OPTIMUM DESIGN OF STRUCTURES

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Preface

Operational research is truly a post-war science. Since the formulation of the simplex method by G. Dantzig in 1946, mathematical programming, both linear and non-linear, has been applied to every aspect of engineering. It deals with the formulation, scientificly, of the tactics for achieving a certain strategy. The main object of this book is to present a reasonably comprehensive account of the various established methods in mathematical programming in a manner acceptable to the engineer. Ideas are familiarised with the aid of simple examples which are solved numerically as well as graphically and the various methods of mathematical optimisation are then used in the design of structures. This is achieved by introducing a large number of examples.

The value of optimisation becomes more apparent with larger structures which require the use of recent advances in matrix and computer methods. For this reason, the entire book is presented using matrix methods. The concept of design is fundamentally different from that of analysis and for this reason the various matrix methods are presented in a form more suitable to the design of structures as opposed to their analysis. A design approach also requires the formulation of theorems which are basically design tools and for this reason a number of recently developed design theorems are included, together with examples of their application.

Chapter 1 gives a brief introduction to the various existing design methods. Commencing with the design of simple beams. Methods of designing more complex structures are then outlined covering the use of elastic, plastic and elasto-plastic theorems. In contrast to these conventional methods, a detailed account of a more direct method of design is given, followed by a discussion of the wider aspects involved in structural design.

Chapter 2 is devoted to mathematical optimisation. This starts with a detailed account of linear programming and the simplex method and concludes with a description of the gradient method. The use of the simplex method in solving discrete problems by integer programming is also covered together with some examples. The various methods of non-linear programming, such as the cutting plane method, the piecewise linearisation method, as well as geometric and dynamic programming are also introduced in this chapter.

Chapter 3 covers the optimum design of structures using the matrix force method. The primary objects of design, such as minimum weight as opposed to minimum cost, are discussed. This is followed by the design of pin jointed and rigidly jointed structures which are either statically determinate or hyperstatic. An example of design by geometric programming is also includent in this Chapter. Chapter 4 deals with the design problem using the matrix displacement method. Cost functions, which are not directly related to the weight of the structure are given together with examples. This chapter also discusses the use of the cutting plane method for solving design examples.

Chapter 5 deals with special aspects of design such as the minimum weight design using plastic theory. Such problems as the design for proportional deflections, for discrete sections, for least mean square deflections as well as design by dynamic programming, integer programming and piecewise linearisation are dealt with in this chapter.

The last chapter uses the new theorems of structural variations when designing structures with variable shape. The theorems themselves are first presented and proved and examples are solved to show their use in the analysis of changing structures. The remainder of the chapter is devoted to the use of the gradient methods for the selection of a particular shape of a structure which best serves the design purpose.

Throughout the book the emphasis has been placed on safety and economy as design requirements. When considering the safety of a structure both stress and deflection limitations are taken into account and particular attention is paid to design problems where these requirements interact. Design exercises with answers, are given at the end of each chapter.

The author wishes to thank members of staff and research students who worked with him on this book or contributed towards the correction of the text. In particular, he wishes to thank Dr. D. W. C. Elliott, of Husband & Co., who carried out research on the subject matter presented in chapter 6. Thanks are due to Mr. P. C. L. Croxton and Dr. D. Anderson for reading the text and checking the examples. The author particularly wishes to thank Miss C. Horton and Miss P. Sage for the preparation of the manuscript and the drawings.

K. I. MAJID

Units

The book is written in SI units. If imperial units are required the following conversion factors may be used for calculating the figures used in the text.

SI	*Imperial*
1 mm	0.0394 in
1 m	3.2808 ft
1 N	0.2248 lbf
1 kN/m^2	20.89 lbf/ft^2
1 N/mm^2	0.648 $tonf/in^2$
1 kN/m^2	9.32×10^{-3} $tonf/ft^2$
1 kNm	8.85×10^3 lbf.in
1 kNm	3.95 tonf.in
1 kNm	736 lbf.ft
1 kNm	0.329 tonf.ft
9.96352 kN	1 tonf

9.81 N = 1 kgf

Modulus of elasticity for steel $\doteq$ 207 kN/mm^2 = 30×10^6 lbf/in^2

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Notation

Symbols are defined when they appear for the first time in the text. Each one is redefined when its meaning changes. Some symbols are used throughout the text with the following meaning:

A, **a**	Area
A	Displacement transformation matrix
$[\mathbf{a}_i]$	Row i of matrix *A*
B	Load transformation matrix
$\mathbf{c}_i$	Cost coefficients in the objective function
$[d]$	Direction vector with a typical element d_j
E	Modulus of elasticity
f	Member flexibility matrix
G	Total number of groups in a structure
H	Horizontal force
I	Second moment of area
J	Total number of joints in a structure
K	The overall stiffness matrix
k	The stiffness matrix of a member
L	The load vector or matrix
$\mathbf{L}_b$	External load on a basic statically determinate structure
$\mathbf{L}_r$	The vector of forces in redundant members
l, l	Span, length
M	Moment
M_p	Full plastic hinge moment
N	Number of members
P	Member force vector or matrix
S	The stress matrix
U	Strain energy
U	Member distortion vector
V	Vertical force
W	Load or weight
X	Joint displacement vector
y	I/z, distance of a point from the neutral Axis

Z	Value of the objective function
z	Section modulus
σ	Stress
$f(x)$	Gradient vector
$\prod_{i=1}^{N} x_i$	Product of N variables $= x_1 x_2 x_3 \ldots x_N$
δ, Δ	Permissible deflections

Contents

Chapter 1

An Outline of existing Design Methods

1.1. Design by repeated analyses

The problem of design often appears to be that of repeated analyses. In the case of statically determinate structures, designed to satisfy a given set of permissible stresses, it may be sufficient to carry out one analysis. This is because the forces in the members of these structures are independent of their cross-sectional properties. Once these forces are determined, by equations of static equilibrium, the cross-sectional properties are selected so that the stresses do not exceed their permissible values. Often the designer selects these sections so that the weight of the structure is minimum.

In every design method the design criteria are often selected to suit the theory employed in the analysis. In the case of design by an elastic theory, for instance, a criterion is that the elastic working stresses should not exceed certain permissible values σ_w laid down in appropriate specifications such as BS 449. Design practice then ensures that the resulting structure is safe. In the case of elastic theory a factor of safety f is applied against yielding anywhere in the structure.

For example, consider the design of a simply supported beam of span l, carrying a uniformly distributed load w per unit length. The maximum bending moment, at the midspan of this beam, can be calculated by statics as $wl^2/8$. From the theory of simple elastic bending we have

$$\left.\begin{aligned} M/I &= \sigma/y \\ \text{or}\quad My/I &= \sigma \end{aligned}\right\} \tag{1.1}$$

where M is the moment at a point in the beam, I is the second moment of area of the cross section, y is the distance of the point from the neutral axis and σ is the stress at the point. In the case of the simply supported beam in order to safeguard against yielding, it is necessary

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to have

$$fwl^2y/8I \leqslant \sigma_y \tag{1.2}$$

where σ_y is the guaranteed yield stress of the material. It is therefore necessary to select a section that has a modulus z, which is the ratio I/y, sufficiently large to satisfy the inequality in equation 1.2.

The lightest section may be acceptable but the design criteria often include many other items that a designer must take into consideration. For instance, it may be required, for architectural reasons, to select a section with the least depth. Furthermore, in many cases the deflection of the beam is restricted to a certain ratio of its span. For example, the BS 449 limits the midspan deflection of this beam to $l/360$. The selected section therefore has to satisfy this deflection requirement which is given by

$$5wl^4/384EI \leqslant l/360 \tag{1.3}$$

It is also necessary to check that the shear stress in the beam does not exceed the permissible values also given in standard specifications. Strictly speaking, the above design is not complete because the self weight of the beam was not included with the load w. It is therefore necessary to add the weight per unit length of the selected section to the load w and repeat the procedure.

Historically, engineers designed statically determinate structures before hyperstatic ones. This is perhaps why they decided to design the latter also by analysing them first. The analysis of hyperstatic structures, however, requires a knowledge of the member properties such as the area or the second moment of area of the sections. Unless these are known, the analysis is unreliable. The oldest, and perhaps the crudest approach, is to assume these sectional properties, analyse the structure and use the results to select a new set of properties. Repeating this cycle of operations often leads to a feasible design. This method however suffers from the fact that it confuses the theory of design by avoiding it altogether and carrying out a number of analyses instead. For realistic hyperstatic structures, this approach is unnecessarily tedious and involves the solution of a large number of simultaneous equations. Furthermore the final set of sections depends to a great extent upon the initial erroneous set. For this reason they are not necessarily the best set and are either heavy or costly to construct. The weakness of this approach becomes apparent when it is realised that most design offices are often short of time and cannot try a number of alternatives to select the most suitable.

These factors played a part in initiating the search for other, much quicker, methods. One outcome was to rely on approximations, thus aggravating the errors involved. One such approximation, which is also commonly practised, is to cast aside the hyperstatic structure and replace it by a statically determinate one that has the same shape. This is achieved by inserting a sufficient number of imaginary hinges at points of counterflexures, which are themselves assumed by the designer. A sufficient number of these hinges render the structure statically determinate and thus it can be analysed by the simple equations of equilibrium. Triangular rigid structures are often treated similarly. The rigid joints are replaced by pins and the resulting structure is then analysed for the member forces.

A second approach assumes that at collapse, the material of the structure yields at a number of sections so that the structure may behave as a mechanism. This approach was developed mainly at Cambridge by Baker *et al.*[1] and is known as the rigid-plastic theory. Unlike the elastic theory, which is concerned with the behaviour of the structure at the working condition, the plastic theory considers the state of failure and, from this, it derives the sections required to sustain the working loads. At the rigid-plastic collapse the structure is also statically determinate. There are a sufficient number of points in the structure where plastic hinges are developed. Each hinge can withstand a constant amount of bending moment known as the 'plastic hinge moment' of the section which can be calculated from the dimensions of the section and the yield stress of the material. Once again, because it is assumed that the collapsing structure is statically determinate, it is possible to use the equations of equilibrium to evaluate the collapse load. A load factor λ is then applied to the working load of the structure and provided that the result is less than the collapse load, it is considered that the structure is safe. This method is now used to design a portal frame.

1.2 Design of a portal by the plastic theory

Consider the frame of Figure 1.1 which is subject to a working vertical load $2W$ at the midspan E of the beam AC and a horizontal load W at the beam level. This frame may be converted into a mechanism in three different ways. To obtain these, it is assumed that the full plastic moment of the beam is M_p and that of the columns is 1.5 M_p. A beam mechanism develops with three hinges, one at E under the vertical load and one at each end of the beam as shown in Figure 1.1b. Because the columns are made stronger than the beam, the plastic hinges tend to develop at the ends of the beam before those at the ends of the columns.

The equation of vertical equilibrium can be used to derive a virtual work equation for the collapse mechanism. As the structure deflects at factored loads $2\lambda w$ acting vertically and λw horizontally, the rotation of the plastic hinges at B and C, at a given instant, is θ while that of the hinge at E is 2θ. The load $2\lambda w$ moves down by an amount $2l\theta$ and the work done by this load is therefore $4\lambda wl\theta$. The horizontal load does no work and is not shown in Figure 1.1b. The work done by the

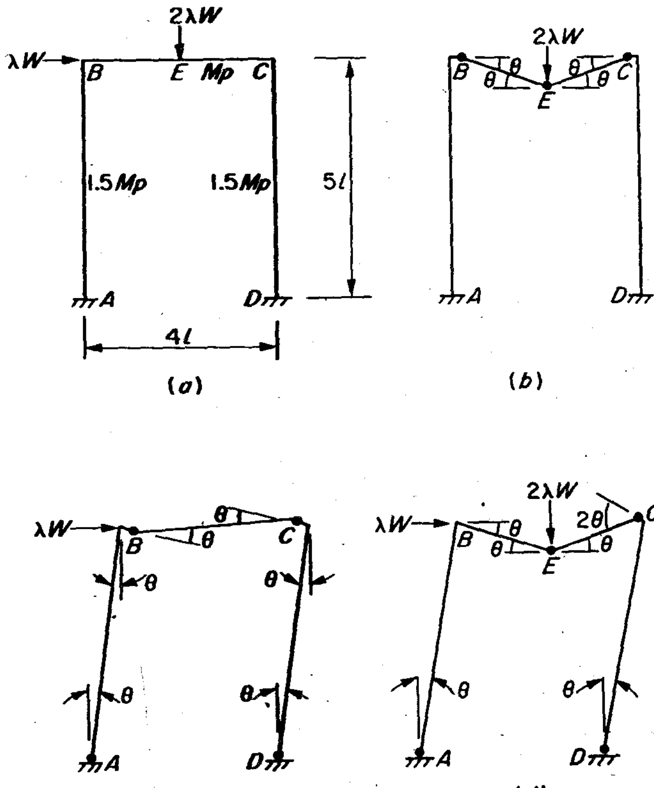

Figure 1.1 Various mechanisms of a portal
(a) Frame and loading
(b) Beam mechanism
(c) Sway mechanism
(d) Combined mechanism

vertical external load is converted into strain energy and consumed by the plastic hinges as they rotate. For instance the hinge at B which is subject to a constant bending moment M_p, rotates by an amount θ and hence consumes $M_p\theta$ of the available energy. Similarly for the other hinges. Equating the work done by the load to the total energy required by the hinges, we obtain the virtual work equation for the collapse mechanism, thus

$$M_p\theta + M_p\theta + 2M_p\theta = 4\lambda wl\theta$$

Hence

$$M_p = \lambda wl \tag{1.4}$$

On the other hand a sway mechanism develops in this frame with four hinges at A, D, B and C as shown in Figure 1.1*c*. The virtual work equation for this collapse mechanism can be derived similarly as

$$3M_p\theta + 2M_p\theta = 5\lambda wl\theta$$

giving

$$M_p = \lambda wl \tag{1.5}$$

Finally a combined mechanism can develop with two hinges at A and D and two other hinges at E and C in the beam. This is shown in Figure 1.1*d*, for which the virtual work equation is:

$$3M_p\theta + 4M_p\theta = 5\lambda wl\theta + 4\lambda wl\theta$$

Thus

$$M_p = 9\lambda wl/7 \tag{1.6}$$

Comparing equations 1.4, 1.5 and 1.6, it is noticed that the largest M_p is required by equation 1.6, for the combined mechanism. To prevent collapse it is therefore necessary to select a section for the beam with a plastic hinge moment which is at least as large as that given by equation 1.6. Accordingly the value of the plastic hinge moment for the columns must be 1.5 times larger.

The weakness of the plastic theory lies in the fact that for most frames, the axial loads in the members are high and because of instability effects, these frames collapse before the development of a mechanism. Therefore at collapse the structure is hyperstatic. Another weakness of the plastic theory is that it assumes that the structure does not deflect until collapse. Because of this assumption, the theory does not consider the deflected shape of the structure in deriving the equations of equilibrium. Furthermore it does not impose any limit on the permissible deflections and thus does not include deflection requirements in the design criteria.

1.3 The elastic-plastic design

According to the elastic-plastic theory, the hinges in a structure do not develop all at once. For this reason, while a load factor is adopted against the collapse of the structure, the load factor at which individual hinges develop is different from one hinge to another. For structures where member axial forces and/or joint deflections are high, an elastic-plastic approach becomes necessary, not only to predict the failure load factor λ_F, but also to investigate exactly where, during the process of loading, the plastic hinges develop.

Like any other theory, the elastic-plastic theory requires its own design criteria and design procedure. This theory considers a frame satisfactory if elastic-plastic analyses under proportional loading reveal that none of the following design criteria are violated:

(1) Under combined dead load, super load and wind load from either side, the frame should not collapse below the permissible load factor λ_1. This is usually taken as 1.4.
(2) Under dead load and vertical superload, the frame should not collapse below the permissible load factor λ_2, which is usually taken as 1.75.
(3) No plastic hinge should develop in a beam below the load factor of unity and the frame should be entirely elastic under the working load.
(4) No plastic hinge should develop in a column below the permissible load factor λ_1 under combined loading, or λ_2 under vertical loading.

A detailed description of the elastic-plastic method of design is given by Majid[2]. It is sufficient therefore to give a summary of the design procedure which is as follows:

(i) A set of lower bound sections is selected for the members;
(ii) The frame is then analysed elastic-plastically, under all the different combined load cases, up to collapse. From these analyses the load factor at which each hinge develops is recorded together with other particulars such as the axial loads in the members;
(iii) From this information, the members are redesigned to satisfy criteria (3) and (4);
(iv) Steps (i) to (iii) are repeated until design criteria (1), (3) and (4) are satisfied with combined loading;
(v) The frame is then analysed under vertical loading to check whether criterion (2) is satisfied and final alterations are made if required.

The elastic-plastic design is more accurate than the plastic method because it considers the behaviour of the frame at every stage of loading up to and including the pre-mechanism collapse. During the various stages of the analysis the effects of both plasticity and instability are considered. It is clear, however, that the method requires a computer to carry out the elastic-plastic analysis. This method is also one of repeated analyses of a structure starting with an assumed lower bound set of sections.

1.4. A direct method of design

Every design method described so far has been one of analysis and not design in its strictest sense. These methods are either approximate because they consider equilibrium only, or lengthy because of carrying out repeated analyses of the structure. The methods all yield a set of feasible sections but none of them can claim to give the best set of sections. For example in the case of the rigid-plastic approach it was assumed, in the example of the portal frame, that the value of M_p for the columns was 1.5 times that for the beam. The relative strength of these members could have been chosen in an infinite number of other ways but only one set would have given the most suitable design in every sense.

A direct method for the design of hyperstatic structures was developed by Pippard[3] as early as 1922. This method should be considered as one of exceptional importance not only because it is simple and requires no solution of simultaneous equations, but also because it demonstrates that the problem of design is different from that of repeated analysis. Furthermore it leaves the designer as the policy maker with a choice of the most suitable design from a large number of alternatives, all of which are obtained easily. This method is now briefly described.

Consider a general hyperstatic pin jointed structure which is subject to a set of external forces $L_b = \{L_1 L_2 \ldots L_m\}$ where the suffix b refers to the basic statically determinate part of the structure. Altogether there are m loading points. Let the forces in the redundant members be $L_r = \{R_1 R_2 \ldots R_n\}$, where there are n redundant forces R. The suffix r refers to the redundant members. The force p_i in any member i is given by

$$p_i = aL_1 + bL_2 + \ldots + mL_m + \alpha R_1 + \beta R_2 + \ldots + \nu R_n \qquad (1.7)$$

or for the whole structure, using matrix notations

$$\mathbf{P} = \mathbf{B}_b\mathbf{L}_b + \mathbf{B}_r\mathbf{L}_r = [\mathbf{B}_b\mathbf{B}_r]\{\mathbf{L}_b\mathbf{L}_r\} \qquad (1.8)$$

where $a, b, n, \alpha, \beta, \ldots \nu$ are numerical constants depending upon the geometry of the structure and can be calculated by joint resolution, i.e. by equilibrium. The matrix $\mathbf{P}$ contains all the member forces including the redundants and matrix $\mathbf{B} = [\mathbf{B}_b\mathbf{B}_r]$ is the force transformation matrix relating the member forces on the one hand to the external loads and the redundant forces on the other. Pippard did not use matrix notations in proposing the method. However, because the rest of this book uses computer orientated matrix methods, it is considered suitable to introduce matrices here and present the method using both notations. The matrices used here will be derived and described in detail in Chapter 3. The strain energy of the structure U is

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