DAVID K. CHENG

FIELD AND WAVE ELECTROMAGNETICS

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Preface

The many books on introductory electromagnetics can be roughly divided into two main groups. The first group takes the traditional development: starting with the experimental laws, generalizing them in steps, and finally synthesizing them in the form of Maxwell's equations. This is an inductive approach. The second group takes the axiomatic development: starting with Maxwell's equations, identifying each with the appropriate experimental law, and specializing the general equations to static and time-varying situations for analysis. This is a deductive approach. A few books begin with a treatment of the special theory of relativity and develop all of electromagnetic theory from Coulomb's law of force; but this approach requires the discussion and understanding of the special theory of relativity first and is perhaps best suited for a course at an advanced level.

Proponents of the traditional development argue that it is the way electromagnetic theory was unraveled historically (from special experimental laws to Maxwell's equations), and that it is easier for the students to follow than the other methods. I feel, however, that the way a body of knowledge was unraveled is not necessarily the best way to teach the subject to students. The topics tend to be fragmented and cannot take full advantage of the conciseness of vector calculus. Students are puzzled at, and often form a mental block to, the subsequent introduction of gradient, divergence, and curl operations. As a process for formulating an electromagnetic model, this approach lacks cohesiveness and elegance.

The axiomatic development usually begins with the set of four Maxwell's equations, either in differential or in integral form, as fundamental postulates. These are equations of considerable complexity and are difficult to master. They are likely to cause consternation and resistance in students who are hit with all of them at the beginning of a book. Alert students will wonder about the meaning of the field vectors and about the necessity and sufficiency of these general equations. At the initial stage students tend to be confused about the concepts of the electromagnetic model, and they are not yet comfortable with the associated mathematical manipulations. In any case, the general Maxwell's equations are soon simplified to apply to static fields, which allow the consideration of electrostatic fields and magnetostatic fields separately. Why then should the entire set of four Maxwell's equations be introduced at the outset?

It may be argued that Coulomb's law, though based on experimental evidence, is in fact also a postulate. Consider the two stipulations of Coulomb's law: that the charged bodies are very small compared with their distance of separation, and that the force between the charged bodies is inversely proportional to the square of their distance. The question arises regarding the first stipulation: How small must the charged bodies be in order to be considered "very small" compared with their distance? In practice the charged bodies cannot be of vanishing sizes (ideal point charges), and there is difficulty in determining the "true" distance between two bodies of finite dimensions. For given body sizes the relative accuracy in distance measurements is better when the separation is larger. However, practical considerations (weakness of force, existence of extraneous charged bodies, etc.) restrict the usable distance of separation in the laboratory, and experimental inaccuracies cannot be entirely avoided. This leads to a more important question concerning the inverse-square relation of the second stipulation. Even if the charged bodies were of vanishing sizes, experimental measurements could not be of an infinite accuracy no matter how skillful and careful an experimentor was. How then was it possible for Coulomb to know that the force was *exactly* inversely proportional to the *square* (not the 2.000001th or the 1.999999th power) of the distance of separation? This question cannot be answered from an experimental viewpoint because it is not likely that during Coulomb's time experiments could have been accurate to the seventh place. We must therefore conclude that Coulomb's law is itself a postulate and that it is a law of nature discovered and assumed on the basis of his experiments of a limited accuracy (see Section 3–2).

This book builds the electromagnetic model using an *axiomatic approach in steps*: first for static electric fields (Chapter 3), then for static magnetic fields (Chapter 6), and finally for time-varying fields leading to Maxwell's equations (Chapter 7). The mathematical basis for each step is Helmholtz's theorem, which states that a vector field is determined to within an additive constant if both its divergence and its curl are specified everywhere. Thus, for the development of the electrostatic model in free space, it is only necessary to define a single vector (namely, the electric field intensity **E**) by specifying its divergence and its curl as postulates. All other relations in electrostatics for free space, including Coulomb's law and Gauss's law, can be derived from the two rather simple postulates. Relations in material media can be developed through the concept of equivalent charge distributions of polarized dielectrics.

Similarly, for the magnetostatic model in free space it is necessary to define only a single magnetic flux density vector **B** by specifying its divergence and its curl as postulates; all other formulas can be derived from these two postulates. Relations in material media can be developed through the concept of equivalent current densities. Of course, the validity of the postulates lies in their ability to yield results that conform with experimental evidence.

For time-varying fields, the electric and magnetic field intensities are coupled. The curl **E** postulate for the electrostatic model must be modified to conform with Faraday's law. In addition, the curl **B** postulate for the magnetostatic model must also be modified in order to be consistent with the equation of continuity. We have, then, the four Maxwell's equations that constitute the electromagnetic model. I believe that this gradual development of the electromagnetic model based on Helmholtz's theorem is novel, systematic, and more easily accepted by students.

In the presentation of the material, I strive for lucidity and unity, and for smooth and logical flow of ideas. Many worked-out examples (a total of 135 in the book) are included to emphasize fundamental concepts and to illustrate methods for solving typical problems. Review questions appear at the end of each chapter to test the students' retention and understanding of the essential material in the chapter. The problems in each chapter are designed to reinforce students' comprehension of the interrelationships between the different quantities in the formulas, and to extend their ability of applying the formulas to solve practical problems. I do not believe in simple-minded drill-type problems that accomplish little more than an exercise on a calculator.

The subjects covered, besides the fundamentals of electromagnetic fields, include theory and applications of transmission lines, waveguides and resonators, and antennas and radiating systems. The fundamental concepts and the governing theory of electromagnetism do not change with the introduction of new electromagnetic devices. Ample reasons and incentives for learning the fundamental principles of electromagnetics are given in Section 1–1. I hope that the contents of this book, strengthened by the novel approach, will provide students with a secure and sufficient background for understanding and analyzing basic electromagnetic phenomena as well as prepare them for more advanced subjects in electromagnetic theory.

There is enough material in this book for a two-semester sequence of courses. Chapters 1 through 7 contain the material on fields, and Chapters 8 through 11 on waves and applications. In schools where there is only a one-semester course on electromagnetics, Chapters 1 through 7, plus the first four sections of Chapter 8 would provide a good foundation on fields and an introduction to waves in unbounded media. The remaining material could serve as a useful reference book on applications or as a textbook for a follow-up elective course. If one is pressed for time, some material, such as Example 2–2 in Section 2–2, Subsection 3–11.2 on electrostatic forces, Subsection 6–5.1 on scalar magnetic potential, Section 6–8 on magnetic circuits, and Subsections 6–13.1 and 6–13.2 on magnetic forces and torques, may be omitted. Schools on a quarter system could adjust the material to be covered in accordance with the total number of hours assigned to the subject of electromagnetics.

The book in its manuscript form was class-tested several times in my classes on electromagnetics at Syracuse University. I would like to thank all of the students in those classes who gave me feedback on the covered material. I would also like to thank all the reviewers of the manuscript who offered encouragement and valuable suggestions. Special thanks are due Mr. Chang-hong Liang and Mr. Bai-lin Ma for their help in providing solutions to some of the problems.

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Right:
Cylindrical coordinates
Spherical coordinates

1 / The Electromagnetic Model

1-1 INTRODUCTION

Stated in a simple fashion, *electromagnetics* is the study of the effects of electric charges at rest and in motion. From elementary physics we know there are two kinds of charges: positive and negative. Both positive and negative charges are sources of an electric field. Moving charges produce a current, which gives rise to a magnetic field. Here we tentatively speak of electric field and magnetic field in a general way; more definitive meanings will be attached to these terms later. A *field* is a spatial distribution of a quantity, which may or may not be a function of time. A time-varying electric field is accompanied by a magnetic field, and vice versa. In other words, time-varying electric and magnetic fields are coupled, resulting in an electromagnetic field. Under certain conditions, time-dependent electromagnetic fields produce waves that radiate from the source. The concept of fields and waves is essential in the explanation of action at a distance. In this book, *Field and Wave Electromagnetics*, we study the principles and applications of the laws of electromagnetism that govern electromagnetic phenomena.

Electromagnetics is of fundamental importance to physicists and electrical engineers. Electromagnetic theory is indispensable in the understanding of the principle of atom smashers, cathode-ray oscilloscopes, radar, satellite communication, television reception, remote sensing, radio astronomy, microwave devices, optical fiber communication, instrument-landing systems, electromechanical energy conversion, and so on. Circuit concepts represent a restricted version, a special case, of electromagnetic concepts. As we shall see in Chapter 7, when the source frequency is very low so that the dimensions of a conducting network are much smaller than the wavelength, we have a quasi-static situation, which simplifies an electromagnetic problem to a circuit problem. However, we hasten to add that circuit theory is itself a highly developed, sophisticated discipline. It applies to a different class of electrical engineering problems, and it is certainly important in its own right.

Two situations illustrate the inadequacy of circuit-theory concepts and the need of electromagnetic-field concepts. Figure 1–1 depicts a monopole antenna of the type we see on a walkie-talkie. On transmit, the source at the base feeds the antenna with a message-carrying current at an appropriate carrier frequency. From a circuit-theory

Fig. 1-1 A monopole antenna.

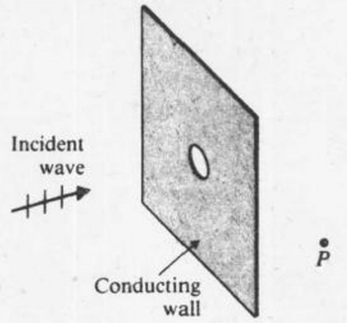

Fig. 1-2 An electromagnetic problem.

point of view, the source feeds into an open circuit because the upper tip of the antenna is not connected to anything physically; hence no current would flow and nothing would happen. This viewpoint, of course, cannot explain why communication can be established between walkie-talkies at a distance. Electromagnetic concepts must be used. We shall see in Chapter 11 that when the length of the antenna is an appreciable part of the carrier wavelength[†], a nonuniform current will flow along the open-ended antenna. This current radiates a time-varying electromagnetic field in space, which can induce current in another antenna at a distance.

In Fig. 1-2 we show a situation where an electromagnetic wave is incident from the left on a large conducting wall containing a small hole (aperture). Electromagnetic fields will exist on the right side of the wall at points, such as *P* in the figure, that are not necessarily directly behind the aperture. Circuit theory is obviously inadequate here for the determination (or even the explanation of the existence) of the field at *P*. The situation in Fig. 1-2, however, represents a problem of practical importance as its solution is relevant in evaluating the shielding effectiveness of the conducting wall.

Generally speaking, circuit theory deals with lumped-parameter systems—circuits consisting of components characterized by lumped parameters such as resistances, inductances, and capacitances. Voltages and currents are the main system variables. For DC circuits, the system variables are constants and the governing equations are algebraic equations. The system variables in AC circuits are time-dependent; they are scalar quantities and are independent of space coordinates. The governing equations are ordinary differential equations. On the other hand, most electromagnetic variables are functions of time as well as of space coordinates. Many are vectors with both a magnitude and a direction, and their representation and manipulation require a knowledge of vector algebra and vector calculus. Even in static cases, the governing equations are, in general, partial differential equations. It

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[†] The product of the wavelength and the frequency of an AC source is the velocity of wave propagation.

is essential that we are equipped to handle vector quantities and variables that are both time- and space-dependent. The fundamentals of vector algebra and vector calculus will be developed in Chapter 2. Techniques for solving partial differential equations are needed in dealing with certain types of electromagnetic problems. These techniques will be discussed in Chapter 4. The importance of acquiring a facility in the use of these mathematical tools in the study of electromagnetics cannot be overemphasized.

1-2 THE ELECTROMAGNETIC MODEL

There are two approaches in the development of a scientific subject: the inductive approach and the deductive approach. Using the inductive approach, one follows the historical development of the subject, starting with the observations of some simple experiments and inferring from them laws and theorems. It is a process of reasoning from particular phenomena to general principles. The deductive approach, on the other hand, postulates a few fundamental relations for an idealized model. The postulated relations are axioms, from which particular laws and theorems can be derived. The validity of the model and the axioms is verified by their ability to predict consequences that check with experimental observations. In this book we prefer to use the deductive or axiomatic approach because it is more elegant and enables the development of the subject of electromagnetics in an orderly way.

The idealized model we adopt for studying a scientific subject must relate to real-world situations and be able to explain physical phenomena; otherwise, we would be engaged in mental exercises for no purpose. For example, a theoretical model could be built, from which one might obtain many mathematical relations; but, if these relations disagree with observed results, the model is of no use. The mathematics may be correct, but the underlying assumptions of the model may be wrong or the implied approximations may not be justified.

Three essential steps are involved in building a theory on an idealized model. *First*, some basic quantities germane to the subject of study are defined. *Second*, the rules of operation (the mathematics) of these quantities are specified. *Third*, some fundamental relations are postulated. These postulates or laws are invariably based on numerous experimental observations acquired under controlled conditions and synthesized by brilliant minds. A familiar example is the circuit theory built on a circuit model of ideal sources and pure resistances, inductances, and capacitances. In this case the basic quantities are voltages (V), currents (I), resistances (R), inductances (L), and capacitances (C); the rules of operations are those of algebra, ordinary differential equations, and Laplace transformation; and the fundamental postulates are Kirchhoff's voltage and current laws. Many relations and formulas can be derived from this basically rather simple model, and the responses of very elaborate networks can be determined. The validity and value of the model have been amply demonstrated.

In a like manner, an electromagnetic theory can be built on a suitably chosen electromagnetic model. In this section we shall take the first step of defining the basic