Undergraduate Textbooks in Physics

EDITED BY R. J. BLIN-STOYLE

VOLUME I

GEOMETRICAL OPTICS

OPTICAL INSTRUMENTATION

by W. T. WELFORD

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Optical Instrumentation

BY

W. T. WELFORD, Ph.D., D.Sc.,

Reader in Applied Optics, Imperial College of Science and Technology, University of London

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EDITOR'S PREFACE

The intention of this series of texts is to provide a uniform and comprehensive account of the basic principles of physics at the undergraduate level. Attention will be concentrated throughout on the discussion and elaboration of fundamental ideas rather than on superficial theoretical and experimental details so that a firm foundation is laid for subsequent specialised treatment. The books will be particularly useful for the degree courses in physics being planned at the new English universities.

They will, furthermore, serve as useful preliminary texts for conventional specialist degree courses in physics.

The volumes in preparation are:

Mechanics of Particles
Mechanics of Solids, Liquids and Gases
Heat and Thermodynamics
Geometrical Optics
Physical Optics
Electricity and Magnetism
Physics of Atoms and Nuclei
Mathematical Methods.

Among the authors are:

Dr. J. B. BIRKS (University of Manchester), Dr. R. CHISHOLM (University of Wales), Dr. R. MORRIS (University of Wales), Dr. E. A. MUSSETT (Northampton College of Technology, London), Dr. D. ROAF (University of Oxford), Professor K. SMITH (University of Sussex), Dr. W. T. WELFORD (Imperial College of Science and Technology, London), and Dr. J. B. WOODWARD (University of Birmingham).

R. J. BLIN-STOYLE

AUTHOR'S PREFACE

This book is intended as an undergraduate text on geometrical optics. The treatment is self-contained but not of the most elementary as I have supposed that all who use the book will have been taught some optics before. Very little of the mathematics which a physics student will normally have by the end of his first year is needed, namely, expansion in power series, three-dimensional coordinate geometry and a few simple concepts of differential geometry and vector algebra; a few mathematical proofs of higher standard have been put in appendices. Also, Chapter 6 and § 5 of Chapter 8 are more difficult than the rest and could be omitted by general degree students.

It must be admitted that, relative to the average present-day physics course, not many fundamental principles are involved in geometrical optics to degree standard — Fermat's principle, the characteristic function, Snell's law and some photometric ideas — and in any case these are all fairly straightforward consequences of electromagnetic theory. I have therefore given a mainly deductive treatment, starting with the laws of geometrical optics and developing the subject using chiefly coordinate geometry. The validity of the laws can be regarded as established either through the electromagnetic theory or through the correct functioning of optical instruments designed according to them. I have not discussed direct experimental proofs because geometrical optics is nowadays seen so much as derived from and approximating to more precise physical theories. I have used a sign convention for conjugate distance equations, etc., based on and following naturally from coordinate geometry; this seems to me to be preferable to the use of an *ad hoc* convention, such as 'real is positive' which, however much it may facilitate the visualizing of object–image relationships, obscures the fact that geometrical optics is a branch of physics and can be treated with the physicist's usual mathematical tools.

Although the basic physics content of geometrical optics is slender it is an important 'applied' subject, since it is the principal design tool in optical instrumentation; I have stressed this aspect, as implied in the sub-title of the book, because I think the physics graduate should understand some of the principles used in designing, say, a microscope, just as from his electronics course he will know what is involved in designing, say, a wide-band stable amplifier; one does not, of course, expect him to be able to carry through either of these in detail, only to know some of the general principles involved.

I am grateful to Dr R. J. Blin-Stoyle and Mr T. Y. Chou, who read the manuscript and made many helpful criticisms and suggestions, to my wife, who read the proofs and compiled the index, and to the Publishers, who have been helpful and cooperative throughout; particularly I should like to thank Mr D. Philippo of the North-Holland Publishing Company, who did the drawings.

W. T. WELFORD

CONTENTS

GEOMETRICAL OPTICS AND THEORIES OF LIGHT

§ 1. The scope of geometrical optics

Geometrical optics is the study of the behaviour and properties of rays of light. A ray of light is a physical abstraction in the sense that, like all concepts dealt with in physics, it represents only a part or aspect of physical reality. It is an aim of physics to state general laws which enable us to predict results of experiments; however, we have to restrict ourselves to laws dealing with abstractions in order to be able to say anything intelligible and useful because the physical world as a whole is too complex to deal with.

Geometrical optics is an approximation to more detailed and complete theories of light. Its study is justified by the fact that it enables us to make extremely accurate and useful predictions about light. It is in fact the chief tool used in the design of optical instruments. In this book geometrical optics is treated partly as a branch of physics and partly as a tool for designing optical instruments and for both these purposes it is necessary to know the relationships between geometrical optics and other theories of light. We start therefore by showing how geometrical optics arises as a result of a series of approximations and simplifications of the most rigorous theory, each allowing us to treat in detail a wider variety of problems.

§ 2. The nature of light

According to modern views light is an electromagnetic wave motion, of the same kind as wireless waves but with shorter wavelength. In fact a complete range or *spectrum* of such radiation is known having wavelengths from several thousands of metres down to about 10^{-13} metre. The electromagnetic spectrum is given in Table 1.1, the first column giving the usual name of the radiation, the second the approximate wavelength range and the third some modes of detection. Geometrical optics is in principle applicable to all of these radiations but in practice lenses, mirrors and prisms are used mainly for the range from the infra-red to the ultra-violet; the range in the centre of these, which can be conveniently be termed 'light', is seen by the eye but the concepts of geometrical optics are applicable to the whole. The wavelength of visible light is given in terms of certain convenient units

TABLE 1.1

The electromagnetic spectrum

Type of radiation	*Wavelength in metres*	*Some modes of detection*
Wireless waves		
long	$10^3 - 10^4$	
medium	$10^2 - 10^3$	
short	$1 - 10^2$	
Microwaves	$10^{-3} - 1$	
Far infra-red	$10^{-5} - 10^{-3}$	Thermocouple, bolometer
Near infra-red	$0.75 \times 10^{-6} - 10^{-5}$	Thermocouple, bolometer, photographic emulsion, photoconductive cell
Visible light	$0.4 \times 10^{-6} - 0.75 \times 10^{-6}$	Photochemical effects (the eye), photo-emissive and photo-conductive cells, photographic emulsion
Ultra-violet	$0.2 \times 10^{-6} - 0.4 \times 10^{-6}$	Photographic emulsion, photo-emissive cell, fluorescence
Vacuum ultra-violet	$5 \times 10^{-9} - 0.2 \times 10^{-6}$	Photographic emulsion, photo-emissive cell
Soft X-rays	$10^{-10} - 5 \times 10^{-9}$	Photographic emulsion, ionization chambers
Hard X-rays	$5 \times 10^{-13} - 10^{-10}$	
Gamma-rays	$5 \times 10^{-14} - 10^{-11}$	

of length of which three are in common use, the Ångstrom (symbol Å $= 10^{-10}$ metre), the micron (symbol $\mu = 10^{-6}$ metre) and the millimicron (symbol $m\mu = 10^{-9}$ metre). In this book we use the Ångstrom; the wavelength of visible light therefore ranges from about 4000 Å to 7000 Å. The symbol for wavelength is λ.

The velocity of electromagnetic waves in vacuum, or, what is substantially the same thing, in air, is constant throughout the spectrum;

it is denoted by c and its value is roughly 300000 km $\sec^{-1}$. The frequency, symbol ν, is related to velocity and wavelength as for all wave motions by the equation

$$\nu = \frac{c}{\lambda}. \tag{1.1}$$

Thus the frequency of visible light is between 10^{14} and 10^{15} $\sec^{-1}$. The frequency of a given radiation is constant but the wavelength and the velocity are less in a material medium such as glass than in vacuum. It would therefore seem more reasonable to plot the electromagnetic spectrum and describe radiations in terms of frequencies rather than wavelengths; we use wavelengths here partly to conform with traditional usage and partly for a reason given at the end of § 3, Ch. 7.

A further refinement is necessary when discussing the details of the interactions of light and matter, as in emission and absorption. All electromagnetic radiation carries energy and according to the quantum theory energy can only be gained or lost by matter in finite amounts called *quanta*. A quantum contains an amount of energy W which depends on the frequency according to the equation

$$W = h\nu \tag{1.2}$$

where h is a universal constant, 6.6×10^{-34} kg m^2 sec^{-1}. The quanta of electromagnetic radiation are called *photons* and they contain about 10^{-19} kg m^2 sec^{-2} or 10^{-12} erg of energy for visible light. The quantized nature of light is of importance in such processes as absorption and fluorescence but in geometrical optics it may be ignored: light is then treated as an electromagnetic wave motion with periodic variations of electric and magnetic field strength just as in wireless waves but on a smaller scale.

The strict treatment according to electromagnetic theory can be used to find the proportions of light reflected and transmitted by a glass surface and also to calculate the way in which light is *scattered* or *diffracted* by objects of a few simple shapes, e.g. spheres and cylinders. The complete electromagnetic theory is too complex for more complicated diffraction problems where irregularly shaped objects scatter the light and a simplifying approximation is made by ignoring the fact that there are really two quantities, the electric and the magnetic field vectors, needed to specify the light; the light is instead represented by a single *scalar wave*. This approximation enables us to solve many more problems of diffraction and also to deal simply with *interference*, the effect observed when waves originally from the same source cross each other.

§ 3. Rays of light

We now have to explain how the concept of a ray of light arises. It is a fact of common observation that light travels at least approximately in straight lines, the 'rays' of everyday language, and it is difficult at first to reconcile this with the preceding statements that light is a wave motion. Waves are known to spread round obstacles, in fact this is the phenomenon of diffraction mentioned above, but in the case of light this is not usually observed without special experimental arrangements. The reason lies in the order of magnitude of the wavelength. For example the wavelengths of sound waves range from about 0.1 m to 20 m and the diffraction takes place on an easily recognisable scale, but light waves are less than 10^{-6} m long and the effects are on a correspondingly smaller scale. A simple example will illustrate this. Suppose we have a point source of light, say a box with a pinhole in one side containing a lamp, as at P in fig. 1.1, and we

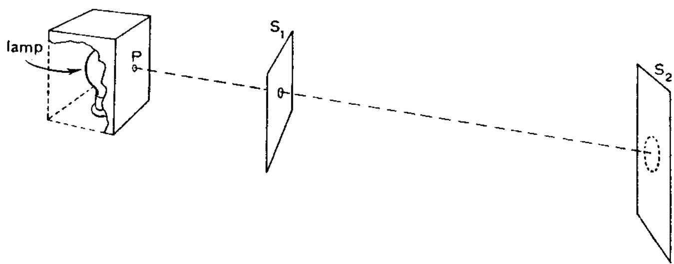

Fig. 1.1. Attempting to isolate a ray of light. As the size of the pinhole in S_1 is decreased the light patch on S_2 at first decreases in extent but then becomes broader and less sharply defined.

attempt to isolate a ray of light from P by placing a screen S_1 with a small hole in it as shown; a second screen S_2 is placed to intercept the ray. Let the distances from P to S_1 and from S_1 to S_2 be each about 1 m. If the hole in S_1 is fairly large a sharply defined bright disc of light will be seen on S_2 of the expected size, i.e. if the hole in S_1 is 1 cm in diameter the disc will be 2 cm diameter. But now as the hole is made smaller the edges of the disc become less sharp and it eventually becomes very faint, diffuse and broad. Table 1.2 shows the approximate

TABLE 1.2

Isolating a ray of light

Diameter of hole *in* mm	*Diameter of light patch* *in* mm
10	20
1	3
0.1	10
0.01	100

sizes which would be found. The light is spread out by diffraction at the hole in S_1 and the wave nature of light makes it impossible to isolate a ray. It can be shown that in such an experiment the light will travel roughly along 'rays', i.e. the diffraction spreading will be small, if the condition

$$\lambda l < d^2 \tag{1.3}$$

is fulfilled, where λ is the wavelength of the light, l is the distance from the hole to the second screen and d is the diameter of the hole. Equation (1.3) thus gives us an indication of the size of a 'ray' in the everyday sense of the term.

We can arrive at the concept of a ray in another way by considering the behaviour of waves as the wavelength is made smaller and smaller. This limiting process is carried out in Appendix A and it is shown there that we are led to the use of two concepts in geometrical optics, namely *rays*, which correspond to the everyday notion discussed above, and *geometrical wavefronts*, which are surfaces orthogonal to rays. The geometrical wavefronts are surfaces in the limiting positions of the surfaces of constant phase in the wave motion as the wavelength tends to zero and it is therefore convenient to call them simply *wavefronts*, although geometrical optics is strictly not concerned with waves.

The rays are the paths along which light energy flows and their positions in given arrangements of lenses can be calculated. Thus geometrical optics can be used to determine the main properties of optical instruments, at any rate those instruments of which the chief function is to form images. It is only in the more refined studies which determine the ultimate limits of performance of instruments that we go back a step in the approximations and use wave theory.

§ 4. Gaussian optics

There is yet another useful approximation which can be made in studying most optical instruments: it is to suppose that the rays all lie very close to the axis of the lens system. This is called *paraxial* or *Gaussian optics* † and it will be described in Ch. 3. It might be supposed that with such a series of approximations, starting from photons, Gaussian optics could not be very useful, but in fact it provides a very simple and neat description of the main properties of an optical system and is an invaluable aid in the early stages of optical design.

We summarise the approximations of optics in Table 1.3. This gives

TABLE 1.3

Optical theories

Quantum theory	Photons	Emission and absorption of light, photoelectric effects
Electromagnetic wave theory	Electromagnetic waves	Reflection and refraction. Diffraction at obstacles of a few simple shapes
Scalar wave theory	Scalar waves	Interference, diffraction at more complicated shapes
Geometrical optics	Rays and geometrical wavefronts	Image-forming optical instruments
Gaussian optics	Rays near the optical axis	Elementary properties of image-forming systems

in the first column the name of the optical theory, in the second the concepts used and in the third examples of problems which are adequately treated by the theory.

† After the mathematician C. F. Gauss who developed the method in detail.