流体力学问题论文集

COLLECTED WORKS ON THE PROBLEMS OF FLUID MECHANICS

VOLUME II

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COLLECTED WORKS ON THE PROBLEMS OF FLUID MECHANICS

The Production and Stability of Converging Shock Waves*

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Converging shock waves offer interesting possibilities of attaining very high temperatures and pressures. A theoretical treatment by G. Guderley which we have confirmed and extended by the method of characteristics indicated that the strength of a strong converging cylindrical or spherical shock varies inversely with a power (0.396 for $\gamma=1.4$) of the surface area of the wave, thus becoming very great close to the center of convergence. The experimental production of high temperatures and pressures by means of these converging shocks depends on their "stability" of form. A converging wave is said to be stable if it approaches perfect cylindrical or spherical shape, thus damping out random disturbances as it propagates. The experimental work of L. G. Smith on Mach reflection is applied to show that these converging waves are stable for the shock range ($M\leq 2.4$) covered by his experiments. Smith's work and the theoretical work of Lighthill indicate that the stability decreases greatly at high Mach numbers.

The simplest experimental method of achieving a cylindrical converging shock is by the use of a shock tube with a converging channel. This, however, results in the hottest region of the gas being in close thermal contact with the cold walls. An axially symmetric shock tube has been designed and constructed which produces a complete converging cylindrical shock rather than just a sector and in which the region of convergence is comparatively well isolated thermally from the walls. It has been found possible to converge a moderate strength shock wave ($M=1.7$) sufficiently to produce considerable luminosity at the center of convergence. Schlieren photographs are presented showing various phases of the formation and stability of these converging waves.

INTRODUCTION

We were first attracted to the study of converging shock waves by the extremely high pressures and temperatures, which apparently could be achieved by their use. The problem of a converging strong cylindrical or spherical shock wave has been analyzed by Guderley.[1] He assumes that a converging shock will approach the center according to some power law, so that if r is the shock radius t seconds before the shock reaches the center, then he assumes $r=at^n$. He then reduces the equations of motion to first order, utilizing the symmetry properties of the problem. The first-order equation is integrated numerically. From the numerical integration it is found that the external boundary conditions are satisfied only for spherical shocks if $n=0.717$ and for cylindrical shocks if $n=0.834$. Knowing this value of n, the pressure and temperature behind the shock are readily computed.

An alternative procedure is to solve the problem by the method of characteristics. A solution by the method of characteristics, the solution of Guderley, and a solution according to acoustic theory, all for the case of spherical shock waves which start from $r=1$ with Mach number $M=1.1$ (ratio of shock velocity to velocity of sound in the undisturbed fluid ahead of the shock), are presented in Fig. 1. It will be noted, as might be

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[1] G. Guderley, Luftfahrt-forsch. 19, Nos. 9, 302 (1942).

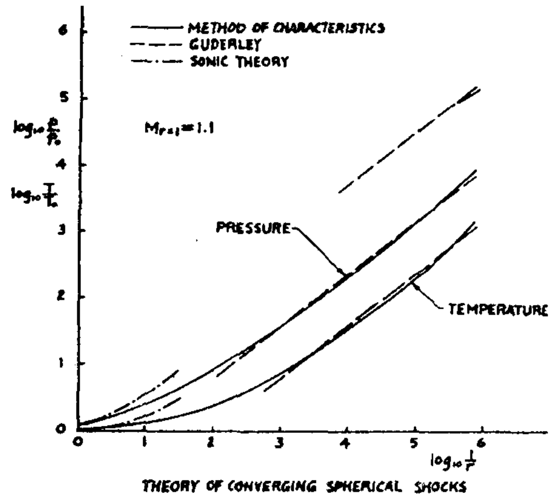

FIG. 1. Comparison between sonic theory, Guderley's solution, and the method of characteristics for spherical converging shock waves ($\gamma = 1.40$). p/p_0 and T/T_0 are the ratios of pressure and temperature immediately behind the shock to the initial values. The short upper curves correspond to the pressures after reflection from the center.

expected, that as the shock approaches the center, infinite values of pressure and temperature are obtained. It should be pointed out that both of these analyses are made for a perfect gas with γ (ratio of specific heats) $= 1.4$; and, of course, for all real gases at the extremely high pressures and temperatures which will be obtained in converging waves, large departures from constant heat capacities and increases in the number of particles due to dissociation and ionization would be expected. A limitation on the temperatures and pressures, which would be obtained even in the absence of these effects, is provided when the converging shock wave reaches radii of the order of several mean free paths.

That shock waves which converge toward a center might be experimentally obtainable was suggested first by the high "stability" of plane shock waves. Thus, it has frequently been noticed that shock waves propagating in a straight channel containing air at rest

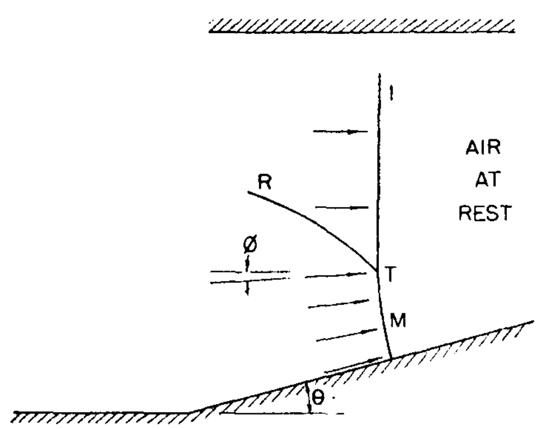

FIG. 2. Sketch of the Mach reflection configuration. *I*—incident shock. *R*—reflected shock. *M*—Mach shock. *T*—triple point.

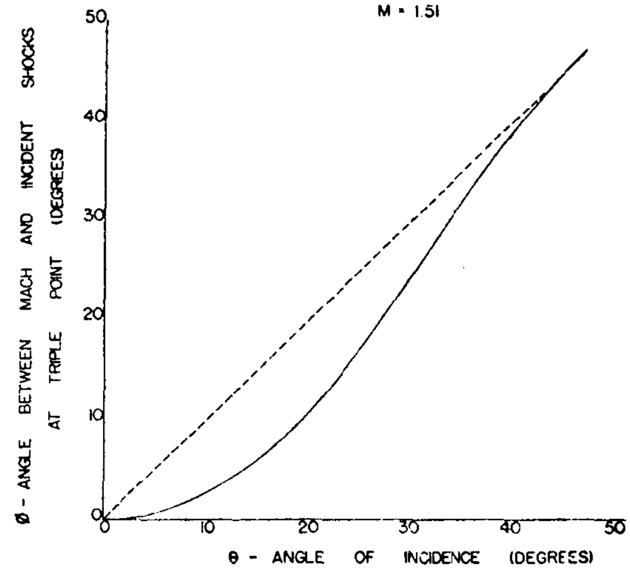

FIG. 3. Angle between the Mach and incident shocks at the triple point *vs* the angle of incidence (from Smith's data). *M*—Mach number of the incident shock.

(a shock tube) tend to become perpendicular to the axis of the channel and very flat. When a shock propagates in any channel, at the intersection with the channel walls the shock must, of course, be normal to the walls. Hence, equal amounts of positive and negative curvature must be distributed along the shock in a parallel-walled channel. For the shock (in spite of disturbances) to become flat, as observed, there must exist some "smoothing" mechanism which redistributes the curvature uniformly.

An analogous phenomenon has been reported in a converging rectangular channel produced by bending one of the walls inward. It has been shown[2] that the shock tends to approach a cylindrical shape with its

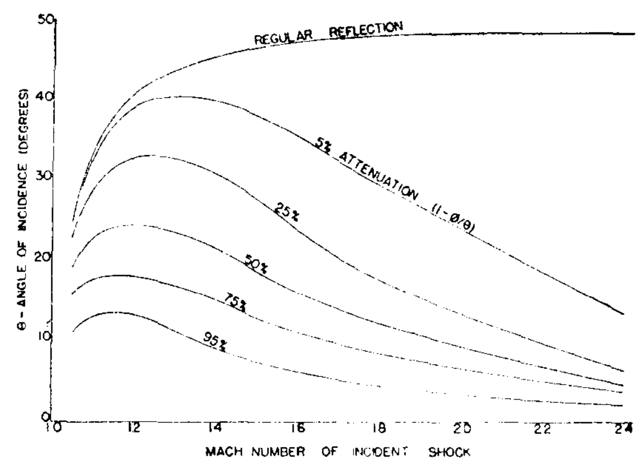

FIG. 4. Attenuation as a function of angle of incidence and Mach number of the incident shock (from Smith's observations—which were very thinly spaced for the higher Mach numbers of the range shown).

[2] A. Hertzberg and A. Kantrowitz, J. Appl. Phys. 21, 874 (1950), Fig. 7c.

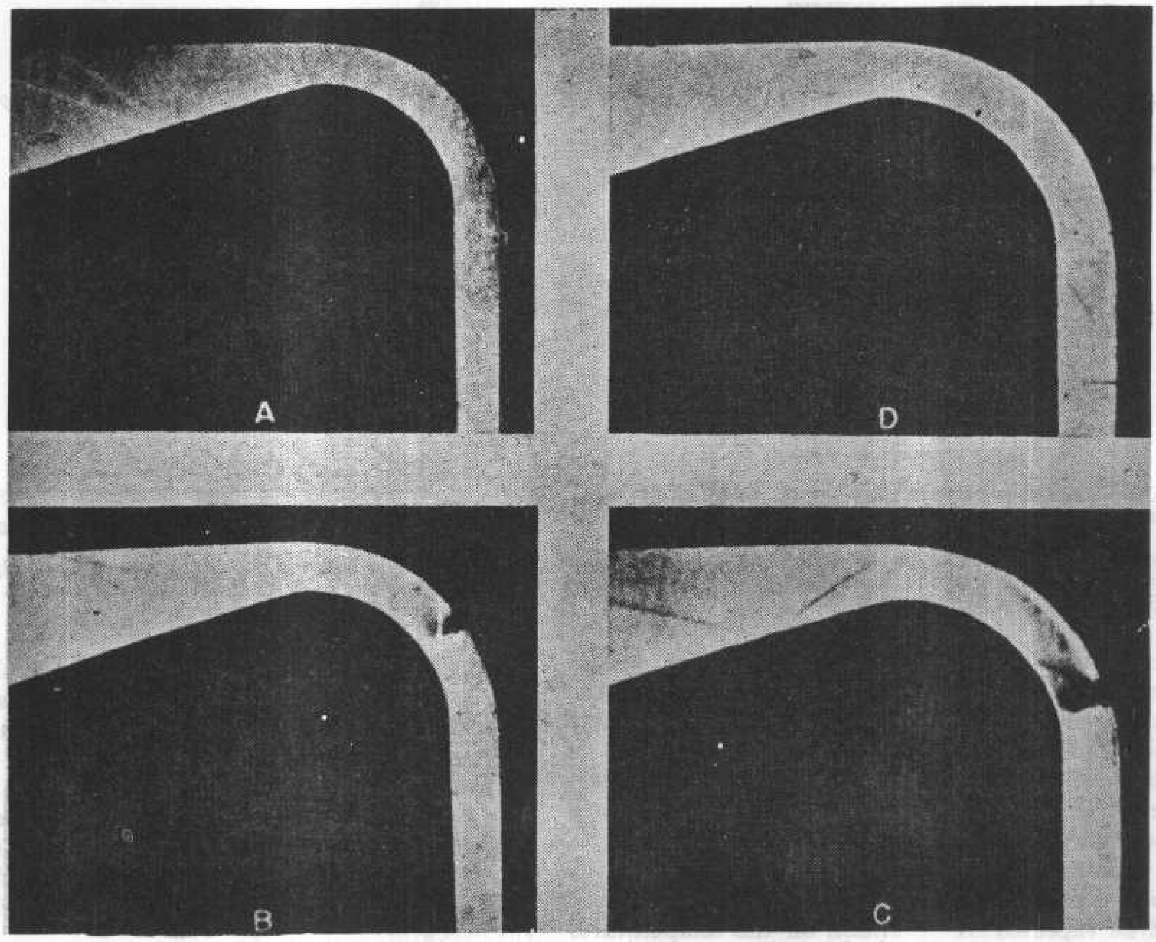

Fig. 5. Schlieren photographs of shock waves turning a corner (obtained from plane shocks with $M=1.1$). *A, B, C, D* indicate the time sequence. Each photograph is of a different shock wave.

ends normal to the channel walls. Here again, the curvature which was initially concentrated near the wall corner has become uniformly distributed along the shock front.

It will be instructive to examine the mechanism of the distribution of curvature found in these experiments. The wall angle used in the experiments of reference 2 was small enough so that Mach reflection was produced. In Mach reflection (see Fig. 2) a portion of the curvature originally concentrated at the corner is spread along the Mach shock, while the remainder appears at the triple point.

Since the local air velocity discontinuity across a shock is normal to the shock, the curved Mach shock leaves the air moving in a direction intermediate to the direction of the walls in Fig. 2. (We reserve for later consideration cases where the Mach shock may have an

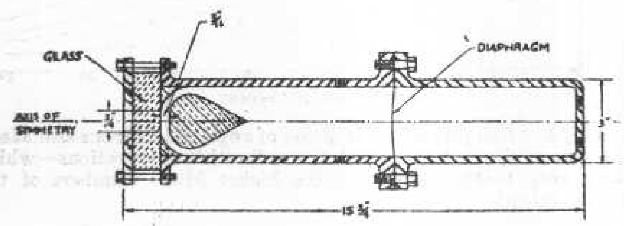

Fig. 6. Sketch of cylindrical shock tube.

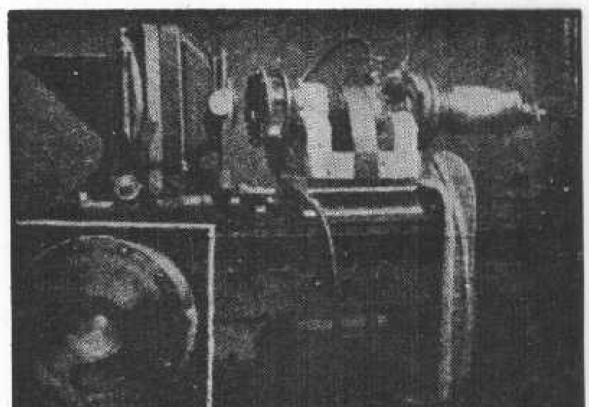

Fig. 7. Photograph of original cylindrical shock tube.

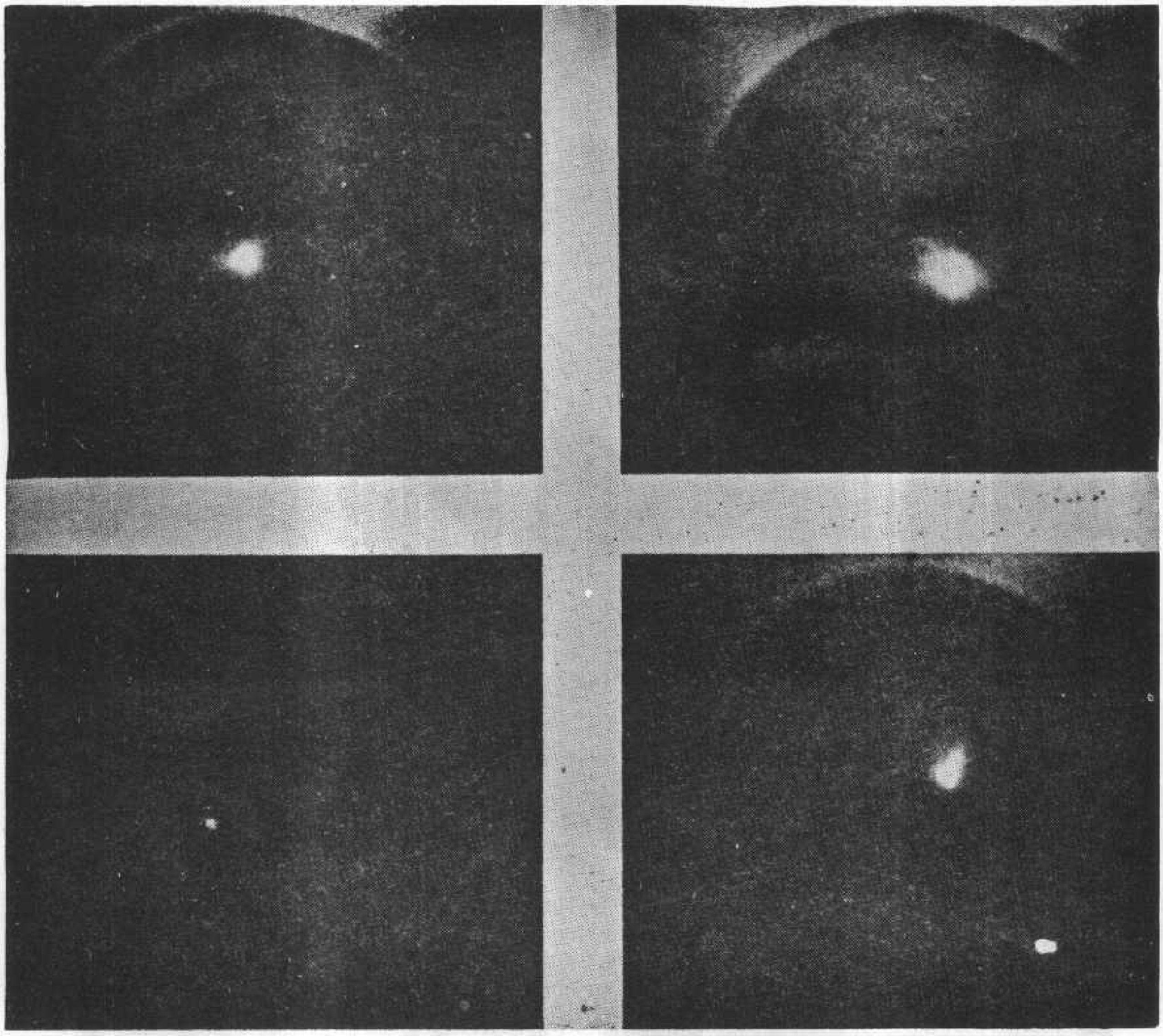

FIG. 8. Luminosity of converging cylindrical shock waves in argon (obtained from plane shocks with $M=1.8$). The glass window, rendered visible by double-exposure, was actually $\frac{3}{4}$ inch in diameter.

inflection point.) For example, from Smith's results[3] it would be expected that for the case where an incident shock with Mach number $M=1.5$ meets a 15° corner the flow immediately behind the triple intersection is moving at only 5° to the original flow. Thus, the reflection produced when the Mach shock reaches the opposite wall is weaker than the original reflection. By repeated reflection the waves following the initial shock become progressively weaker and the original shock approaches a cylindrical form.

Since all the elementary (one-dimensional) shock forms, cylindrical and spherical as well as plane, are characterized by such a uniformity of curvature, Mach reflection can give to each of these simple shapes a

[3] Lincoln G. Smith, "Photographic investigation of the reflection of plane shocks in air," NDRC No. A-350, OSRD No. 6271.

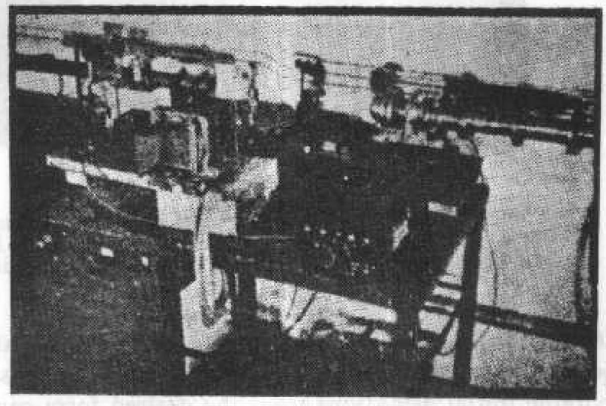

FIG. 9. Equipment for schlieren investigation of converging cylindrical shock wave.

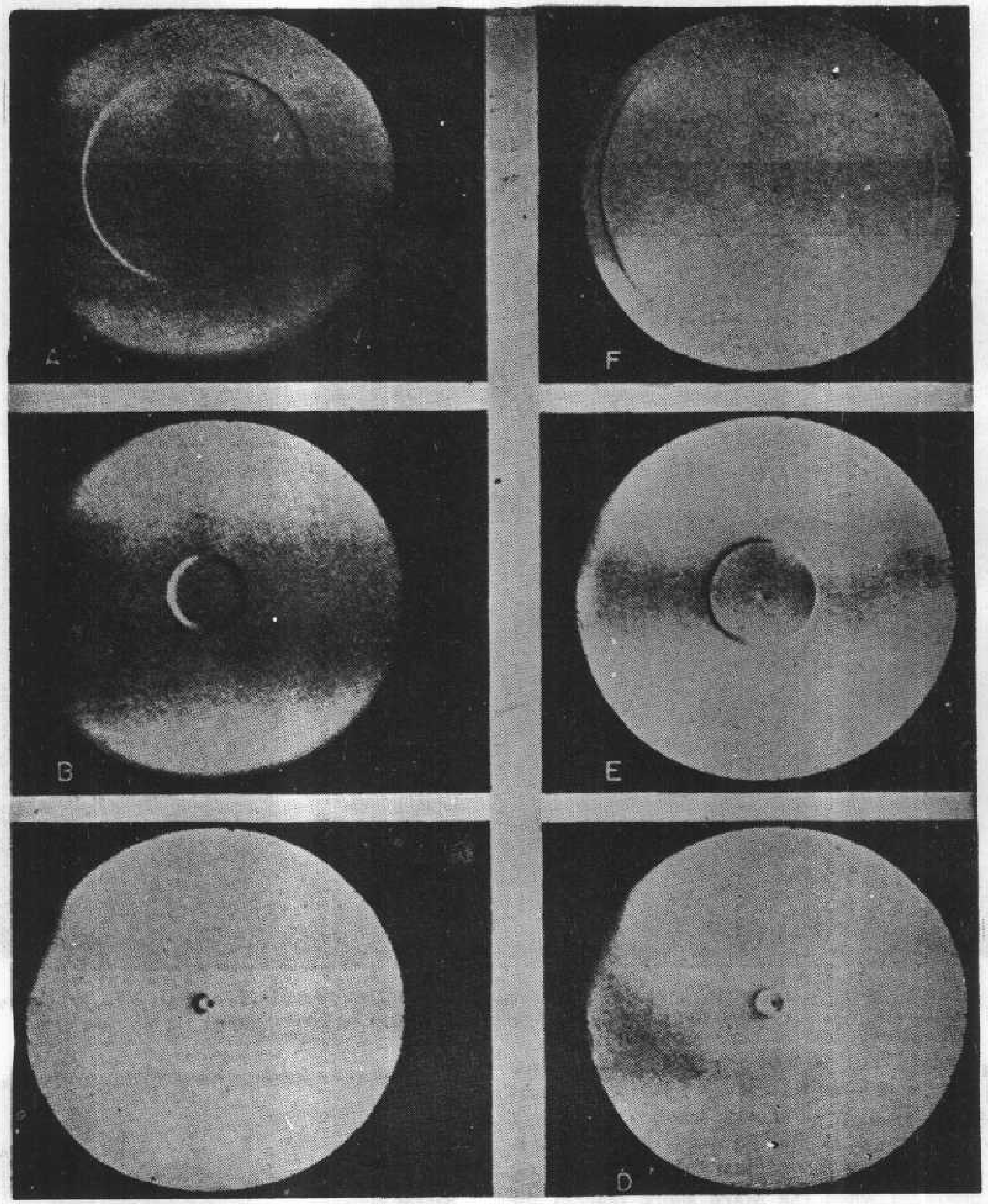

FIG. 10. Schlieren photographs of converging cylindrical shock waves in air (obtained from plane shocks with $M=1.1$). A, B, C indicate the incident waves and D, E, F indicate the reflected waves. Each photograph is of a different shock wave. The glass window was $1\frac{1}{2}$ inch in diameter.

"stability of form." Thus, in experimental attempts at production of converging cylindrical or spherical shocks, inevitable initial deviations from the desired form can be smoothed and random disturbances encountered in the course of propagation can be damped out, if this tendency towards uniform distribution of curvature (stability) exists.

Some quantitative information on the rate of attenuation of corners can be obtained from analysis of Smith's results. In Fig. 3 we show a plot of the angle

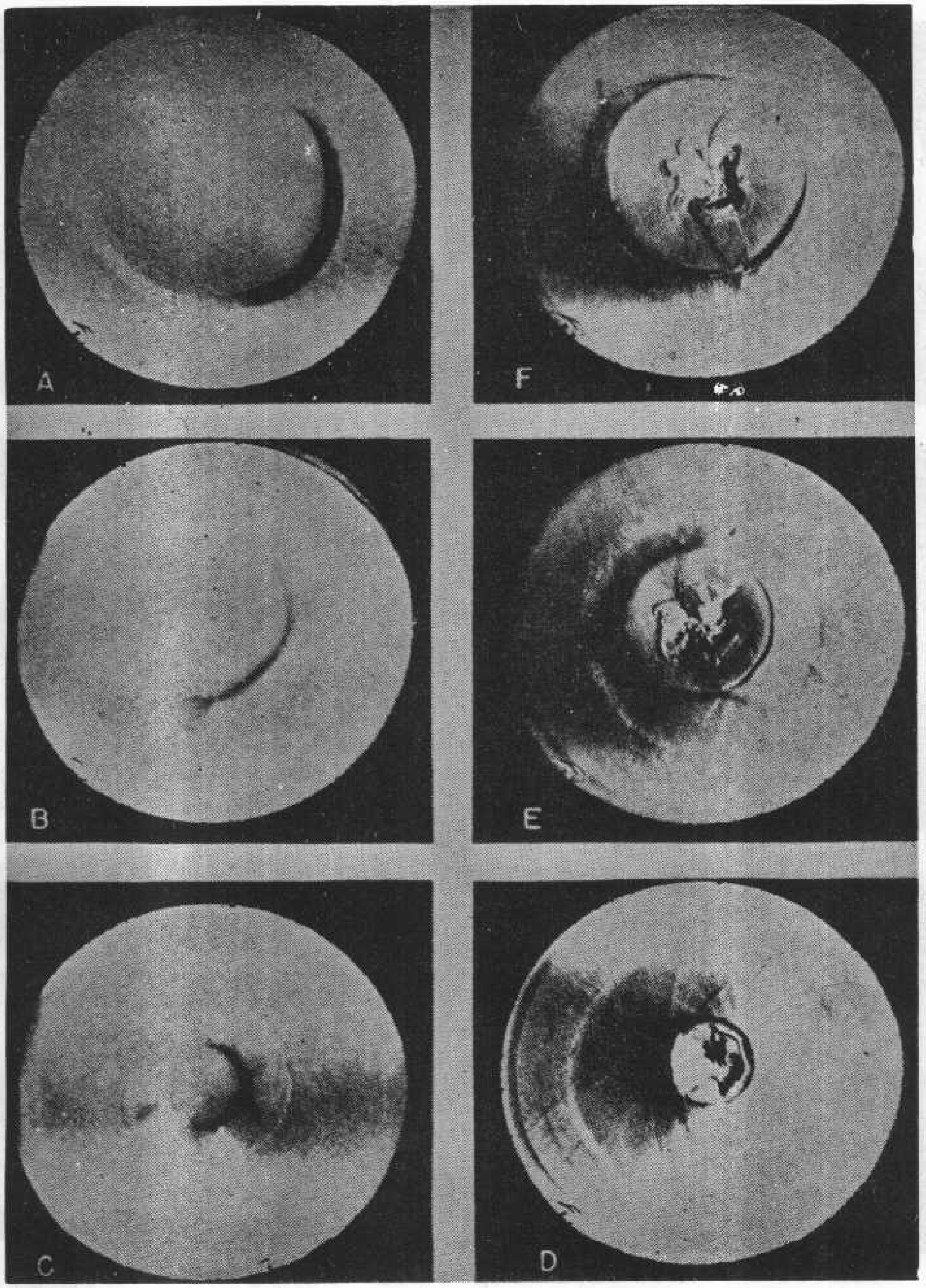

FIG. 11. Schlieren photographs of converging cylindrical shock waves in air (obtained from plane shocks with $M=1.8$). A, B, C indicate the incident waves and D, E, F indicate the reflected waves. Each photograph is of a different shock wave. The glass window was 1¼ inch in diameter.

between the incident and Mach shocks, ϕ, *vs* the incident angle, θ.‡ When the triple point reaches the opposite wall the reflected Mach reflection will be weaker if $\phi<\theta$. It will be seen from Fig. 3 that for shocks of the strength used in that plot considerable attenuation is obtained for small incident angles. Contours of constant attenuation, $1-\phi/\theta$, for the shock

‡ In Smith's notation, using α and μ, we find

$$\theta=\tfrac{1}{2}\pi-\alpha,$$
$$\phi=\theta-\mu.$$

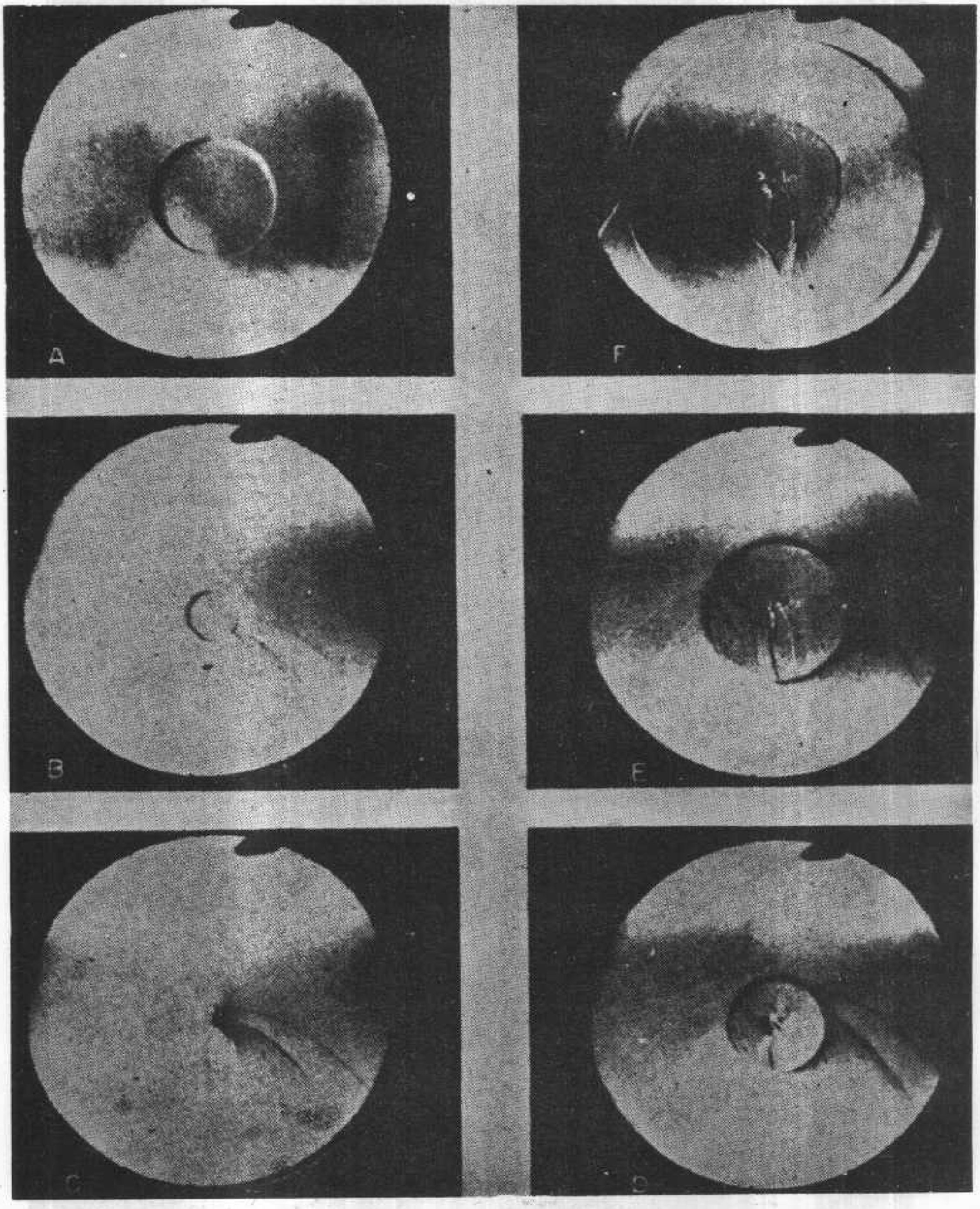

Fig. 12. Schlieren photographs of deliberately disturbed converging cylindrical shock waves in air (obtained from plane shocks with $M=1.4$). *A*, *B*, *C* indicate the incident waves and *D*, *E*, *F* indicate the reflected waves. Each photograph is of a different shock wave. The disturbance was a rod $\frac{1}{8}''$ in diameter placed in path of shock about $\frac{1}{2}$ inch before shock appears at bottom of glass window ($1\frac{1}{2}$ inch in diameter).

strengths and incident angles used in Smith's experiments are plotted in Fig. 4. It is clear from Fig. 4 that the effects of corners will be rapidly smoothed out for low Mach number shocks with low incident angles, such as the case of reference 2, Fig. 7c. On the other hand, it is also clear from Fig. 4 that the effects of corners in strong shocks will be attenuated much more slowly, if at all.

In view of the lack of experimental Mach reflection data for very strong shocks, an attempt was made to extract some stability information from Lighthill's linearized analysis[4] of Mach reflection at small incident angles. Lighthill's solution shows the Mach shock always tangent to the incident shock at the triple point (i.e., $\phi = 0$ or 100 percent attenuation—compare Fig. 4). However, he shows that for $M > 2.531$ the Mach shock will have an inflection point and a region near the wall where the curvature is opposite to that shown in Fig. 2.

Thus, for $M > 2.531$, in the region between the triple point and the inflection point, the Mach shock bends through an angle greater than the incident angle. The maximum deflection for $M = \infty$ is 1.35 times the incident angle. Consider now the secondary reflected compression waves, which will come from an upper wall parallel to the flow in the region IR. The total deflection in all these waves is greater than the deflection at the original corner. If all these waves coalesced into a single shock (without any weakening by expansion waves which are present in adjacent regions), its strength would therefore be greater (by a factor of 1.35 for $M = \infty$) than the strength of the original reflection. This indicates the possibility of the occurrence of a slowly divergent series of Mach reflections, i.e., instability. Whether or not this instability really exists cannot be deduced from Lighthill's solution. A solution for the large disturbance problem or experimental studies of the Mach reflection of strong shocks will be necessary to decide this question.

EXPERIMENTAL PRODUCTION OF CONVERGING SHOCK WAVES

If we consider now experimental methods of producing these converging shocks, it is clear that a conical converging passage may produce a portion of a spherical wave and a plane converging passage could produce a sector of a cylindrical wave. However, it seems likely that wall cooling would prevent the attainment of very high temperatures in this way, for in this case relatively large areas of cold wall would be immediately adjacent to the highest temperature region.

To overcome this objection, we search for methods of producing the complete wave rather than just a sector of it. First, we might consider an explosion or detonation near the center of a spherical or cylindrical cavity and observe the converging shock which is reflected from the walls of the cavity. This method has the disadvantage that the center of convergence is in a fluid of variable and unknown characteristics; and also, because of the decrease of stability of the shock with increasing strength, it seems desirable to avoid disturbances in the vicinity of the center. Alternatively, we could attempt to initiate an inward-traveling shock at a cylindrical or spherical surface by bursting a diaphragm or some explosive process, but the experimental difficulty of producing either pure cylindrical or spherical convergence in this manner seems very great.

[4] M. J. Lighthill, Proc. Roy. Soc. (London) A198, 454 (1949).

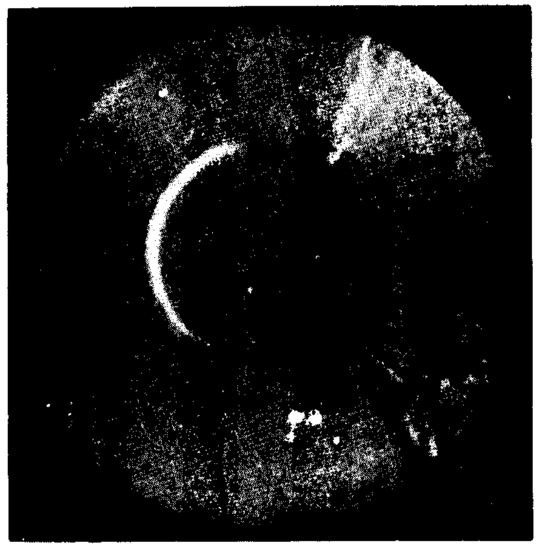

FIG. 13. Example of the onset of regular reflection caused by an excessively large random disturbance.

We have already mentioned how a plane shock can be distorted into a sector of a spherica' or cylindrical shock by bending of the channel walls at a sharp corner. Approximately a quadrant of a cylinder or an octant of a sphere could be obtained in this way before the onset of regular reflection would be encountered (see Fig. 4). However, by decreasing the curvature of the corner, we can delay the advent of regular reflection. Thus, if the walls turn inward slowly and smoothly enough, a plane shock may be converted to a more nearly complete cylindrical or spherical shock.

To create a cylindrical converging shock there remains still another alternative, for in the above method we have warped a plane surface into a cylindrical surface with its axis parallel to the plane, and we must also examine the case of a plane converted into a cylinder with its axis perpendicular to the plane. If this latter geometrical distortion could be simply achieved experimentally, it offered the possibility of producing a complete converging cylindrical shock (rather than just a sector).

Indeed, we did find this possible, so our further studies of converging shocks have been restricted to the cylindrical form because of the simplicity of its production under controllable conditions and the ease of observation of the results—though the spherical form could yield higher temperatures.

Essential to our design was the assumption that a shock may be guided nearly at will by a relatively narrow passage. This was tested in a conventional shock tube and the resulting schlieren pictures are shown in Fig. 5. The shock tube, schlieren system, and spark source used for this predesign experiment are those described in reference 4. Thus, we have shown that the shock negotiates the corner successfully, ending up flat and normal to the walls directing it. If such a cross section is rotated about an axis, we obtain something similar to a child's toy top mounted within a capped pipe, as indicated in the sketch of Fig. 6. It is clear that such a cylindrical shock tube must generate something approaching a cylindrical converging shock. The apparatus shown in Fig. 7 was therefore constructed. The high pressure chamber was filled with helium and the low pressure chamber with air. The helium generated a shock of Mach number roughly 1.8. If perfect cylindrical convergence were obtained to a shock diameter of say five mean free paths (i.e., mean free paths in the undisturbed air), we would expect a temperature sufficient to dissociate and possibly even to ionize at least some of the components of the air. We looked through the glass window in a darkened room, expecting to see light given out by the converging shock wave as it approached the center. The expected light appeared. It proved rather difficult to photograph, however, probably because most of the light given off by air when highly compressed is not in a spectral region easily transmitted by glass. On the other hand, shock waves of the same strength in argon produced easily photographable luminosity. In Fig. 8 are presented several such photographs. Also the conductivity of the argon in the region of convergence was roughly checked and found to indicate the occurrence of ionization.

To study the stability of the converging shocks we undertook a schlieren investigation with the equipment shown in Fig. 9. Because a spark source could not be mounted internally owing to the construction of the cylindrical shock tube and its rather small size, it seemed necessary to use a folded optical path with the light reflected by the first surface mirror on the end of the teardrop and twice traversing the region to be examined. The knife-edge is placed as close as feasible alongside the virtual line source at the focus of the schlieren lens to insure that the incident and reflected beams of parallel light traverse nearly the same path through the gas.

The spark source used was similar to one previously described,[5] being merely the discharge through a suitable gap of three Glass mike condensers each rated at 0.05 μf for 7500 v and each placed at the corner of a triangle enclosing the spark gap. The main spark was triggered at the desired time by a teaser electrode between the main electrodes.

Actuating a solenoid concealed within the "teardrop" caused a needle to pierce the cellulose acetate diaphragm separating the two chambers of the shock tube, releasing the pressurized gas and initiating a plane shock wave. As this shock proceeded along the tube, it broke an electrical contact mounted above a tiny orifice in the side of the tube. Breaking the contact activated an RC delay circuit, causing a thyratron (after the pre-set delay) to fire through the primary of a transformer, the secondary of which was connected in the teaser electrode.

Schlieren photographs of the converging cylindrical shock were obtained for two different pressure ratios across the diaphragm and a few of the results are displayed in Figs. 10 and 11. In these photographs it is also seen that, as our previous stability considerations would lead us to expect, the approach to cylindrical form even in the visible region becomes poorer as initial shock strength increases. This is due partly to the slower dying away of the effect of the corner and partly to slower attenuation of disturbances resulting from bursting of the diaphragm (though the magnitude and irregularity of these diaphragm bursting disturbances did also increase for the stronger shocks due to poorer diaphragm material).

If perfect cylindrical convergence were to continue even through the microscopic region, then irregular vortices, such as are to be noted in both series of photographs, would not occur. We take the intensity of these irregularities as a measure of the departure from perfect convergence. Previously, on the basis of Fig. 4 we concluded that the cylindrical shock should be completely formed and the large disturbances necessary to produce and shape it should be nearly fully attenuated while the shock is still weak, before any appreciable convergence occurs. The experimental results seem to support this viewpoint.

To verify visually our ideas of Mach reflection and shock stability which have already been outlined, we placed a small obstacle in the path of the shock just before it reached the region of observation. As nearly as possible we duplicated all other conditions under which the pictures of Figs. 10 and 11 were obtained and, for an intermediate pressure ratio across the diaphragm, secured the results presented in Fig. 12 The net result of the artificial disturbance is seen to be merely a displacement of the center of convergence towards the disturbed side.

In Fig. 13 we present a case in which the disturbance produced by incomplete bursting of the diaphragm was so great that regular reflection occurred, destroying the cylindrical convergence.

[5] L. S. G. Kovásznay, Rev. Sci. Instr. **20**, 696 (1949).

A theory of the stability of plane shock waves

By N. C. Freeman

(*Communicated by M. J. Lighthill, F.R.S.—Received* 1 *September* 1954)

The stability of form of a plane shock, obtained when a 'corrugated' piston is moved impulsively from rest with constant velocity, is investigated mathematically. Linearization of the problem is accomplished by assuming the corrugations to be small. The solution is built up by methods of Fourier analysis from 'conefield' solutions of the analogous 'wedge'-shaped piston problem, solved by methods due to Lighthill. The plane shock is shown to be stable, perturbations from plane decaying with time in an oscillatory manner like $t^{-\frac{3}{2}}$ for large ta_1/λ (where a_1 is the velocity of sound behind the shock and λ the wave-length of the corrugations). The stability, measured by the amplitude of this oscillation after the shock has traversed a given distance, decreases both as the shock Mach number increases above and decreases below the value 1·14. Shocks of this strength exhibit strongest stability.

Asymptotic forms for large time are given for both the shock shape and pressure distribution for shocks of moderate strength in §4. A more complicated asymptotic form for the shock shape holds at large Mach numbers (§5) which in the limiting case of infinite Mach number gives the result that the perturbations of shape decay like $t^{-\frac{1}{2}}$ only. Complete solutions are obtained for weak shocks in terms of Bessel functions (§6).

1. Introduction and discussion of results

This paper is concerned with the attenuation of small perturbations in shape of plane shock waves during propagation into a stationary fluid. This phenomena has been studied experimentally by Professor A. R. Kantrowitz, together with the analogous problem for a cylindrically converging shock. The rapid attenuations of perturbations for the plane case has led Kantrowitz & Perry (1951) to call the phenomena the 'stability' of shock waves. Quite generally, therefore, we may, following Kantrowitz, define the *stability* of form of a plane, cylindrical or spherical shock wave as the ability to approach perfect plane, cylindrical or spherical shape as it propagates, owing to a gradual equalization of curvature between neighbouring portions.

It will be realized that stability will play an important part in the production of shocks in a shock tube. This method is recognized as a very efficient way of producing plane shocks of any given strength in spite of the fact that the shock, initially produced, is far from being uniformly plane. The rupture of the diaphragm in the tube causes the shock to originate as a spherical perturbed wave becoming plane only after it has travelled several shock tube diameters down the tube. Nevertheless, the distance travelled before the shock is completely plane is comparatively small.

For a cylindrical converging shock the problem becomes much more complicated. Kantrowitz has shown that this stabilizing effect is not nearly so marked, the stability apparently decreasing rapidly with increase of Mach number. This problem will not, however, be considered in this paper. It is hoped that the study of a fundamentally simpler problem, that of a plane perturbed shock, may prove useful in understanding the essentially more difficult mathematical problem of the cylindrical shock.

Physically, the problem is essentially a three-dimensional one, but it will be shown to be sufficient to solve, mathematically, the problem in two dimensions. The three-dimensional solution can then be deduced from it. A mathematical approach to this problem necessitates solving the equations of inviscid fluid theory in two-dimensional unsteady flow. It will, however, be sufficient to neglect squares of perturbations to the steady flow behind a uniform shock. This type of problem has been solved by Lighthill in his papers 'Diffraction of blast, I and II' (1949, 1950). The only assumptions in these solutions are that the inviscid fluid equations are valid behind the shock and that the perturbations of the flow field are small. The first paper is concerned with a shock moving along a plane wall, which suddenly changes in direction by a small amount δ. The perturbations on the original plane shock, after it has passed the corner, are then of order δ. The results show the well-known Mach-type reflexion. Blackburn (1953) has extended this work to the case of a shock moving along a 'wavy' wall, having a sinusoidal profile. The method, in effect, necessitates obtaining the Fourier transforms of the plane-wall solution. Blackburn actually obtains the solution by solving the wave equation satisfied by the perturbation pressure under boundary conditions given on the shock and the wall. The results can be expressed as asymptotic expansions valid for large distances from the wall. It is found that for finite Mach number the perturbation ξ, defined as the distance the shock is displaced from plane, decays like $Y^{-\frac{3}{2}}$ when Y is the distance from the wall. As the Mach number becomes very large the decay becomes more like $Y^{-\frac{1}{2}}$. The physical significance of this result will become evident later.

In Lighthill's second paper (1950), the problem of a shock undergoing normal reflexion from a similar slightly wedge-shaped wall is considered using the above method on the perturbation of the solution of shock reflexion from a plane wall. The shock shape after reflexion and also the wall pressure distribution are found. This work has again been extended by Blackburn to the case of shock reflexion from a 'wavy' wall, having a profile $\epsilon\, e^{i\omega Y}$, by considering the Fourier transform of the Lighthill reflexion problem. In this work, Blackburn shows that the perturbation of the reflected shock oscillates with time, for large time, these oscillations decaying like $t^{-\frac{3}{2}}$. This, in itself, is a valuable contribution to shock stability theory; but there are two objections to this approach, if it were intended as a complete solution of the problem. First, the initial shape of the reflected shock is not known, and secondly, the reflected shock has a maximum strength of $2\gamma/(\gamma-1)$ ($=7$ for $\gamma=\frac{7}{5}$), restricting the theory to perturbed shocks of Mach number between 1 and 2·646.

In an attempt to find a more satisfactory model for a discussion of stability, the author decided to investigate the motion of a corrugated piston into a stationary inviscid fluid, this being the most realistic model consistent with mathematical simplicity. This may be considered mathematically as a perturbation of the solution for a plane piston moving into a stationary inviscid fluid. If the plane piston is considered to start impulsively from rest and subsequently maintain a constant velocity V, a shock will be produced on the piston and will propagate forward at a velocity U, greater than V. The solution of this problem is well known —the pressure, velocity and density of the fluid being constant behind the shock, which moves into the stationary fluid with constant velocity. The magnitudes of the constants are obtained from the following formulae expressed in terms of the piston speed; U, p, ρ are the velocity, pressure and density, respectively, of the gas:

$$M = \frac{U}{a_0} = \frac{3}{5}\frac{V}{a_0}\left[1+\sqrt{\left\{1+\left(\frac{3}{5}\frac{V}{a_0}\right)^{-2}\right\}}\right],$$

$$\frac{p}{p_0} = \frac{7M^2-1}{6}, \quad \frac{\rho}{\rho_0} = \frac{6M^2}{M^2+5}.$$

Here, the suffix zero refers to conditions in front of the shock and $a_0 = \sqrt{(7p_0/5\rho_0)}$ is the sound speed; the ratio of specific heats is taken to be 7/5. Hence M, the Mach number of the shock, varies over the whole range from one to infinity.

The equations of motion for unsteady two-dimensional inviscid flow, satisfied behind the shock, are expressible as linear equations in the perturbation pressure, velocities and density. Elimination of all variables except the pressure yields the wave equation. Physically, this equation asserts that all pressure variations behind the shock front propagate with the local mean speed of sound. In addition, they satisfy boundary conditions at the shock wave and at the piston.

It is found convenient to attack this problem indirectly. Following Blackburn, we build up the solution of this problem as a combination of conical-field solutions of the wave equation. These can be obtained by conformal mapping techniques as in Lighthill's part II. Each satisfies a boundary condition at the surface of a wedge-shaped piston, and they can be combined to produce a piston of corrugated shape by expressing the slope of the piston surface as a combination of small discontinuities a small distance apart.

A piston of wedge shape is considered first, having a single discrete change in slope of 2δ at the origin and symmetrical about the X-axis, extending to infinity in the positive and negative Y directions. As the piston extends to infinity, there is no fundamental length in this problem. The equations can therefore be transformed into equations in the two new variables obtained by dividing the space variables by time. The perturbation pressure, as before, satisfies a simple equation in terms of these variables; in words, it is a 'conical field' solution of the wave equation. Boundary conditions are satisfied at the shock wave and at the piston, which occupy fixed positions in terms of these new variables, since the mean speed of each is constant. The region perturbed by the discontinuity of piston slope at the origin will expand with the sound speed behind the shock. Outside this region the shock will move away from the piston exactly as in the plane case. The perturbed region is therefore bounded by the shock, the piston and a steady flow region. Mathematically, it is necessary to solve a second-order linear differential equation with two independent variables with given boundary conditions. In a closely similar case, the solution has already been obtained by Lighthill in the form of derivatives of pressure over the perturbed region. The solution, which is easily adapted to the present problem, is complicated to deal with in this form, the shock shape having to be deduced by numerical methods; but returning to our original problem of the corrugated piston, it is possible to obtain a complete solution in terms of these results. If the piston shape is given by $X = \epsilon\, e^{i\omega Y}$, where Y is measured in the plane of the piston and X perpendicular to it, then the change in slope between points $Y = \eta$ and $Y = \eta + d\eta$ is $-\epsilon\omega^2 e^{i\omega\eta}\, d\eta$. Thus, replacing 2δ in the 'wedge' solution by $-\epsilon\omega^2 e^{i\omega\eta}\, d\eta$, Y by $Y-\eta$ and integrating all along the wall with respect to η, the complete solutions are obtained. An explicit relation is therefore obtained for the shock shape as the Fourier transform of the 'wedge' solution.

The behaviour of the Fourier transform of a function for large argument is determined by the singularities of the function. Hence, asymptotic expressions for large time (compared with the time taken for a sound wave to traverse one wave-length of the wall) can be obtained for the 'corrugated' solution from consideration of the singularities of the 'wedge' case. Discussion of the singularities will be a profitable way of studying the stability of the perturbed shock. The perturbed region is bounded by the shock, the piston and the wave front (on which $r = R/a_1 t$ takes the value unity, where R is the distance from origin and a_1 the speed of sound behind the shock). Now, the behaviour of the pressure on the wave front, since it is that of a cylindrical wave, must be like $(1-r)^{\frac{1}{2}}$. This corresponds to a decay with time like $t^{-\frac{3}{2}}$ for the Fourier transform. The pressure behind the shock decays like $t^{-\frac{3}{2}}$, therefore, for a shock produced by a 'corrugated' piston. However, the shock boundary condition in the 'wedge' case (see equation (2·9) below) is found to imply that the gradient of pressure in a particular direction is zero on the shock. This is compatible with the singularity at the wave front, which makes the gradient infinite for any direction except a tangential one, only if the particular direction is tangential to the wave front. This is the case only for infinite Mach number. For finite-shock Mach number, therefore, the strength of the cylindrical wave must vanish at the shock-wave front intersection, and this is found to be the case, the singularity on the