be so simply related in terms of absolute temperature occurs because air, at modest pressures and temperatures well above critical, closely obeys the ideal gas law of

$$p/\rho = \frac{R}{\bar{m}} T \quad (18)$$

where  $R$  is the universal gas constant equal to 1545.32 ft-lbf/lb mole-°R, and  $\bar{m}$  is the molal mass for the gas of interest having units of lbm/lb mole. For air the value of

$$\frac{R}{\bar{m}} = R = 53.342 \text{ ft-lbf/lbm-R}$$

For a gas which obeys equation 18 it is possible, with the use of the first and second laws of thermodynamics, to demonstrate that internal thermal energy is a function only of temperature and thus the enthalpy is a function only of temperature. If equation 18 is now combined with equation 12 and applied to a reversible process in which the entropy is held constant, then

$$\frac{R}{J} \frac{dp}{p} = \frac{dh}{T}$$

and introducing the definition of specific heat at constant pressure gives

$$\frac{dp}{p} = \frac{J}{R} c_p \frac{dT}{T}$$

which can now be integrated between some reference state to a state point of interest as

$$\ln \frac{p}{p_s} = \frac{J}{R} \int_{T_s}^T c_p \frac{dT}{T} = \ln Pr \quad (19)$$

when the experimentally determined variation of  $c_p$  as a function of temperature is introduced. Thus the pressure ratio ( $Pr = p/p_s$ ) becomes the property of thermodynamic interest and is a function only of the absolute temperature. The pressure ratio values are listed as  $Pr$  in the charts and equations 15 and 17. Use of the charts will be made in following examples.

### PROCESSES

A system undergoes a process with the change of any property of the system. While it would be impractical to attempt to describe all possible thermodynamic processes, certain processes serve as measures and ideals for comparisons with actual processes. An ideal process is one which is internally or externally reversible, that is, one which can retrace the original path (statepoints) without leaving evidence to the system or its surroundings that the initial process ever occurred. Internal irreversibilities are those associated with fluid friction, mixing, heat transfer and combustion within the system boundaries. External irreversibilities are those associated with mechanical friction and uncontrolled heat transfer by the system with its surroundings.

The ideal processes of most thermodynamic interest are those in which one property is held constant while all others are allowed to undergo change. Some of these ideal processes will be illustrated with reference only to changes in properties in the ideal gas law for the present.

For a constant pressure process (isobaric)

$$\frac{T_2}{T_1} = \frac{\rho_1}{\rho_2} \quad (20)$$

For a constant density process (this is also a constant "specific volume" or isometric process since the specific volume,

$v$ , is also the reciprocal of density)

$$\frac{T_2}{T_1} = \frac{p_2}{p_1} \quad (21)$$

For a constant temperature process (isothermal)

$$\frac{p_2}{p_1} = \frac{\rho_2}{\rho_1} \quad (22)$$

For a constant entropy process (isentropic), with constant specific heats

$$\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{k-1}{k}} \quad (23)$$

or

$$\frac{p_2}{p_1} = \left( \frac{\rho_2}{\rho_1} \right)^k \quad (24)$$

Virtually all thermodynamic cycles are composed of combinations of these four processes. It is one of the engineer's jobs to commit to hardware devices which can approximate as closely as possible these ideals, then analyze the device to determine this proximity (irreversibilities) so that changes can be made for improvement.

### BASIC GAS TURBINE CYCLE

The basic gas turbine cycle is named for the Boston engineer, George Brayton, who proposed it in the late nineteenth century. This cycle consists of an isentropic compression process followed by a constant pressure heat addition process, then an isentropic expansion process and completion of the cycle with a constant pressure heat rejection process. This also happens to be the same cycle proposed by Rankine in the middle of the nineteenth century. The difference between the Rankine cycle and the Brayton cycle is that the latter was proposed to work with a single phase gaseous substance while the former utilized a two-phase (liquid and gas) substance, with the changes of phase occurring during the two constant pressure processes.

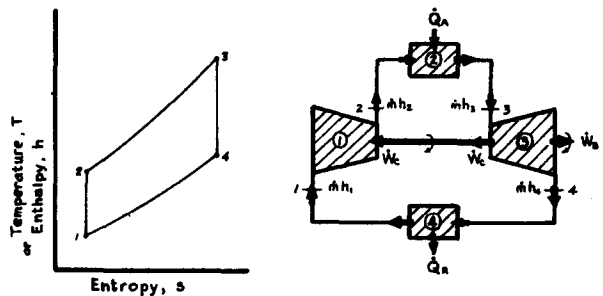


Fig. 3.

The ideal Brayton cycle and an energy diagram are shown in Fig. 3 with the thermodynamic coordinates of temperature (or enthalpy) vs. entropy for an ideal gas. The four systems within this energy diagram are indicated (1), (2), (3), (4). For this ideal cycle the pressure ratios of compression and expansion are equal

$$\frac{p_2}{p_1} = r_{pC} = \frac{p_3}{p_4} = r_{pT}$$

Now applying the first law of thermodynamics for each steady state process of each system shown in the energy

diagram, and neglecting potential energy of position changes:

for system (1), work rate of compression

$$\dot{W}_C = \dot{m}(h_2 - h_1), \text{ Btu/sec}$$

for system (2), heat addition rate

$$\dot{Q}_A = \dot{m}(h_3 - h_2), \text{ Btu/sec}$$

for system (3), work rate of expansion

$$\dot{W}_C + \dot{W}_B = \dot{m}(h_3 - h_4), \text{ Btu/sec}$$

$$\dot{W}_T = \dot{W}_C + \dot{W}_B \quad \text{total work rate of the turbine}$$

for system (4), heat rejection rate

$$\dot{Q}_R = \dot{m}(h_4 - h_1), \text{ Btu/sec}$$

And if the system boundary is taken to include all four sub-systems, the brake work rate is

$$\dot{W}_B = \dot{Q}_A - \dot{Q}_R = \dot{m}(h_3 - h_2 + h_1 - h_4), \text{ Btu/sec.}$$

The thermal efficiency for the cycle is defined as

$$\eta = \frac{\dot{W}_B}{\dot{Q}_A} \quad (25)$$

so that

$$\eta = \frac{h_3 - h_2 + h_1 - h_4}{h_3 - h_2} = 1 - \frac{h_4 - h_1}{h_3 - h_2}$$

Now assuming some typical numbers and using air for the ideal gas, let

$$\dot{m} = 10 \text{ lbm/sec}, \quad T_1 = 530 \text{ R}, \quad T_3 = 2060 \text{ R}$$

and

$$\frac{p_2}{p_1} = \frac{p_3}{p_4} = 6.$$

From the properties of air charts, Table 1 can be readily filled in by noting

$$\frac{p_2}{p_1} = \frac{Pr_2}{Pr_1} \quad \text{or} \quad Pr_2 = Pr_1 \left( \frac{p_2}{p_1} \right) = (2.679)(6) = 16.074$$

and

$$\frac{p_3}{p_4} = \frac{Pr_3}{Pr_4} \quad \text{or} \quad Pr_4 = Pr_3 \left( \frac{p_4}{p_3} \right) = \frac{406.65}{6} = 67.775$$

TABLE 1

State	T, R	Pr	h, Btu/lbm
1	530*	2.679	31.21
2	880.4	16.074*	116.06
3	2060*	406.65	426.22
4	1303.8	67.775*	222.60

\*key quantity for entering Gas Charts.

Then the following quantities can be evaluated as

$$\dot{W}_C = 10(116.06 - 31.21) = 848.5 \text{ Btu/sec} = 1200 \text{ HP}$$

$$\dot{Q}_A = 10(426.22 - 116.06) = 3101.6 \text{ Btu/sec}$$

$$\dot{W}_T = 10(426.22 - 222.60) = 2036.2 \text{ Btu/sec} = 2880 \text{ HP}$$

$$\dot{Q}_R = 10(222.60 - 31.21) = 1913.9 \text{ Btu/sec}$$

$$\dot{W}_B = 3101.6 - 1913.9 = 1187.7 \text{ Btu/sec} = 1680 \text{ HP}$$

and the thermal efficiency is

$$\eta = \frac{1187.7}{3101.6} = 38.2\%$$

These then are the ideal values for a gas turbine operating under the given conditions.

Consider now introducing some "losses" or non-ideal processes into the cycle. For the compression process, define the compressor efficiency as

$$\eta_C = \frac{\text{ideal work rate of compression}}{\text{actual work rate of compression}} = \frac{\dot{W}_C}{\dot{W}_{Ca}} \quad (26)$$

For the expansion process, define the turbine efficiency as

$$\eta_T = \frac{\text{actual work rate of expansion}}{\text{ideal work rate of expansion}} = \frac{\dot{W}_{Ta}}{\dot{W}_T} \quad (27)$$

For the cycle thermal efficiency equation 25 becomes

$$\eta = \frac{\text{actual output work rate}}{\text{actual heat addition rate}} = \frac{\dot{W}_{Ba}}{\dot{Q}_{Aa}} \quad (28)$$

with the actual output work rate or shaft horse power being

$$\dot{W}_{Ba} = \dot{W}_{Ta} - \dot{W}_{Ca}$$

Some simple pressure losses during the heat addition and heat rejection processes will be introduced as

$$\frac{p_2 - p_{2'}}{p_2} = \frac{p_3 - p_{3'}}{p_3} = \frac{\Delta p_A}{p_2}$$

during heat addition and

$$\frac{p_4' - p_1}{p_1} = \frac{p_4 - p_1}{p_1} = \frac{\Delta p_R}{p_1}$$

during heat rejection. Figure 4 shows the various state points plotted to the thermodynamic coordinates with the following values of the "losses" assumed for this example as

$$\eta_C = 84\%, \quad \eta_T = 88\%, \quad \frac{\Delta p_A}{p_2} = 4\% \quad \text{and} \quad \frac{\Delta p_R}{p_1} = 6\%$$

with all other quantities as before. Notice that while the compressor pressure ratio stays the same

$$\frac{p_2}{p_1} = \frac{p_2'}{p_1} = \frac{Pr_2}{Pr_1} = r_{pC} = 6,$$

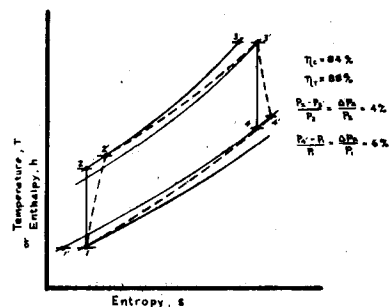


Fig. 4.

the expansion ratio for the turbine becomes

$$\frac{p_3}{p_4} = \frac{p_3'}{p_4'} = \left( \frac{p_2}{p_1} \right) \left( \frac{1 - \frac{p_2 - p_3'}{p_2}}{1 + \frac{p_3 - p_4'}{p_1}} \right)$$

$$= \left(\frac{p_2}{p_1}\right) \left(\frac{1 - \frac{\Delta p_A}{p_2}}{1 + \frac{\Delta p_R}{p_1}}\right) = \frac{Pr_{3'}}{Pr_4} = r_{pT}$$

$$= 6 \left(\frac{1 - 0.04}{1 + 0.06}\right) = 5.43$$

Table 2 can now be filled in with the use of the properties of air charts as before after applying the first law of thermodynamics for each system

TABLE 2

State	T, R	Pr	h, Btu/lbm
1	530*	2.679	31.21
2	880.4	16.074*	116.06
2'	946.8	20.875	132.41*
3	2060*	406.65	426.22
3'	2060*	406.65	426.22
4	1339.2	75.0*	231.78
4'	1428.7	95.95	255.12*
1'	530*	2.679	31.21

\*key quantity for entering Properties of Air Charts.

(1)

$$\dot{W}_C = \dot{m}(h_2 - h_1)$$

$$Pr_2 = Pr_1 \cdot r_{pC} = (2.679)(6) = 16.074$$

$$h_2 = 116.06 \text{ Btu/lbm}, \quad T_2 = 880.4 \text{ R}$$

$$\eta_C = \frac{\dot{W}_C}{\dot{W}_{Ca}} = \frac{\dot{m}(h_2 - h_1)}{\dot{m}(h_{2'} - h_1)}$$

$$h_{2'} = \frac{h_2 - h_1}{\eta_C} + h_1 = \frac{116.06 - 31.21}{.84} + 31.21$$

$$= 132.41 \text{ Btu/lbm}$$

$$T_{2'} = 946.8 \text{ R}$$

$$\dot{W}_{Ca} = 10(132.41 - 31.21) = 1012.1 \text{ Btu/sec}$$

$$= 1431 \text{ HP}$$

(2)

$$\dot{Q}_{Aa} = \dot{m}(h_3 - h_{2'}) = \dot{m}(h_3 - h_2) = 10(426.22 - 132.41)$$

$$= 2938.1 \text{ Btu/sec}$$

(3)

$$\dot{W}_T = \dot{m}(h_3 - h_4), \quad Pr_{3'} = Pr_3$$

$$Pr_4 = \frac{Pr_{3'}}{r_{pT}} = \frac{406.65}{5.43} = 75.0$$

$$h_4 = 231.78 \text{ Btu/lbm}, \quad T_4 = 1339.2 \text{ R}$$

$$\eta_T = \frac{\dot{W}_{Ta}}{\dot{W}_T} = \frac{\dot{m}(h_3 - h_4)}{\dot{m}(h_3 - h_4)}$$

$$h_4' = h_3 - \eta_T(h_3 - h_4)$$

$$= 426.22 - 0.88(426.22 - 231.78)$$

$$= 255.12 \text{ Btu/lbm}$$

$$T_{4'} = 1428.7 \text{ R}$$

$$\dot{W}_{Ta} = 10(426.22 - 255.12) = 1711.0 \text{ Btu/sec}$$

$$= 2420 \text{ HP}$$

(4)

$$\dot{Q}_{Ra} = \dot{m}(h_4' - h_1) = 10(255.12 - 31.21)$$

$$= 2239.1 \text{ Btu/sec}$$

(5)

$$\dot{W}_{Ba} = \dot{Q}_{Aa} - \dot{Q}_{Ra} = 2938.1 - 2239.1$$

$$= 699.0 \text{ Btu/sec} = 989 \text{ HP}$$

$$\eta = \frac{\dot{W}_{Ba}}{\dot{Q}_{Aa}} = \frac{699.0}{2938.1} = 23.8\%$$

Now with a second law of thermodynamics analysis, it will be possible to determine how the irreversibilities have affected the performance and where "improvements" would produce the greatest return. Apply equation 10 for each system and use equation 12 for the evaluation of entropy between the state points as shown in Fig. 4.

For system (1)

$$I_{rrev1} = \dot{m}(s_{2'} - s_2) = \dot{m} \int_{T_2}^{T_{2'}} \frac{dh}{T}$$

$$= \dot{m} \int_{T_2}^{T_{2'}} c_p \frac{dT}{T} \approx \dot{m} c_p \ln \frac{T_{2'}}{T_2}$$

and with

$$\eta_C = \frac{\dot{W}_C}{\dot{W}_{Ca}} = \frac{\dot{m}(h_2 - h_1)}{\dot{m}(h_{2'} - h_1)} \approx \frac{T_2 - T_1}{T_{2'} - T_1}$$

then

$$\frac{T_{2'}}{T_2} = \frac{T_{2'}/T_1}{T_2/T_1} = \frac{1}{\eta_C} \left( \frac{T_2}{T_1} - 1 \right) + 1$$

so that

$$I_{rrev1} = \dot{m} c_p \ln \left[ \frac{1}{\eta_C} \left( \frac{T_2}{T_1} - 1 \right) + 1 \right]$$

$$= (10)(.245) \ln \left[ \frac{1}{0.84} \left( \frac{880.4}{530} - 1 \right) + 1 \right]$$

$$= 0.1790 \text{ Btu/sec} \cdot \text{R}$$

For system (2), which is also adiabatic inasmuch as all energy entering the system as a heat addition goes to increase the temperature of the working substance

$$I_{rrev2} = \dot{m}(s_{3'} - s_3) = - \frac{\dot{m}R}{J} \int_{p_3}^{p_{3'}} \frac{dp}{p}$$

$$= \frac{\dot{m}R}{J} \ln \frac{p_3}{p_{3'}} = \frac{\dot{m}R}{J} \ln \left( \frac{1}{1 - \frac{\Delta p_A}{p_2}} \right)$$

$$= \frac{(10)(53.342)}{778.16} \ln \left[ \frac{1}{1 - 0.04} \right] = 0.0280 \text{ Btu/sec} \cdot \text{R}$$

For system (3)

$$\begin{aligned} \text{Irrev}_3 &= \dot{m}(s_4' - s_4) = \dot{m} \int_{T_4}^{T_4'} \frac{dh}{T} \\ &= \dot{m} \int_{T_4}^{T_4'} c_p \frac{dT}{T} \approx \dot{m} c_p \ln \frac{T_4'}{T_4} \end{aligned}$$

and with

$$\eta_T = \frac{\dot{W}_{Ta}}{\dot{W}_T} = \frac{\dot{m}(h_3 - h_4)}{\dot{m}(h_3 - h_4')} \approx \frac{T_3 - T_4}{T_3 - T_4'}$$

then

$$\frac{T_4'}{T_4} = (1 - \eta_T) \frac{T_3}{T_4} + \eta_T$$

so that

$$\begin{aligned} \text{Irrev}_3 &= \dot{m} c_p \ln \left[ (1 - \eta_T) \frac{T_3}{T_4} + \eta_T \right] \\ &= (10)(.260) \ln \left[ (1 - 0.88) \frac{2060}{1339.2} + 0.88 \right] \\ &= 0.1635 \text{ Btu/sec} \cdot \text{R} \end{aligned}$$

For system (4), also adiabatic,

$$\begin{aligned} \text{Irrev}_4 &= \dot{m}(s_1 - s_4') = -\frac{\dot{m}R}{J} \int_{p_1}^{p_4'} \frac{dp}{p} \\ &= \frac{\dot{m}R}{J} \ln \frac{p_1}{p_4'} = \frac{\dot{m}R}{J} \ln \left( \frac{1}{1 - \frac{\Delta p_R}{p_1}} \right) \\ &= \frac{(10)(53.342)}{778.16} \ln \left( \frac{1}{1 - 0.06} \right) = 0.0424 \text{ Btu/sec} \cdot \text{R} \end{aligned}$$

Determining the thermodynamic significance (i.e., the power losses) associated with these irreversibilities requires the use of some temperature for each process. As a matter of judgment, it should be the temperature which will predict the same losses as found from the first law analysis. For the compressor this is the average temperature of ideal and actual at the compressor discharge, in this case

$$T_{C\text{avg}} = \frac{T_2 + T_2'}{2} = \frac{880.4 + 946.8}{2} = 914 \text{ R}$$

Then the increase in compressor power required is

$$T_{C\text{avg}} \text{Irrev}_1 = (914)(0.1790) = 163.5 \text{ Btu/sec} = 231 \text{ HP}$$

which compares with the first law analysis as

$$\begin{aligned} \dot{W}_{Ca} - \dot{W}_C &= \dot{m}(h_2' - h_2) = 10(132.41 - 116.06) \\ &= 163.5 \text{ Btu/sec} = 231 \text{ HP} \end{aligned}$$

For the pressure loss during the heat addition process, the average temperature should be the same as that for the compressor. The power loss during this process is

$$T_{C\text{avg}} \text{Irrev}_2 = (914)(0.0280) = 25.6 \text{ Btu/sec} = 36.2 \text{ HP}$$

For the turbine, the average temperature is that of the ideal and actual at the turbine exhaust, in this case

$$T_{T\text{avg}} = \frac{T_4 + T_4'}{2} = \frac{1339.2 + 1428.7}{2} = 1384 \text{ R}$$

so the loss in turbine power is

$$T_{T\text{avg}} \text{Irrev}_3 = (1384)(0.1635) = 226.5 \text{ Btu/sec} = 320 \text{ HP}$$

which compares with the first law analysis as

$$\begin{aligned} \dot{W}_T - \dot{W}_{Ta} &= \dot{m}(h_4' - h_4) = 10(255.12 - 231.78) \\ &= 233.4 \text{ Btu/sec} = 330 \text{ HP} \end{aligned}$$

For the pressure loss during the heat rejection process, the average temperature should be that for the compressor inlet temperature so the power loss during this process is

$$T_1 \text{Irrev}_4 = (530)(0.0424) = 22.45 \text{ Btu/sec} = 31.8 \text{ HP}$$

The decrease in the cycle efficiency can also be determined as

$$\begin{aligned} \Delta \eta &= \frac{\sum T \text{Irrev}}{\dot{Q}_A} = \frac{163.5 + 25.6 + 226.5 + 22.45}{3101.6} \\ &= \frac{438.05}{3101.6} = 14.1\% \end{aligned}$$

or

$$\eta = 38.2 - 14.1 = 24.1\%$$

which compares with

$$\eta = 23.8\%$$

from the first law analysis.

When all of the quantities from the second law analysis are written in a single equation and the use of the pressure ratio relationships are introduced from equation 23

$$\begin{aligned} \frac{T_2}{T_1} &= r_{pC}^{\frac{k-1}{k}} = r_{pC}^{\alpha} \\ \frac{T_3}{T_4} &= r_{pT}^{\frac{k-1}{k}} = r_{pT}^{\alpha} \end{aligned}$$

the decrement in thermal efficiency becomes

$$\begin{aligned} \Delta \eta &= \frac{1}{\dot{Q}_A} \left\{ \frac{\dot{m} T_1}{2} \left[ r_{pC}^{\alpha} \left( 1 + \frac{1}{\eta_C} \right) + \right. \right. \\ &\quad \left. \left. \left( 1 - \frac{1}{\eta_C} \right) \right] c_p \ln \left[ \frac{\frac{1}{\eta_C} (r_{pC}^{\alpha} - 1) + 1}{r_{pC}^{\alpha}} \right] + \right. \\ &\quad \left. \text{compression process} \right\} \end{aligned}$$

$$\frac{\dot{m} T_1}{2} \left[ r_{pC}^{\alpha} \left( 1 + \frac{1}{\eta_C} \right) + \right.$$

$$\left. \left( 1 - \frac{1}{\eta_C} \right) \right] \frac{R}{J} \ln \left[ \frac{1}{1 - \Delta p_A / p_2} \right] +$$

pressure loss of heat addition process

$$\frac{\dot{m} T_3}{2} \left[ \frac{1}{r_{pT}^{\alpha}} (1 + \eta_T) + \right.$$

$$\left. (1 - \eta_T) \right] c_p \ln \left[ (1 - \eta_T) r_{pT}^{\alpha} + \eta_T \right] +$$

expansion process



$$\dot{m} T_1 \frac{R}{J} \ln \left[ \frac{1}{1 - \Delta p_R / p_1} \right] \quad (29)$$

pressure loss of heat rejection process

If average values of  $c_p$  and  $k$  are used for each process, the approximations of equation 23 in equation 29 check closely with the previous calculations,

$$\Delta \eta = \frac{1}{3101.6} \left\{ \frac{(10)(530)}{2} \left[ 6^{.286} \left( 1 + \frac{1}{.84} \right) + \left( 1 - \frac{1}{.84} \right) \right] (.248) \ln \left[ \frac{1}{.84} \frac{(6^{.286} - 1) + 1}{6^{.286}} \right] + \frac{(10)(530)}{2} \left[ 6^{.286} \left( 1 + \frac{1}{.84} \right) + \left( 1 - \frac{1}{.84} \right) \frac{53.342}{778.16} \ln \left[ \frac{1}{1 - .04} \right] + \frac{(10)(2060)}{2} \left[ \frac{1}{5.43^{.254}} (1 + .88) + (1 - .88) \right] (.260) \ln [(1 - .88) 5.43^{.254} + .88] \right\}$$

$$(10)(530) \frac{53.342}{778.16} \ln \left[ \frac{1}{1 - .06} \right]$$

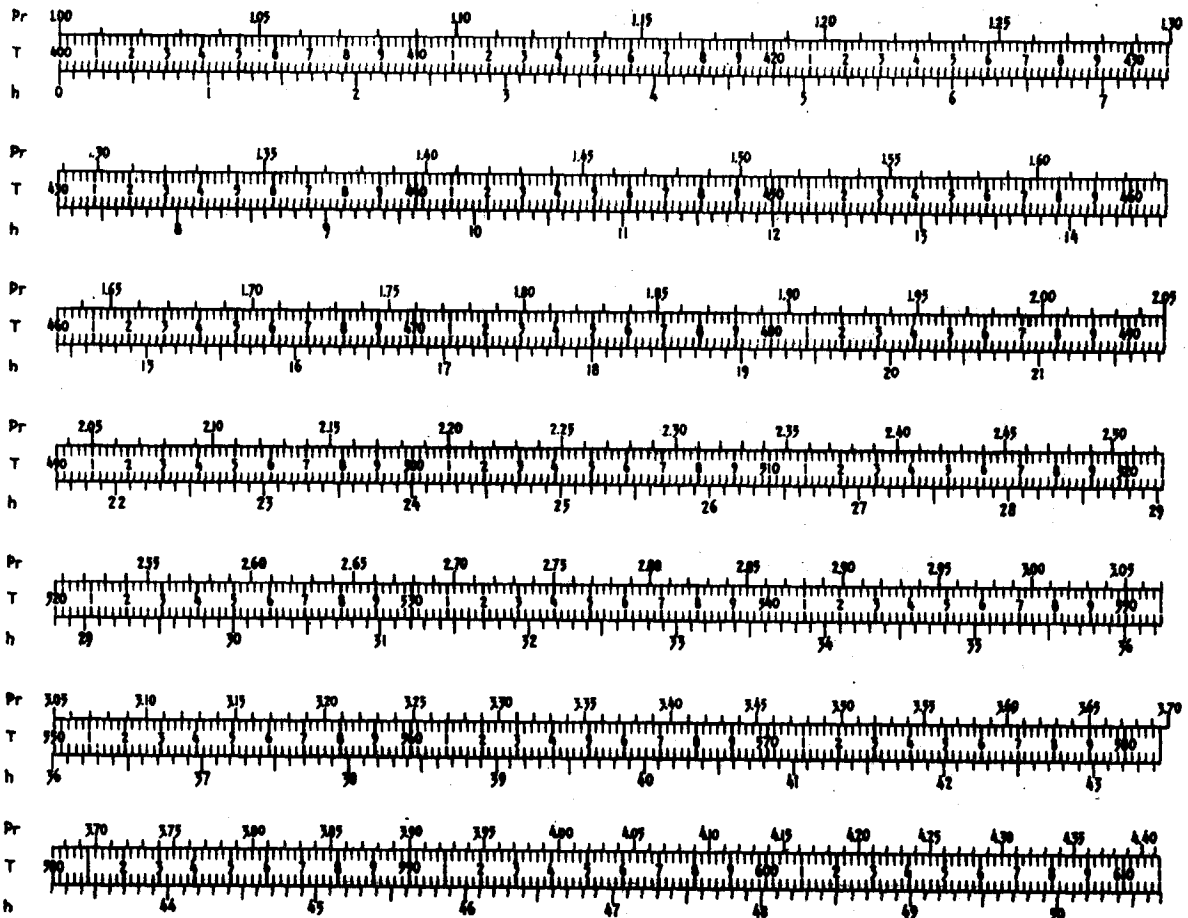
$$= \frac{168.5 + 25.8 + 224 + 22.5}{3101.6} = \frac{440.8}{3101.6} = 14.2\%$$

The explicit use of entropy is a very straightforward and powerful method for evaluating irreversibility effects on the cycle as all "losses" can be examined independently with superposition of losses to determine overall effect.

#### REFERENCES

- Thermodynamics, Faires, V. M., Macmillan Co.
- Thermodynamics, Keenan, J. H., John Wiley & Sons, Inc.
- Principles of Engineering Thermodynamics, Kiefer, P. J., Kinney, G. F., and Stuart, M. C., John Wiley & Sons, Inc.
- Thermodynamics, Obert, E. F., McGraw-Hill Book Co.
- Thermodynamics, Reynolds, W. C., McGraw-Hill Book Co.
- Thermodynamics, Van Wylen, G. J., John Wiley & Sons, Inc.
- Heat and Thermodynamics, Zemansky, M. W., McGraw-Hill Book Co.
- Gas Tables, Keenan, J. H. and Kaye, J., John Wiley & Sons, Inc.
- Gas Turbine Gas Charts, Bureau of Ships, U. S. Navy, Research Memorandum No. 6-44, Government Printing Office, Washington, D.C., prepared by A. Amorosi.

## PROPERTIES OF AIR



Pr \_\_\_\_\_ 1.00 to 4.41  
 T (degrees R) \_\_\_\_\_ 400 to 611  
 h (BTU/lb.) \_\_\_\_\_ 0.0 to 90.7