INTRODUCTION TO RANDOM PROCESSES

With Applications to Signals and Systems

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Second Edition

MCGRAW-HILL PUBLISHING COMPANY
New York St. Louis San Francisco Auckland Bogotá
Caracas Hamburg Lisbon London Madrid Mexico
Milan Montreal New Delhi Oklahoma City
Paris San Juan São Paulo Singapore
Sydney Tokyo Toronto

Library of Congress Cataloging-in-Publication Data

Gardner, William A.
Introduction to random processes: with applications to signals and systems / William A. Gardner. — 2nd ed.
p. cm.
Includes bibliographical references.
ISBN 0-07-022855-8
1. Stochastic processes. 2. Stochastic systems. 3. Signal processing. I. Title.
QA274.G375 1989
519.2—dc20

89-37273
CIP

1234567890 DOC/DOC 89432109

ISBN 0-07-022855-8

The editors for this book were Daniel A. Gonneau and Susan Thornton and the production supervisor was Richard A. Ausburn. It was set in Times Roman by Techna Type, Inc.

Printed and bound by R. R. Donnelley & Sons Company.

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Preface to the Second Edition

In this second edition, the pedagogical value of the exercises at the ends of the chapters has been enhanced by the addition of nearly 100 new or revised exercises for a total of over 350 exercises. This provides a broad selection ranging from drill problems and applications to verifications of theoretical results as well as extensions and generalizations of the theory. To further enhance the value of the exercises as a study aid, detailed exemplary solutions to over 50 selected exercises (designated by ★) have been included at the back of the book. In addition, a supplement containing detailed solutions to all exercises in the book can be purchased from the publisher. This manual, entitled *The Random Processes Tutor: A Comprehensive Solutions Manual for Independent Study*, is essential for full command of the subject and is highly recommended for all students and professionals.

To reflect recent advances in the application of random process theory to problems in the area of statistical signal processing, a substantial expansion of the section in Chapter 12 on applications of the theory of cyclostationary processes has been incorporated in this edition. This includes new solutions to the problems of detection and classification of weak modulated signals, selective location of multiple sources of interfering modulated signals, and distortion reduction and interference suppression for modulated signals using adaptive frequency-shift filters and blind-adaptive antenna arrays.

Other changes in this edition include a few corrections and numerous minor revisions to enhance clarity.

Acknowledgments for the Second Edition

I am grateful to Dr. Chih-Kang Chen, for his contribution to the preparation of the solutions manual from which the selection of solutions at the back of the book was obtained.

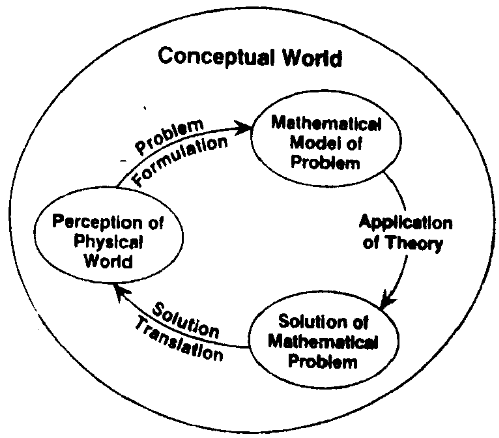
Preface to the First Edition

For the Instructor

This book is intended to serve primarily as a first course on random processes for graduate-level engineering and science students, especially those with an interest in the analysis and design of signals and systems. It is also intended to serve as a technical reference for practicing engineers and scientists.

Like any theory in engineering and science, the theory of random processes is a *tool* for solving practical problems. However, this theory has been developed into such a highly technical abstraction that expert utilization of this tool requires specialization. Moreover, even within an area of specialization, successful application of the abstract theory to practical problems in engineering and science presents a challenge that can be met only with substantial effort to appropriately conceptualize the representation of the empirical components of a problem by the abstract mathematical models with which the theory deals. A simplistic view of how a theory is used to solve a practical problem is illustrated, diagrammatically, in the figure on page xii. The point to be made is that, in practice, one does not make only one mental excursion around this circuit. For any nontrivial problem, the first excursion yields an inadequate solution. However, learning that occurs during the first excursion enables one to improve and refine the problem formulation and resultant mathematical model. Typically, it is only after many excursions of the circuit that an adequate model and a practical solution are obtained.

Although this process of conceptualization in problem solving is familiar to instructors of engineering and science, it too often receives only cursory



recognition in textbooks on theoretical subjects. Although a theoretical textbook that attends to this crucial problem of conceptualization and modeling can become cluttered, a textbook that all but ignores it, while clean and self-consistent, presents the student with a very limited picture of the subject as it relates to engineering and science. Thus, it is my intention, in this textbook on random processes, to strike a balance between a clean treatment of theory, on one hand, and discussion of its relation to the empirical world of signals and systems, on the other hand. The stage is set for this balance in the expository discussions throughout the book. The balance is played out by the particular selection and emphasis of topics in each chapter. For example, the concept of ergodicity is not played down—considerable attention is given to relationships between probabilistic (ensemble) averages and nonprobabilistic or deterministic (time) averages. For, as any engineer or scientist with practical signal-processing experience knows, the actual performance of signal-processing systems is often determined by time-averaging instantaneous performance measurements, whereas theoretical performance is measured by ensemble averages or *expected values*. Just as important as, but distinct from, the concept of ergodicity is the duality (described mathematically by Herman O. A. Wold's isomorphism) between those theories built on time averages and those built on ensemble averages. For example, the original theory of optimum filtering, developed by Norbert Wiener, is based on time averages, whereas the currently more popular theory of optimum filtering originated by Andrei Nikolaevich Kolmogorov, is based on ensemble averages. Similarly, the theory of spectral analysis of random processes, developed by Wiener (generalized harmonic analysis), is based on time averages, whereas the currently popular theory of spectral analysis—originated by Kolmogorov, Harald Cramér, Michel Moise Loève, and Joseph Leo Doob—is based on ensemble averages. As another example, Rudolf Emil Kalman's theory of optimum recursive filtering, which is based on ensemble averages, has as its time-average counterpart the earlier theory of recursive least-squares estimation, initiated by Carl Friedrich Gauss. Although the probabilistic theories have enjoyed more popularity in the last few decades, the earlier deterministic theories are currently gaining in popularity. This can be seen, for example, in the field of *adaptive* signal processing. In summary, a theme of this book is the exploitation of the duality that exists between probabilistic and deterministic theories, for the purpose of narrowing the gap between theory and practice. In historical perspective, this book can be viewed as a blend of the earliest and latest treatments of the subject.

Part 1 is a tutorial review which has two purposes: (1) to introduce notation and definitions for probability and random variables and (2) to refresh the reader's memory of this prerequisite material. The body of the book consists of Part 2, which is a self-contained introduction to the subject of random processes, with emphasis on the so-called *second-order theory* that deals with autocorrelation and spectral density.

With Part 1 treated as review, the entire book, with possibly some topics from the last three chapters omitted, can be covered in a 15-week course (preferably 4 hours of lecture per week). If the last two chapters are omitted, and a few selected topics from the remainder of Part 2 are only briefly treated, then a 10-week course is adequate.

For the Student

There are many physical phenomena in which the observed quantities are uniquely related, by simple laws, to a number of initial conditions and the forces causing the phenomena. Examples are the response voltage or current in an electrical circuit due to a periodic or transient excitation voltage, and the motion of a component in a mechanical device under the action of a force.

On the other hand, there is an important class of phenomena in which the observed quantities cannot be related, by simple laws, to the initial conditions and the actuating forces. Consider, for example, thermal noise in a circuit. If we attempted to relate the instantaneous values of the fluctuating noise voltage to the motions of the individual electrons, it would immediately become clear that we are faced with an impossible task. Even if we could obtain a solution to the problem by this method, its complexity would be an obstacle in the analysis of systems where thermal noise is present. For handling noise effectively, new concepts and methods are necessary.

In a dice game the prediction of the outcome of a throw, if it were possible, would require much detailed information that we do not possess. The physical properties of the dice and of the surface to which they are thrown and the initial conditions of motion of the dice are critical factors in the prediction. These factors are never fully known. Furthermore, if we assume that we have this information and are able to apply classical mechanics to the problem, it is easy to imagine how utterly complicated the solution would be. We must also remember that since the phenomenon of chance is the essence of a dice game, we really do not seek the law for the exact outcome of the game. What we desire is a mathematical description of the game in which chance is a fundamental characteristic; that is, we desire a *probabilistic* description.

In mechanics, the analysis of the behavior of a gas in a container is a problem relevant to this discussion. If the gas is assumed to be a large number of minute particles moving under their mutual interaction, then theoretically, in accordance with the principles of dynamics, the course of any particle of the gas for the infinite future is determined once the initial positions and velocities of every particle are known. But such initial conditions are impossible to obtain. Even if they were available and the solution were possible, it is beyond imagination that this extremely complex formulation of the problem would serve a useful purpose. Meaningful results in the analysis of a gas have been obtained by exploiting probabilistic concepts in addition to the concepts of classical mechanics. This has led to the *statistical theory of mechanics.*

The formation of speech waves is another example of a highly complex process that is not governed by simple laws. In fact, since the purpose of speech is to transmit information, it must have the characteristic that its variation as time passes cannot be predicted exactly by the listener. The information which the listener is to receive is unknown to him before its transmission. It is made known to him through a sequence of words selected by the sender from a collection of all possible words which are known to both sender and receiver. The message, in the form of speech, is clearly under the command of the speaker, and the listener can only make observations and cannot predict exactly, at any moment, its future course. It is true that, at times, a word or two, or even a short sequence of words, are predictable because of the rules of grammar or the occurrence of common phrases and idioms. However, precise prediction is impossible. Clearly, if speech does not possess this characteristic of unpredictability and is determinable exactly and completely, then indeed it conveys no information.

All information-bearing functions, which we shall refer to as *signals*, in the form of fluctuating quantities, such as voltages, velocities, temperatures, positions, or pressures, which are to be processed, transmitted, and utilized for the attainment of an objective, necessarily possess the characteristic that they are not subject to precise prediction. Also, in view of the foregoing discussion, thermal noise, impulse noise, errors in measurement, and other similar forms of disturbances in the transmission of a signal should be considered to have the same characteristic of unpredictability. These quantities, the signals and noises in a communication system, vary with time, either continuously or at discrete points, and are referred to as random time functions or *random processes*. Meaningful results on the analysis of random processes in communication systems have been obtained by exploiting probabilistic concepts in addition to the concepts of classical function analysis, such as harmonic analysis. This has led to the *statistical theory of communication*, and more generally to the *statistical theory of signal processing*.

The objective of this book is to present an introductory treatment of random processes, with applications to the analysis and design of signals and systems. It is assumed that the reader is familiar with Fourier analysis and the elementary theories of linear systems and of probability and random variables, at the level of typical introductory courses in a contemporary undergraduate electrical-engineering curriculum. The tutorial review of probability and random variables in Part 1 of this book is intended to serve primarily as a refresher.

Acknowledgments

It is a pleasure to express my gratitude to Professor Thomas Kailath, who read the first draft of this book and made valuable suggestions for improvement. It is also a pleasure to thank my students, who have helped to shape the style of this book. In addition, it is a pleasure to express my appreciation to Mrs. Jill M. Rojas and Mrs. Patty A. Gemulla for their excellent job of typing the entire manuscript. My deepest gratitude is expressed to my wife, Nancy, for her constant support and encouragement.

William A. Gardner

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Part 1

REVIEW OF PROBABILITY, RANDOM VARIABLES, AND EXPECTATION