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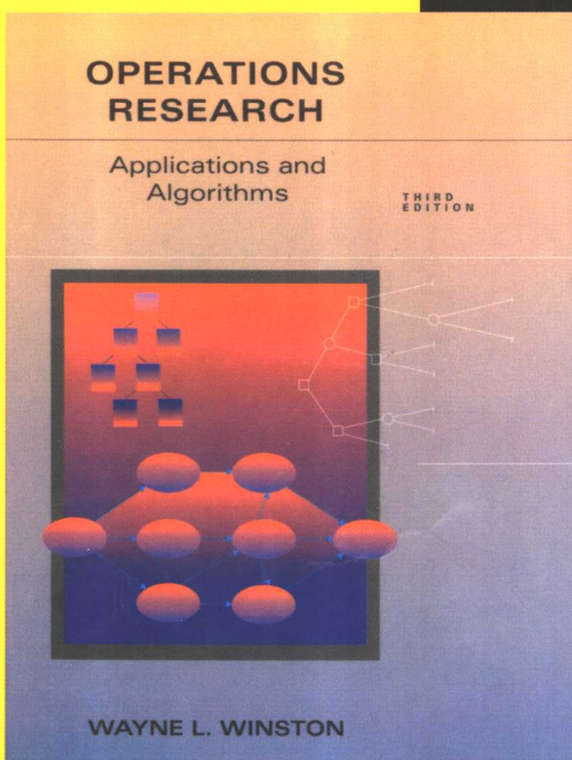
国外大学优秀教材——工业工程系列（影印版）

WAYNE L. WINSTON

运筹学 数学规划（第3版）

OPERATIONS RESEARCH
Mathematical Programming
(THIRD EDITION)



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Forward

This textbook series is published at a very opportunity time when the discipline of industrial engineering is experiencing a phenomenal growth in China academia and with its increased interests in the utilization of the concepts, methods and tools of industrial engineering in the workplace. Effective utilization of these industrial engineering approaches in the workplace should result in increased productivity, quality of work, satisfaction and profitability to the cooperation.

The books in this series should be most suitable to junior and senior undergraduate students and first year graduate students, and to those in industry who need to solve problems on the design, operation and management of industrial systems.

Gavriel Salvendy

Department of Industrial Engineering, Tsinghua University

School of Industrial Engineering, Purdue University

April, 2002

前　　言

本教材系列的出版正值中国学术界工业工程学科经历巨大发展、实际工作中对工业工程的概念、方法和工具的使用兴趣日渐浓厚之时。在实际工作中有效地应用工业工程的手段将无疑会提高生产率、工作质量、合作的满意度和效果。

该系列中的书籍对工业工程的本科生、研究生和工业界中需要解决工程系统设计、运作和管理诸方面问题的人士最为适用。

加弗瑞尔・沙尔文迪
清华大学工业工程系
普渡大学工业工程学院（美国）
2002 年 4 月

†Indicates that section may be omitted without loss of continuity.

3 The Simplex Algorithm

4 Sensitivity Analysis and Duality*

[‡]Indicates that material on use of the computer has been added to this edition.

* Note: Instructors should cover Chapter 5 or Chapter 6, but not both.

5 Integer Programming

6† Advanced Topics in Linear Programming

7 Nonlinear Programming

1

Introduction to Operations Research

During World War II, British military leaders asked scientists and engineers to analyze several military problems: the deployment of radar and the management of convoy, bombing, antisubmarine, and mining operations. The application of mathematics and the scientific method to military operations was called operations research. Today, the term *operations research* (or, often, *management science*) means a scientific approach to decision making, which seeks to determine how best to design and operate a system, usually under conditions requiring the allocation of scarce resources.

1.1 The Methodology of Operations Research

When operations research is used to solve a problem of an organization, the following seven-step procedure should be followed (Figure 1).

Step 1. Formulate the Problem

The operations research (or OR) analyst first defines the organization's problem. Defining the problem includes specifying the organization's objectives and the parts of the organization (or system) that must be studied before the problem can be solved.

To illustrate the idea of problem formulation, suppose that a bank manager hires an operations research analyst to help the bank reduce expenditures on tellers' salaries while still maintaining an adequate level of customer service. After discussion with the manager of the bank, how might the analyst describe the bank's objectives? Here are three possibilities:

1 The bank may want to minimize the weekly salary cost needed to ensure that the average time a customer waits in line is at most 3 minutes.

2 The bank may want to minimize the weekly salary cost required to ensure that only 5% of all customers wait in line more than 3 minutes for a teller.

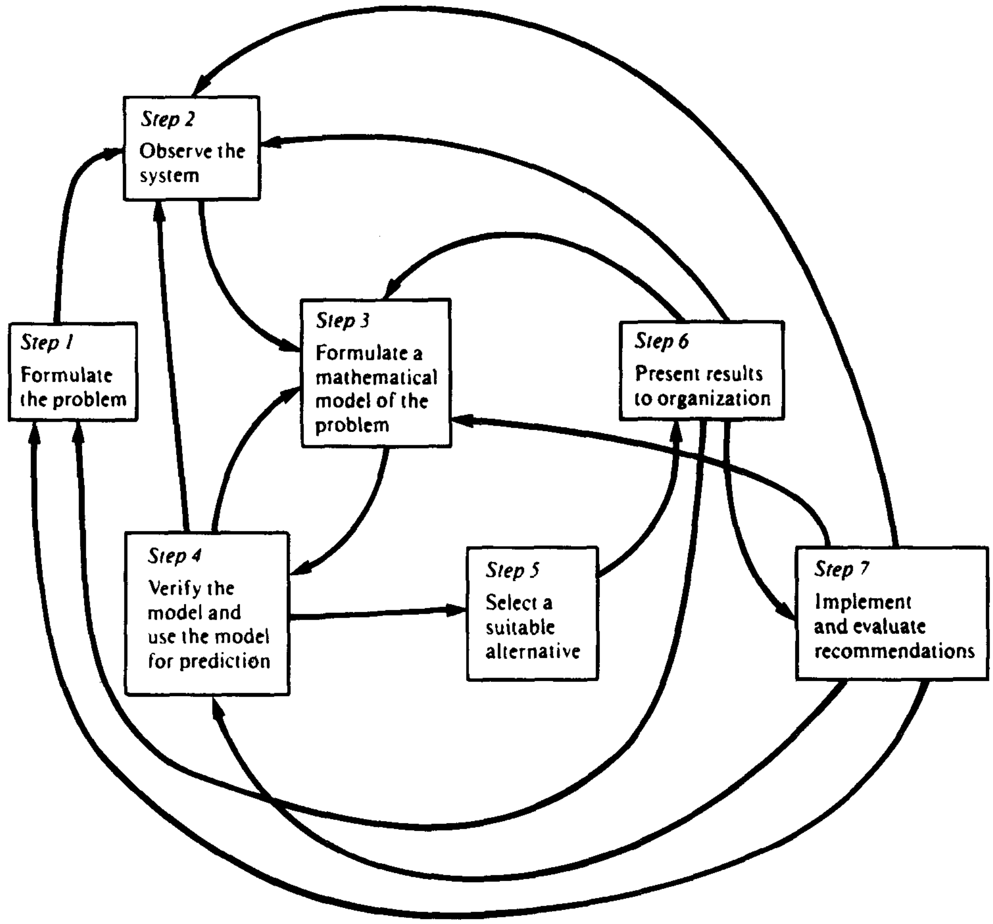
3 The bank may be willing to spend up to $1000 per week on tellers' salaries and minimize the average time a customer must wait in line for a teller, given the salary constraint.

The analyst must also identify the aspects of the bank's operation that affect the achievement of the bank's objectives. For instance,

1 On the average, how many customers arrive at the bank each hour? The more customers, the more tellers will be needed to provide adequate service.

2 On the average, how many customers can a teller serve per hour?

FIGURE 1
The Operations Research Methodology



3 How does having customers wait in a single line rather than in several tellers' lines affect the achievement of the bank's objectives?

4 If each teller has one line, will customers always join the shortest line? If one or another line becomes much longer, will customers jockey (or switch) between lines?

Step 2. Observe the System

Next, the analyst collects data to estimate the values of parameters that affect the organization's problem. These estimates are used to develop (in Step 3) and evaluate (in Step 4) a mathematical model of the organization's problem.

In our bank example, the analyst would collect data to estimate the following system parameters (among others):

1 On the average, how many customers arrive each hour? Does the arrival rate of customers depend on the time of day?

2 On the average, how many customers can a teller serve in one hour? Does the speed with which a teller serves customers depend on the number of customers waiting in line?

3 Do customers always join the shortest line? When at the end of a long line, do customers frequently switch to a shorter line?

Step 3. Formulate a Mathematical Model of the Problem

In this step, the analyst develops a mathematical model (or idealized representation) of the problem. In this book, we describe many mathematical techniques that can be used to model systems.

In our bank example, a mathematical model might be developed to predict the values of

W_q = average time customer waits in line
P = probability that a customer will spend at least 3 minutes waiting in line

based on knowing the following parameters:

λ = average number of customers arriving at the bank each hour
μ = average number of customers a teller can serve in an hour
s = number of bank tellers
Configuration of tellers and the manner in which customers join lines

A mathematical model that yields an equation relating these four parameters to W_q or P would be an example of an **analytic model.** For instance, it can be shown that if there is only one teller, then under certain conditions

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} \quad (1)$$

Thus, the analyst knowing these parameters could use Equation (1) to determine how well the system (the bank) is controlling W_q.

Many situations, however, are so complex that no analytic model exists. For example, if customers switch between lines, then in most situations there is no "tractable" mathematical equation that can be used to relate the four parameters to W_q or P. When no tractable analytic model exists, one must often develop a **simulation model**, which enables a computer to approximate the behavior of the actual system.

Step 4. Verify the Model and Use the Model for Prediction

The analyst now tries to determine if the mathematical model developed in Step 3 is an accurate representation of reality. To determine how well the model fits reality, one determines how valid the model is for the current situation. For instance, in the bank example, suppose one believes the situation satisfies the conditions for which Equation (1) is valid. Then current estimates of the parameters might be substituted into (1), and one would see how well (1) predicts the observed values of W_q. If the model's predictions of W_q and P are not close to the actual values (found by observing the bank), a new model is surely needed. In this case, one would return (note the feedback arrows in Figure 1) to either Step 2 or Step 3 and develop a new model that better describes the actual situation. Even if the first model does fit the current situation, one must beware of blindly applying it. For example, one might develop a model for the bank example that accurately predicts W_q and P when the bank is not crowded. If the bank attracts more customers and becomes more congested, however, the validity of the model under the new situation must be carefully checked.

Step 5. Select a Suitable Alternative

Given a model and a set of alternatives, the analyst now chooses the alternative (if there is one) that best meets the organization's objectives. For instance, in our bank example, one might determine the employment policy that minimizes the weekly salary cost needed to ensure that the average customer waits in line at most three minutes.

Sometimes the set of available alternatives is subject to certain restrictions or constraints. For example, suppose the bank can hire both full-time tellers and part-time tellers (who must work four consecutive hours per day). Assume that part-time tellers are less costly but work more slowly than full-time tellers. One might want to minimize the average time a customer waits in line, subject to the following constraints:

1 At most $2000 per week can be spent on tellers' salaries.
2 Part-time tellers can work at most one quarter of all hours worked each week.

In many situations, the best alternative may be impossible or too costly to determine. For example, a firm considering purchasing a microcomputer might want to buy the one that best satisfies the following criteria:

1 Most economical computer
2 Ease of use
3 Most useful software available
4 Speed of operation

It is likely that no single computer will be the best with respect to all four criteria. Suppose three computers are under consideration. Computer A may be best with respect to criteria 1 and 3 and worst with respect to criteria 2 and 4; Computer B may be best on criteria 2 and 4 but less good for the others; and Computer C may be second best with respect to all the criteria. Which computer would best meet the firm's objectives? This is a difficult question that can be answered using the theory of multiattribute decision making.

Even if a best alternative does exist, it may often be extremely costly to determine. Consider a food-processing company, Foodco, that produces canned foods and wants to minimize the cost of its warehousing and distribution operations. There are 200 sites under consideration for location of warehouses. Foodco's goal is to minimize the sum of warehousing costs (including the costs of building and operating warehouses) plus the total cost of shipping the company's products from manufacturing plants to warehouses and from warehouses to customers. Clearly, Foodco would be interested in answers to the following questions:

1 How many warehouses should be built?
2 Where should the warehouses be built?
3 What should be the size (storage capacity) of each warehouse?
4 From which warehouse should a given customer receive shipments?

It is not difficult (see Chapter 5) to formulate a mathematical model of Foodco's problem, but sometimes an exact solution to this problem is beyond the capacity of today's computers. Solution methods do exist, however, to determine (at reasonable computational cost) a warehousing and distribution strategy that comes close (within 1–2%) to the minimum possible cost.

Step 6. Present the Results and Conclusions of the Study to the Organization

In this step, the analyst presents the model and recommendations from Step 5 to the decision-making individual or group. In some situations, one might present several alternatives and let the organization choose the one that best meets its needs.

After presenting the results of the operations research study to the organization, the analyst may find that the organization does not approve of the recommendations. This may result from incorrect definition of the organization's problems or from failure to involve the decision maker from the start of the project. In this case, the analyst should return to Step 1, 2, or 3 (see Figure 1).

Step 7. Implement and Evaluate Recommendations

If the organization has accepted the study, the analyst aids in implementing the recommendations. The system must be constantly monitored (and updated dynamically as the environment changes) to ensure that the recommendations are enabling the organization to meet its objectives. For instance, in the bank example, suppose the objective is to ensure that at most 5% of customers wait more than 3 minutes. Suppose that after the analyst's suggestions are implemented, 80% of customers spend more than three minutes in line. Then the bank's objective is clearly not being met, and the analyst should return to Step 1, 2, or 3 and reexamine the model (see Figure 1).

1.2 Successful Applications of Operations Research

In this section, we list several applications of operations research. In many of them, the business or government agency involved saved millions of dollars by successfully applying operations research models.

1 *Police Patrol Officer Scheduling in San Francisco.* Using linear programming (see Chapter 2), goal programming, and integer programming (see Chapter 5), Taylor and Huxley (1989) devised a method to schedule patrol officers for the San Francisco Police Department. By using their method, the department has saved $11 million per year, improved response times by 20%, and increased revenue from traffic citations by $3 million per year.

2 *Reducing Fuel Costs in the Electric Power Industry.* By using probabilistic dynamic programming and a simulation model, Chao et al. (1989) saved 79 electric utilities over $125 million in purchasing, inventory, and shortage costs.

3 *Designing an Ingot Mold Stripping Facility at Bethlehem Steel.* Using integer programming (see Chapter 5), Vasko et al. (1989) helped Bethlehem Steel design an ingot mold stripping facility. The integer programming model has saved Bethlehem $8 million per year in operating costs.

4 *Gasoline Blending at Texaco.* Using the blending models discussed in Section 2.8 and nonlinear programming (discussed in Chapter 7), Dewitt et al. (1989) devised a model that is used by Texaco's refineries to determine how to blend incoming crude oils into leaded regular, unleaded regular, unleaded plus, and super unleaded gasolines. It is estimated that this model saves Texaco over $30 million annually. The model also allows Texaco to answer many what-if questions, such as what an increase of 0.01% in the sulfur content of regular gasoline will do to the cost of producing regular gasoline. The method used to answer such what-if questions is called **sensitivity analysis** and is discussed in Chapters 4.

5 *Scheduling Trucks at North American Van Lines.* Using network models and dynamic programming (see Chapter 8), Powell et al. (1988) developed a model that is used to assign loads to North American Van Lines drivers. Use of this model has provided better service to customers and reduced costs by $2.5 million per year.

6 *Inventory Management at Blue Bell.* Blue Bell manufactures jeans, sportswear, and garments worn by white-collar workers. Using linear programming (see Chapter 2) and probabilistic inventory models, Edwards, Wagner, and Wood (1985) reduced Blue Bell's average inventory level by 31%.

7 *Using Linear Programming to Determine Bond Portfolios.* Linear programming (see Chapter 2) has been used by several people (see Chandy and Kharabe (1986)) to determine bond portfolios that maximize expected return subject to constraints on the level of risk and diversification in the portfolio.

8 *Using Linear Programming to Plan Creamery Production.* Sullivan and Secrest (1985) used linear programming (see Chapter 2) to determine how a creamery should process buttermilk, raw milk, sweet whey, and cream into cream cheese, cottage cheese, sour cream, and whey cream. Use of the model has increased the profitability of the creamery by $48,000 per year.

9 *Equipment Replacement at Phillips Petroleum.* How old should a car or truck be before a company should replace it? Phillips Petroleum (see Waddell (1983)) used equipment replacement models (discussed in Sections 8.5) to answer this question. These equipment replacement models are estimated to save Phillips $90,000 per year.

10 *Where Should a City Locate a New Airport?* Many objectives must be considered when determining the location of a new airport, including:

- **a** Cost of building the airport
- **b** Capacity it will provide
- **c** Access time to the airport
- **d** Safety of the system
- **e** Social disruption caused in building it
- **f** Noise pollution caused by airport operations

If no possible location site is best with regard to all objectives, where should the city choose to locate the new airport? Using the technique of multiattribute utility theory, Keeney (1973) helped Mexico City determine where a new airport should be located.

1.3 Where to Read More About Operations Research

Many journals publish articles involving the theory and applications of operations research: *Operations Research, Management Science, Interfaces, Mathematics of Operations Research, Marketing Science, AIIE Transactions, Decision Sciences, Mathematical Programming, European Journal of Operations Research, Production and Inventory Management, Omega*, and *Naval Research Logistics*, among others. For the reader who is particularly interested in present-day applications of OR, we heartily recommend *Interfaces*.

1.4 About This Book

In this book, we discuss many of the mathematical models commonly used by operations research analysts in modeling systems and organizations. In most chapters, we allocate at least as much space to the art of formulating mathematical models as to the solution of a mathematical model (selecting the best alternative). Even if the reader does not use most of the models discussed in this book, the emphasis on formulating mathematical models will be helpful in other quantitatively oriented courses and in any job in which quantitative reasoning is important.

We have tried to make this book as self-contained as possible. Certain chapters require a knowledge of basic differential and integral calculus and elementary probability theory. Others require a knowledge of linear algebra.

Problems are given at the end of each section, with additional problems at the end of each chapter. We urge the reader to solve as many of the problems as possible. The best way to keep up with the material is to do some problems at the end of each section after attending a lecture. Remember that the only way to master the material covered in this book is by doing the problems!

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2

Introduction to Linear Programming

Linear programming (LP) is a tool for solving optimization problems. In 1947, George Dantzig developed an efficient method, the simplex algorithm, for solving linear programming problems (also called LP). Since the development of the simplex algorithm, LP has been used to solve optimization problems in industries as diverse as banking, education, forestry, petroleum, and trucking. In a survey of Fortune 500 firms, 85% of those responding said that they had used linear programming. As a measure of the importance of linear programming in operations research, approximately 40% of this book will be devoted to linear programming and related optimization techniques.

In Section 2.1, we begin our study of linear programming by describing the general characteristics shared by all linear programming problems. In Sections 2.2 and 2.3, we learn how to solve graphically those linear programming problems that involve only two variables. Solving these simple LPs will give us some useful insights for solving more complex LPs. The remainder of the chapter explains how to formulate linear programming models of real-life situations.

2.1 What Is a Linear Programming Problem?

In this section, we introduce linear programming and define some important terms that are used to describe linear programming problems.

EXAMPLE 1 Giapetto's Woodcarving, Inc., manufactures two types of wooden toys: soldiers and trains. A soldier sells for $27 and uses $10 worth of raw materials. Each soldier that is manufactured increases Giapetto's variable labor and overhead costs by $14. A train sells for $21 and uses $9 worth of raw materials. Each train built increases Giapetto's variable labor and overhead costs by $10. The manufacture of wooden soldiers and trains requires two types of skilled labor: carpentry and finishing. A soldier requires 2 hours of finishing labor and 1 hour of carpentry labor. A train requires 1 hour of finishing and 1 hour of carpentry labor. Each week, Giapetto can obtain all the needed raw material but only 100 finishing hours and 80 carpentry hours. Demand for trains is unlimited, but at most 40 soldiers are bought each week. Giapetto wants to maximize weekly profit (revenues – costs). Formulate a mathematical model of Giapetto's situation that can be used to maximize Giapetto's weekly profit.

Solution In developing the Giapetto model, we explore characteristics shared by all linear programming problems.

Decision Variables We begin by defining the relevant **decision variables.** In any linear programming model, the decision variables should completely describe the decisions to be