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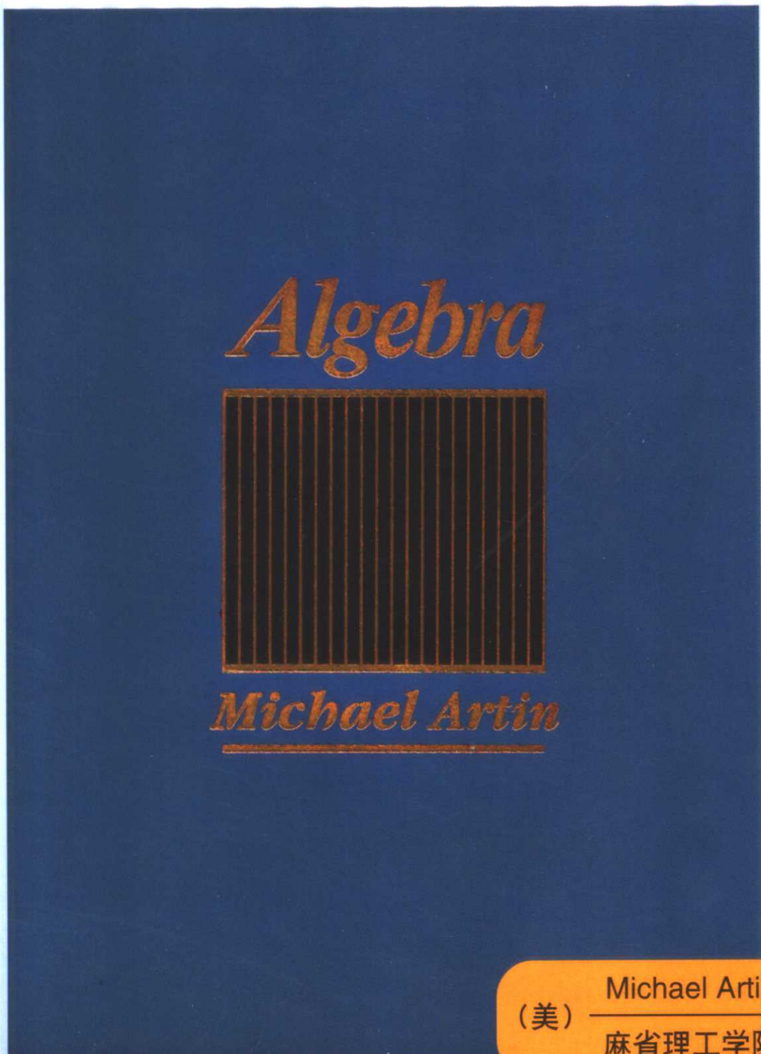

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麻省理工学院

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Algebra

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Preface

Important though the general concepts and propositions may be with which the modern and industrious passion for axiomatizing and generalizing has presented us, in algebra perhaps more than anywhere else, nevertheless I am convinced that the special problems in all their complexity constitute the stock and core of mathematics, and that to master their difficulties requires on the whole the harder labor.

Herman Weyl

This book began about 20 years ago in the form of supplementary notes for my algebra classes. I wanted to discuss some concrete topics such as symmetry, linear groups, and quadratic number fields in more detail than the text provided, and to shift the emphasis in group theory from permutation groups to matrix groups. Lattices, another recurring theme, appeared spontaneously. My hope was that the concrete material would interest the students and that it would make the abstractions more understandable, in short, that they could get farther by learning both at the same time. This worked pretty well. It took me quite a while to decide what I wanted to put in, but I gradually handed out more notes and eventually began teaching from them without another text. This method produced a book which is, I think, somewhat different from existing ones. However, the problems I encountered while fitting the parts together caused me many headaches, so I can't recommend starting this way.

The main novel feature of the book is its increased emphasis on special topics. They tended to expand each time the sections were rewritten, because I noticed over the years that, with concrete mathematics in contrast to abstract concepts, students often prefer more to less. As a result, the ones mentioned above have become major parts of the book. There are also several unusual short subjects, such as the Todd–Coxeter algorithm and the simplicity of PSL_2.

In writing the book, I tried to follow these principles:

1. The main examples should precede the abstract definitions.
2. The book is not intended for a "service course," so technical points should be presented only if they are needed in the book.
3. All topics discussed should be important for the average mathematician.

Though these principles may sound like motherhood and the flag, I found it useful to have them enunciated, and to keep in mind that "Do it the way you were taught" isn't one of them. They are, of course, violated here and there.

The table of contents gives a good idea of the subject matter, except that a first glance may lead you to believe that the book contains all of the standard material in a beginning algebra course, and more. Looking more closely, you will find that things have been pared down here and there to make space for the special topics. I used the above principles as a guide. Thus having the main examples in hand before proceeding to the abstract material allowed some abstractions to be treated more concisely. I was also able to shorten a few discussions by deferring them until the students have already overcome their inherent conceptual difficulties. The discussion of Peano's axioms in Chapter 10, for example, has been cut to two pages. Though the treatment given there is very incomplete, my experience is that it suffices to give the students the flavor of the axiomatic development of integer arithmetic. A more extensive discussion would be required if it were placed earlier in the book, and the time required for this wouldn't be well spent. Sometimes the exercise of deferring material showed that it could be deferred forever—that it was not essential. This happened with dual spaces and multilinear algebra, for example, which wound up on the floor as a consequence of the second principle. With a few concepts, such as the minimal polynomial, I ended up believing that their main purpose in introductory algebra books has been to provide a convenient source of exercises.

The chapters are organized following the order in which I usually teach a course, with linear algebra, group theory, and geometry making up the first semester. Rings are first introduced in Chapter 10, though that chapter is logically independent of many earlier ones. I use this unusual arrangement because I want to emphasize the connections of algebra with geometry at the start, and because, overall, the material in the first chapters is the most important for people in other fields. The drawback is that arithmetic is given short shrift. This is made up for in the later chapters, which have a strong arithmetic slant. Geometry is brought back from time to time in these later chapters, in the guise of lattices, symmetry, and algebraic geometry.

Michael Artin
December 1990

A Note for the Teacher

There are few prerequisites for this book. Students should be familiar with calculus, the basic properties of the complex numbers, and mathematical induction. Some acquaintance with proofs is obviously useful, though less essential. The concepts from topology, which are used in Chapter 8, should not be regarded as prerequisites. An appendix is provided as a reference for some of these concepts; it is too brief to be suitable as a text.

Don't try to cover the book in a one-year course unless your students have already had a semester of algebra, linear algebra for instance, and are mathematically fairly mature. About a third of the material can be omitted without sacrificing much of the book's flavor, and more can be left out if necessary. The following sections, for example, would make a coherent course:

> Chapter 1, Chapter 2, Chapter 3: 1–4, Chapter 4, Chapter 5: 1–7,
> Chapter 6: 1,2, Chapter 7: 1–6, Chapter 8: 1–3,5, Chapter 10: 1–7,
> Chapter 11: 1–8, Chapter 12: 1–7, Chapter 13: 1–6.

This selection includes some of the interesting special topics: symmetry of plane figures, the geometry of SU_2, and the arithmetic of imaginary quadratic number fields. If you don't want to discuss such topics, then this is not the book for you.

It would be easy to spend an entire semester on the first four chapters, but this would defeat the purpose of the book. Since the real fun starts with Chapter 5, it is important to move along. If you plan to follow the chapters in order, try to get to that chapter as soon as is practicable, so that it can be done at a leisurely pace. It will help to keep attention focussed on the concrete examples. This is especially important in the beginning for the students who come to the course without a clear idea of what constitutes a proof.

Chapter 1, matrix operations, isn't as exciting as some of the later ones, so it should be covered fairly quickly. I begin with it because I want to emphasize the general linear group at the start, instead of following the more customary practice of basing examples on the symmetric group. The reason for this decision is Principle 3 of the preface: The general linear group is more important.

Here are some suggestions for Chapter 2:

1. Treat the abstract material with a light touch. You can have another go at it in Chapters 5 and 6.
2. For examples, concentrate on matrix groups. Mention permutation groups only in passing. Because of their inherent notational difficulties, examples from symmetry such as the dihedral groups are best deferred to Chapter 5.
3. Don't spend too much time on arithmetic. Its natural place in this book is Chapters 10 and 11.
4. Deemphasize the quotient group construction.

Quotient groups present a pedagogical problem. While their construction is conceptually difficult, the quotient is readily presented as the image of a homomorphism in most elementary examples, and so it does not require an abstract definition. Modular arithmetic is about the only convincing example for which this is not the case. And since the integers modulo n form a ring, modular arithmetic isn't the ideal motivating example for quotients of groups. The first serious use of quotient groups comes when generators and relations are discussed in Chapter 6, and I deferred the treatment of quotients to that point in early drafts of the book. But fearing the outrage of the algebra community I ended up moving it to Chapter 2. Anyhow, if you don't plan to discuss generators and relations for groups in your course, then you can defer an in-depth treatment of quotients to Chapter 10, ring theory, where they play a central role, and where modular arithmetic becomes a prime motivating example.

In Chapter 3, vector spaces, I've tried to set up the computations with bases in such a way that the students won't have trouble keeping the indices straight. I've probably failed, but since the notation is used throughout the book, it may be advisable to adopt it.

The applications of linear operators to rotations and linear differential equations in Chapter 4 should be discussed because they are used later on, but the temptation to give differential equations their due has to be resisted. This heresy will be forgiven because you are teaching an algebra course.

There is a gradual rise in the level of sophistication which is assumed of the reader throughout the first chapters, and a jump which I've been unable to eliminate occurs in Chapter 5. Had it not been for this jump, I would have moved symmetry closer to the beginning of the book. Keep in mind that symmetry is a difficult concept. It is easy to get carried away by the material and to leave the students behind.

Except for its first two sections, Chapter 6 contains optional material. The last section on the Todd–Coxeter algorithm isn't standard; it is included to justify the discussion of generators and relations, which is pretty useless without it.

There is nothing unusual in the chapter on bilinear forms, Chapter 7. I haven't overcome the main problem with this material, that there are too many variations on the same theme, but have tried to keep the discussion short by concentrating on the real and complex cases.

In the chapter on linear groups, Chapter 8, plan to spend time on the geometry of SU_2. My students complained every year about this chapter until I expanded the sections on SU_2, after which they began asking for supplementary reading, wanting to learn more. Many of our students are not familiar with the concepts from topology when they take the course, and so these concepts require a light touch. But I've found that the problems caused by the students' lack of familiarity can be managed. Indeed, this is a good place for them to get an idea of what a manifold is. Unfortunately, I don't know a really satisfactory reference for further reading.

Chapter 9 on group representations is optional. I resisted including this topic for a number of years, on the grounds that it is too hard. But students often request it, and I kept asking myself: If the chemists can teach it, why can't we? Eventually the internal logic of the book won out and group representations went in. As a dividend, hermitian forms got an application.

The unusual topic in Chapter 11 is the arithmetic of quadratic number fields. You may find the discussion too long for a general algebra course. With this possibility in mind, I've arranged the material so that the end of Section 8, ideal factorization, is a natural stopping point.

It seems to me that one should at least mention the most important examples of fields in a beginning algebra course, so I put a discussion of function fields into Chapter 13.

There is always the question of whether or not Galois theory should be presented in an undergraduate course. It doesn't have quite the universal applicability of most of the subjects in the book. But since Galois theory is a natural culmination of the discussion of symmetry, it belongs here as an optional topic. I usually spend at least some time on Chapter 14.

I considered grading the exercises for difficulty, but found that I couldn't do it consistently. So I've only gone so far as to mark some of the harder ones with an asterisk. I believe that there are enough challenging problems, but of course one always needs more of the interesting, easier ones.

Though I've taught algebra for many years, several aspects of this book are experimental, and I would be very grateful for critical comments and suggestions from the people who use it.

"One, two, three, five, four..."
"No Daddy, it's one, two, three, four, five."
"Well if I want to say one, two, three, five, four, why can't I?"
"That's not how it goes."

Acknowledgments

Mainly, I want to thank the students who have been in my classes over the years for making them so exciting. Many of you will recognize your own contributions, and I hope that you will forgive me for not naming you individually.

Several people have used my notes in classes and made valuable suggestions—Jay Goldman, Steve Kleiman, Richard Schafer, and Joe Silverman among them. Harold Stark helped me with the number theory, and Gil Strang with the linear algebra. Also, the following people read the manuscript and commented on it: Ellen Kirkman, Al Levine, Barbara Peskin, and John Tate. I want to thank Barbara Peskin especially for reading the whole thing twice during the final year.

The figures which needed mathematical precision were made on the computer by George Fann and Bill Schelter. I could not have done them by myself.

Many thanks also to Marge Zabierek, who retyped the manuscript annually for about eight years before it was put onto the computer where I could do the revisions myself, and to Mary Roybal for her careful and expert job of editing the manuscript.

I've not consulted other books very much while writing this one, but the classics by Birkhoff and MacLane and by van der Waerden from which I learned the subject influenced me a great deal, as did Herstein's book, which I used as a text for many years. I also found some good ideas for exercises in the books by Noble and by Paley and Weichsel.

Some quotations, often out of context, are scattered about the text. I learned the Leibnitz and Russell quotes which end Chapters 5 and 6 from V. I. Arnold, and the Weyl quote which begins Chapter 8 is from Morris Klein's book *Mathematical Thought from Ancient to Modern Times*.

Contents

Chapter 1

Matrix Operations

Erstlich wird alles dasjenige eine Größe genennt,
welches einer Vermehrung oder einer Verminderung fähig ist,
oder wozu sich noch etwas hinzusetzen oder davon wegnehmen läßt.

Leonhard Euler

Matrices play a central role in this book. They form an important part of the theory, and many concrete examples are based on them. Therefore it is essential to develop facility in matrix manipulation. Since matrices pervade much of mathematics, the techniques needed here are sure to be useful elsewhere.

The concepts which require practice to handle are *matrix multiplication* and *determinants*.

1. THE BASIC OPERATIONS

Let m, n be positive integers. An $m \times n$ matrix is a collection of mn numbers arranged in a rectangular array:

$$(1.1) \qquad m \text{ rows} \overbrace{\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}}^{n \text{ columns}}$$

For example, $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 5 \end{bmatrix}$ is a 2×3 matrix.

The numbers in a matrix are called the *matrix entries* and are denoted by a_{ij}, where i, j are indices (integers) with $1 \le i \le m$ and $1 \le j \le n$. The index i is called the *row index*, and j is the *column index*. So a_{ij} is the entry which appears in the ith row and jth column of the matrix:

$$\begin{array}{c} \qquad j \\ i\begin{bmatrix} \cdots & a_{ij} & \cdots\cdots \\ & \vdots & \\ & \vdots & \end{bmatrix} \end{array}$$

In the example above, $a_{11} = 2$, $a_{13} = 0$, and $a_{23} = 5$.

We usually introduce a symbol such as A to denote a matrix, or we may write it as (a_{ij}).

A $1 \times n$ matrix is called an n-dimensional *row vector*. We will drop the index i when $m = 1$ and write a row vector as

(1.2) $$A = [a_1 \cdots a_n], \quad \text{or as} \quad A = (a_1, \ldots, a_n).$$

The commas in this row vector are optional. Similarly, an $m \times 1$ matrix is an m-dimensional *column vector*:

(1.3) $$B = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}.$$

A 1×1 matrix $[a]$ contains a single number, and we do not distinguish such a matrix from its entry.

(1.4) *Addition* of matrices is vector addition:

$$(a_{ij}) + (b_{ij}) = (s_{ij}),$$

where $s_{ij} = a_{ij} + b_{ij}$ for all i, j. Thus

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 3 \\ 4 & -3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 3 \\ 5 & 0 & 6 \end{bmatrix}.$$

The sum of two matrices A, B is defined only when they are both of the same shape, that is, when they are $m \times n$ matrices with the same m and n.

(1.5) *Scalar multiplication* of a matrix by a number is defined as with vectors. The result of multiplying a number c and a matrix (a_{ij}) is another matrix:

$$c(a_{ij}) = (b_{ij}),$$

where $b_{ij} = ca_{ij}$ for all i, j. Thus

$$2\begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 4 & 6 \\ 4 & 2 \end{bmatrix}.$$

Numbers will also be referred to as *scalars*.