Third Edition

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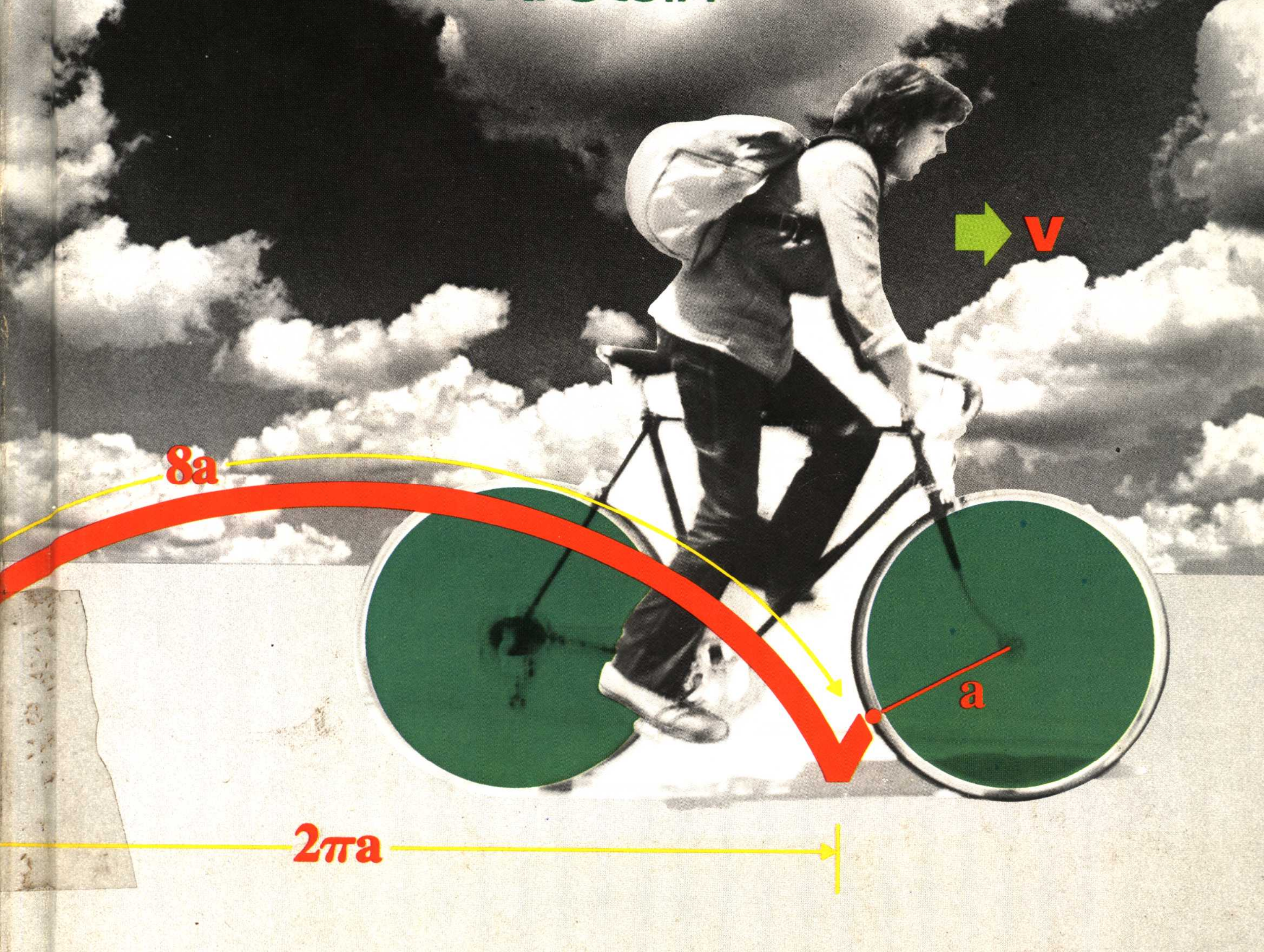
Sherman K. Stein



CALCULUS AND ANALYTIC GEOMETRY

Third Edition

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To
Joshua,
Rebecca,
and
Susanna

Arnold Toynbee, *Experiences*, Oxford University Press, pp. 12–13, 1969.

. . . at about the age of sixteen, I was offered a choice which, in retrospect, I can see that I was not mature enough, at the time, to make wisely. This choice was between starting on the calculus and, alternatively, giving up mathematics altogether and spending the time saved from it on reading Latin and Greek literature more widely. I chose to give up mathematics, and I have lived to regret this keenly after it has become too late to repair my mistake. The calculus, even a taste of it, would have given me an important and illuminating additional outlook on the Universe, whereas, by the time at which the choice was presented to me, I had already got far enough in Latin and Greek to have been able to go farther with them unaided. So the choice that I made was the wrong one, yet it was natural that I should choose as I did. I was not good at mathematics; I did not like the stuff . . . . Looking back, I feel sure that I ought not to have been offered the choice; the rudiments, at least, of the calculus ought to have been compulsory for me. One ought, after all, to be initiated into the life of the world in which one is going to have to live. I was going to have to live in the Western World . . . and the calculus, like the full-rigged sailing ship, is . . . one of the characteristic expressions of the modern Western genius.

PREFACE

The goal of this edition remains the same as that of the first two editions: To provide the student and the instructor with a readable, flexible text that covers the important topics of single and multivariable calculus as simply and clearly as possible.

Organizational changes

When both users and nonusers of the second edition made the same suggestions in the survey conducted by the publisher, I accepted their advice. As a result, the antiderivative and the definite integral are treated much earlier. Limits precede the derivative. An optional section on ε, δ has been added. The treatment of the number e has been drastically revised.

Plane curves, applications of the definite integral, series, and multiple integrals are now covered in single chapters. The discussion of line integrals and Green's theorem and its generalizations has been reorganized and expanded. Vectors are treated before partial derivatives so that directional derivatives and the gradient can be treated with partial derivatives.

There have been some deletions and many additions, such as optional sections on complex numbers, the relation between the exponential function and the trigonometric functions, separable and linear differential equations, and the role of the Jacobian in change of variables. Two new overview sections, "What to do in the face of an integral" (Sec. 7.9) and "How to set up a definite integral" (Sec. 8.3), should prove helpful. Optional sections on Kepler's laws and Maxwell's equations have been added, with the former developed through a sequence of exercises. The appendixes now include a review of algebra and a treatment of change of coordinates.

The derivatives of the trigonometric, exponential, and logarithmic functions are still done quite early because of their importance in applications and the students' need for extensive practice with them. Furthermore, the arguments that obtain the derivatives of these functions reveal more clearly the idea of a limit than does the algebra that produces the derivative of a polynomial (in which Δx can be set equal to 0 with impunity). Also, L'Hôpital's rule is presented early in order to make it available throughout the course.

Pedagogical changes

Once again I have strengthened the chapter summaries, which students find very helpful; they provide an emphasis and perspective that individual sections cannot. Also I have added many more asides in the margin to guide the student. Figures have been revised and many new ones added; they are now numbered.

Applications

The number and variety of applications have been increased. This has been done primarily through exercises since I wanted to keep the main exposition uncluttered. Applications vary in length from a brief mention in an exercise to one- or two-page presentations in the text. They are listed in the index under "applications."

Exercises

I have not counted the exercises, but there are more than enough of all degrees of difficulty. Exercises before the single box (■) are routine. These generally now come in pairs, with each odd-numbered exercise comparable to the following even-numbered exercise. They focus principally on definitions and drill, and so should not constitute a full homework assignment. Exercises between the single box and double box may involve more steps or computations. Exercises after the double box may be more challenging or offer alternative perspectives or further applications. Often the most interesting (but not necessarily the most difficult) problems are to be found here. The back of the book contains answers to the odd-numbered exercises and guide quizzes. Calculator exercises are included when appropriate.

Epsilon, delta

Section 2.4, which is new, is devoted to the ε, δ definition of limits. It begins with the definition of $\lim_{x\to\infty} f(x) = \infty$, for which the concept, the diagrams, and the details are easiest. Then, after dealing with $\lim_{x\to\infty} f(x) = L$, it turns to $\lim_{x\to a} f(x) = L$. This section may be omitted (it is marked "optional") or it may be covered in one to three lectures, depending on the depth of treatment.

Level of difficulty

The level of difficulty is controlled by the choice of sections and exercises and by the pace. The exposition has been kept as simple as possible, with a strong emphasis on motivation. The text can serve students of widely varying abilities and interests, such as those in engineering, the physical sciences, mathematics, economics, and biology.

Differential equations

The text now contains solutions to two types of differential equations: separable and linear with constant coefficients. The first are included because of their use in the differential equations of natural growth and inhibited growth, the second, because they suffice for almost all elementary physical applications.

Complex numbers

Students who do not meet complex numbers in calculus could easily bypass them completely. In subsequent courses that do make use of complex numbers it is often assumed that the student has "surely" met them somewhere—in high school or in calculus. Therefore Sec. 10.9 is devoted to the complex numbers. The following section obtains the equation $e^{i\theta} = \cos\theta + i\sin\theta$, thus giving a major application of series and demonstrating the connection between the exponential and the trigonometric functions. I encourage the instructor to include these sections, though they are marked optional, even at the sacrifice of some traditional material.

Duration

There is enough material for a three-semester course. Since the number of class meetings per week ranges from three to five, it is impossible to give a uniform guide to what should be covered each quarter or semester. As a rule of thumb one section corresponds to one class meeting, though several are longer and some shorter. There are 121 non-summary sections in Chapters 0 to 15. Of these, sixteen are marked optional.

The following table describes a maximum (complete) and a minimum (core only) treatment, with remarks on certain sections. Most instructors will steer a course somewhere between the two listed. The instructor's manual has a more detailed commentary as well as answers to the even-numbered exercises, including sketches of solutions to the non-routine exercises.

	MAXIMUM		MINIMUM	
	Lectures	Comment	Lectures	Comment
Chapter				
0	2	Survey of calculus and text	0	Left to student to read
1	4		1	Precalculus material, but mention Secs. 1.3, 1.4
2	10	Two days on Sec. 2.2; three days on Sec. 2.4, perhaps a bit of Appendix F	6	Omit Sec. 2.4
3	6		6	
4	8	Two days on Sec. 4.5	7	
5	6		5	Omit Sec. 5.6
6	13	Two days on each of Secs. 6.7, 6.8, 6.9	8	Omit Secs. 6.3 and 6.10
7	9		6	Omit Secs. 7.3 and 7.9; coalesce Secs. 7.6, 7.7, 7.8
8	9		7	Omit Sec. 8.7; touch Sec. 8.8 lightly
9	7		6	Omit Sec. 9.6
10	13		8	Omit Secs. 10.9 to 10.13
11	6		5	Assume Sec. 11.5
12	9		7	Omit Secs. 12.8, 12.9
13	10	Two days on Sec. 13.3	7	Omit Secs. 13.5, 13.6
14	8		5	Omit Secs. 14.7, 14.8
15	9	Two days on Sec. 15.3	7	Omit Sec. 15.8
Total	129		91	
Appendix				
A	1	Precalculus material, some to be treated early in course (A–E)	0	Precalculus material, used as reference by students (A–E)
B	2		0	
C	1		0	
D	2		0	
E	4		0	
F	3	ε, δ continued	0	
G	3	A sample of advanced calculus	0	
Total	16			

Options

Chapter 0 may be left to the student to read. If the class is adequately prepared, Chap. 1 may be omitted or the last two sections emphasized. In Chap. 2 there is the choice of omitting Sec. 2.4 on ε, δ.

Chapter 6 offers a choice in the way logarithms are treated. The approach in Secs. 6.1, 6.2, and 6.4 assumes the exponential functions as given and that $\lim_{h \to 0} (1 + h)^{1/h}$ exists. It grows naturally out of the student's precalculus experience and provides an opportunity to review the manipulations of exponents and logarithms. However, instructors who wish to define the logarithm as an integral are free to follow Secs. 6.3 and 6.10 and de-emphasize Secs. 6.1 and 6.2. (If the informal approach in Secs. 6.1, 6.2, and 6.4 is followed, most of Chap. 6 can be done before Chap. 5, that is, Secs. 6.1, 6.2, 6.4 to 6.6, 6.8 and 6.9.)

The next choice is how much attention will be given the special integration techniques in Secs. 7.6 to 7.9.

In Chap. 10, after completing the standard topics in series, there are several optional sections. Sections 10.9 and 10.10, taken together, introduce complex numbers and exhibit a major application of series. Section 10.11, on linear differential equations with constant coefficients, depends at one point on Sec. 10.10.

Sections 13.5 and 13.6 present an optional unit, an intuitive treatment of the Jacobian, its significance as a measure of local magnification, and its use in the change of variables in an integral.

Only Secs. 14.1, 14.2, 14.3, and 14.6 in Chap. 14 are needed in Chap. 15.

Solutions manual

Complete solutions to all odd-numbered exercises and guide quizzes are available to the student in a manual prepared to accompany this text.

Acknowledgments

At each stage of this revision two former graduate students at Davis, Anthony Barcellos and Dean Hickerson, scrutinized every sentence, every formula, every diagram, every marginal note (adding, incidentally, many of their own), and every line of type. Harsh taskmasters, both conscientiously represented instructor and student; their dedication and thoroughness significantly improved much of the exposition.

The revision also benefits from suggestions I received from colleagues at Davis, in particular Henry Alder, Carl Carlson, G. Donald Chakerian, David Mead, Washek Pfeffer, and Evelyn Silvia.

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Several students worked on parts of the text and the accompanying solutions manual, reviewing manuscript, doing exercises, and checking answers. For their labors I wish to thank Dana Reneau, Judy Clarke, Colin Missel, Mark O'Donnell, Karen Thomason, Kevin Zumbrun, and Ed Bazo.

My appreciation also goes to Shelly Langman, Carol Napier, and Stephen Wagley at McGraw-Hill for their enthusiastic and skillful support in this revision. In spite of the little leeway in choice and order of topics granted any author of a basic text, the publisher has encouraged me to offer fresh options if they are needed by users of calculus.

One final remark. Special care has been taken to keep errors to a minimum. Galleys and page proofs received four independent readings. Each exercise was worked by at least three people; answers in the back of the book were checked against page proofs. Though it is every author's dream to produce the error-free book, no one, to my knowledge, has ever achieved that aspiration. I would therefore appreciate your calling to my attention any errors that may still remain.

Sherman K. Stein

CONTENTS

AN OVERVIEW OF CALCULUS AND THIS BOOK

0

There are two main concepts in calculus: the derivative and the integral. Underlying both is the theme of limits. This chapter introduces these ideas informally, tells where they appear in the text, and offers a glimpse into their history. The reader may wish to turn back to these pages from time to time to maintain a broad perspective, which is otherwise too easily lost in the day-by-day details of definitions, theorems, and applications.

0.1 The derivative

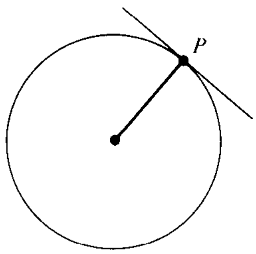

Figure 0.1

The tangent line to a circle at a given point P can be found as follows. First draw the radius from the center of the circle to P and then construct the line through P perpendicular to that radius. That line is tangent to the circle. (See Fig. 0.1.)

But how would we construct the tangent line at a point P on a curve that is not a circle? For instance, how would we find the tangent line at the point P on the curve in Fig. 0.2 which is described by the equation $y = x^2$?

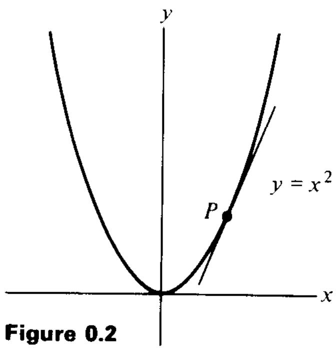

Figure 0.2