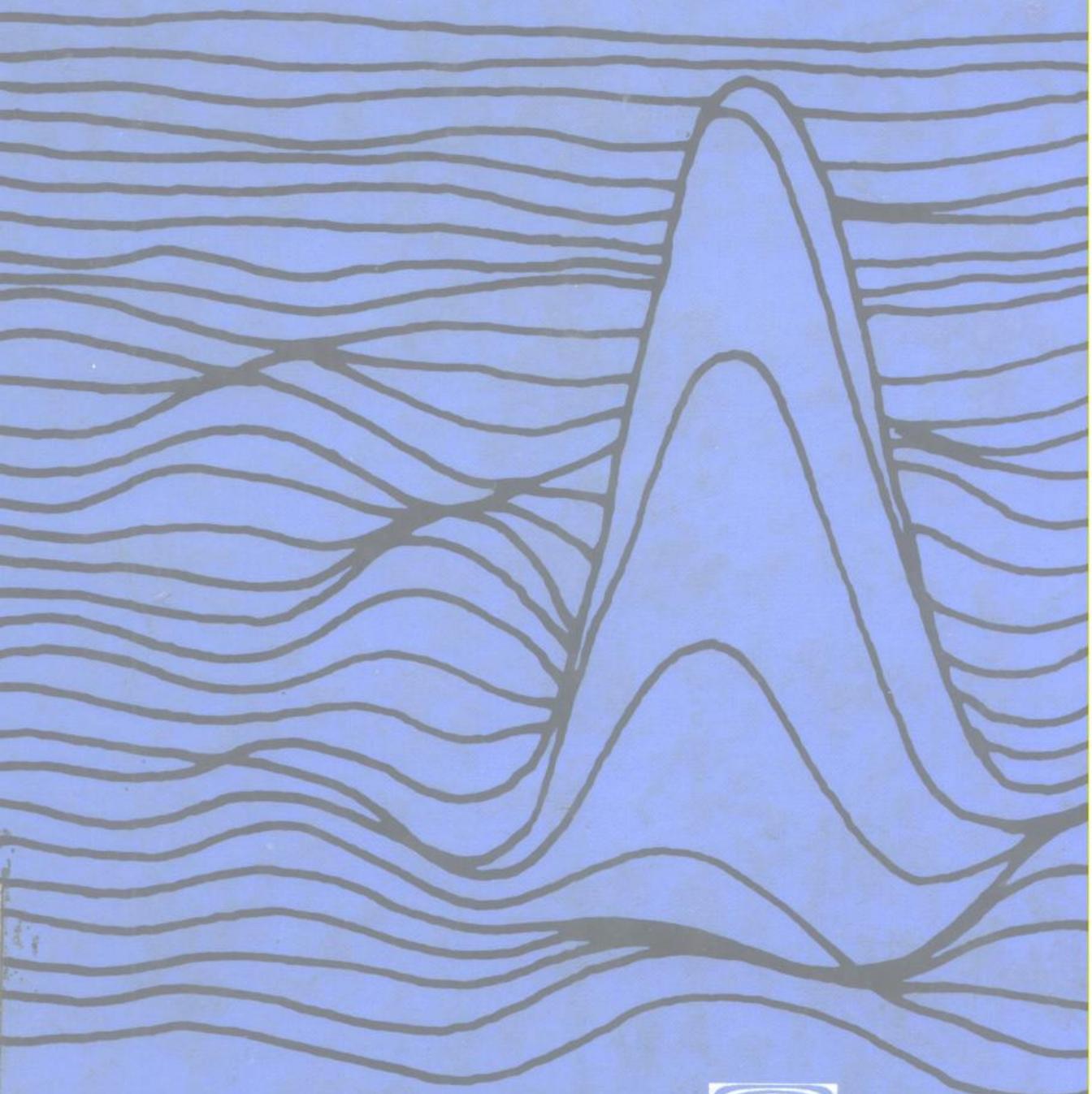


# Array Signal Processing

**S. Haykin, Editor**

J.H. Justice · N.L. Owsley · J.L. Yen · A.C. Kak



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# ARRAY SIGNAL PROCESSING

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# *Preface*

This is the first book to be devoted completely to array signal processing, a subject that has become increasingly important in recent years. The book consists of six chapters. Chapter 1, which is introductory, reviews some basic concepts in wave propagation. The remaining five chapters deal with the theory and applications of array signal processing in (a) exploration seismology, (b) passive sonar, (c) radar, (d) radio astronomy, and (e) tomographic imaging. The various chapters of the book are self-contained.

The book is written by a team of five active researchers, who are specialists in the individual fields covered by the pertinent chapters.

Much of the material covered in the book has not appeared in book form before. It is hoped that it will be found useful by researchers in array signal processing, its theory and applications, and to newcomers to the subject. Also, by bringing the various applications of array signal processing under one cover, it is hoped that the book will lead to further cross-fertilization among the various disciplines of interest.

*Simon Haykin*

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# 1

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## *Introduction*

### 1.1 ARRAY PROCESSING

*Array processing* deals with the processing of signals carried by propagating wave phenomena [1]. The received signal is obtained by means of an *array of sensors* located at different points in space in the field of interest. The aim of array processing is to extract useful characteristics of the received signal field (e.g., its signature, direction, speed of propagation).

The sources of energy responsible for illuminating the array may assume a variety of different forms. They may be *noncoherent* (i.e., independent of each other) or *coherently* related to each other. Equally, as seen from the location of the array, the radiation may be from diffused media and therefore distributed in nature, or it may be from isolated sources of finite angular extent.

The array itself takes on a variety of different geometries depending on the application of interest [2, 3]. The most commonly used configuration is the *linear array*, in which the sensors (all of a common type) are uniformly spaced along a straight line. Another common configuration is a *planar array*, in which the sensors form a rectangular grid or lie on concentric circles.

In this book we will study theoretical aspects of array signal processing and its application in exploration seismology, sonar, radar, radio astronomy, and tomography.

In *exploration seismology*, array processing is used to unravel the physical characteristics of a limited region of the interior of the earth which may have potential for trapping commercial quantities of hydrocarbons. In particular, we use it to image the interior of the object from some finite aperture on the surface, with the measurements being made at a finite number of points within the limited aperture. The earth tends to act like an elastic medium for the propagation of acoustic energy. Accordingly, for the source of acoustic energy we use a *shot* (e.g., a stick of dynamite) that applies an impulse to the earth, and to record the received signal we use a *geophone*. Typically, a number of shots are laid out at equally spaced intervals along a surveyed line, and geophones are located at equally spaced intervals on one or both sides of each shot. The resultant geophone outputs are due to signals reflected, diffracted, or refracted back to the earth surface from the original source of disturbance.

In *passive, listening-only sonar*, the received signal is externally generated, and the primary requirement of array processing is to estimate both the temporal and spatial structure of the received signal field. The array sensors consist of sound pressure-sensing electromechanical transducers known as *hydrophones*, which are immersed in the underwater medium. Basically, the processing applied to the sensor outputs involves some form of spectral and/or wavenumber analysis so as to determine a directional map of the background sound power, with emphasis on the detection of signals that are characterized by low signal-to-noise ratios and extended duration.

In *radar* array processing, a transmitting antenna is used to floodlight the environment surrounding the radar site, and a receiving array of *antenna* elements is used to listen to the radar returns caused by reflections from targets located in the path of the propagating electromagnetic wave. Here again, we may use array processing to estimate the wavenumber power spectrum of the received signal, with emphasis on spatial resolution.

In *radio astronomy*, the interest is in radio emission from celestial sources. The emission, depending on the radiation mechanism and the state of the emitting region, shows broad continuum spectral features, narrow band, or absorption line structures. The arrays used here consist of tens of *antenna* elements that extend from hundreds of meters to nearly the diameter of the earth. The requirement is to use array processing for *image reconstruction* of unpolarized or partially polarized continuum and spectral line radio sources, with emphasis on resolution, ambiguity, and dynamic range of the reconstructed maps.

In *tomography*, array processing is used to obtain *cross-sectional images* of objects from either transmission or reflection data. In most cases, the object is illuminated from many different directions either sequentially or simultaneously, and the image is reconstructed from data collected either in transmission or reflection. The most spectacular success of tomography has thus far been in medical imaging with x-rays. There is also active interest today in extending tomographic imaging to ultrasound and microwaves for use in medical imaging, seismic exploration, and nondestructive testing.

Before going on to a detailed discussion of these topics, one by one, we will complete this introductory chapter by reviewing some concepts in wave propagation [4] which are basic to the development of array signal processing theory.

## 1.2 WAVE PROPAGATION

Whenever a driving force is coupled to an open medium, we find that *traveling waves* are generated. They are so-called because they travel away from the source of the disturbance. Traveling waves have the important property that they transport energy, the form of which depends on the physical nature of the driving force. Also, in the *far field* or a large distance away from the source, the waves become essentially plane. As a matter of fact, the *plane wave* is probably the most common of all the different forms of wave propagation.

Suppose we have a harmonic traveling wave that propagates through a homogeneous dispersive medium in the direction of the unit vector  $\hat{z}$ , along the  $z$ -axis of the Cartesian coordinate system. At the plane defined by a fixed value of  $z$ , the wave function has the time dependence

$$g(t, z) = A \cos [2\pi(ft - vz)] \quad (1.1)$$

where  $A$  is the amplitude,  $f$  the frequency, and  $t$  the time. The parameter  $v$  is called the *wavenumber*. The physical meaning of the wavenumber  $v$  is that in a distance  $z$ , measured along the propagation direction  $\hat{z}$ , the phase accumulates by  $2\pi vz$  radians. (Frequently, the wavenumber is defined as  $k = 2\pi v$ , so that the phase accumulated in a distance  $z$  along the propagation direction  $\hat{z}$  equals  $kz$  radians [4].)

Let  $\mathbf{r}$  denote a point in space as measured from the origin of the Cartesian coordinate system. The plane  $z = \text{constant}$  is described by the plane  $z = \hat{z} \cdot \mathbf{r} = \text{constant}$ , where  $\hat{z} \cdot \mathbf{r}$  is the *dot product* of the unit vector  $\hat{z}$  along the propagation direction and the vector  $\mathbf{r}$ . Thus we may express the quantity  $-vz$  in Eq. (1.1) as

$$\begin{aligned} -vz &= -v(\hat{z} \cdot \mathbf{r}) \\ &= \mathbf{v} \cdot \mathbf{r} \end{aligned} \quad (1.2)$$

where

$$\mathbf{v} = -v\hat{z} \quad (1.3)$$

We refer to  $\mathbf{v}$  as the *wavenumber vector*. Note that this vector points in the opposite direction to the direction of propagation  $\hat{z}$ .

Using this new notation, we may now express the traveling wave of Eq. (1.1) in the equivalent form

$$g(t, \mathbf{r}) = A \cos [2\pi(ft + \mathbf{v} \cdot \mathbf{r})] \quad (1.4)$$

The argument of the sinusoidal wave function is called the *phase*  $\phi(t, z)$ ; that is,

$$\begin{aligned} \phi(t, z) &= 2\pi(ft - vz) \\ &= 2\pi(ft + \mathbf{v} \cdot \mathbf{r}) \end{aligned} \quad (1.5)$$

At fixed time  $t$ , the points with equal phase  $\phi$  define a plane called a *wavefront*, which is described by

$$\begin{aligned} d\phi &= 2\pi(f dt + \mathbf{v} \cdot d\mathbf{r}) \\ &= 2\pi(0 + \mathbf{v} \cdot d\mathbf{r}) && \text{at fixed time} \\ &= 0 && \text{only if } d\mathbf{r} \text{ is perpendicular to } \mathbf{v} \end{aligned} \quad (1.6)$$

Accordingly, at fixed time  $t$  the phase will have the same value at all points reached by adding up vectors  $d\mathbf{r}$  that are perpendicular to the vector wavenumber  $\mathbf{v}$ . That is, the phase increment  $d\phi$  equals zero in moving from one such point to another, which means moving about in a plane. It is for this reason that such a wave is referred to as a *plane wave*.

Another quantity used in describing wave propagation is the *phase velocity*, which is defined as  $dz/dt$  for a fixed phase  $\phi(t, z)$ . Using Eq. (1.5), we may write

$$d\phi = 2\pi(f dt - v dz)$$

Putting  $d\phi = 0$ , and solving for  $dz/dt$ , we get the following formula for the phase velocity:

$$v_\phi = \frac{dz}{dt} = \frac{f}{v} \quad (1.7)$$

Thus a plane wave propagates with respect to a fixed point in space in a direction defined by the vector wavenumber  $-\mathbf{v}$  and with a phase velocity or speed equal to  $f/v$ , where  $v = |\mathbf{v}|$  is the magnitude of  $\mathbf{v}$ .

In the case of electromagnetic plane waves the electric and magnetic fields are transverse to the direction of propagation [4]. Assuming that the direction of propagation is along  $\hat{z}$ , there are two transverse directions,  $\hat{x}$  and  $\hat{y}$ , and the fields with one orientation with respect to  $\hat{x}$  and  $\hat{y}$  are independent of those with an orientation differing by  $90^\circ$ . Accordingly, we may have various amplitudes of the fields in each of the two transverse directions and various possible relative phases. We refer to a specific relation of the amplitudes and phases of the two independent transverse fields as a state of *polarization* [4].

All the waves that we study consist of some physical quantity whose displacement from its equilibrium value varies with both position and time. Thus for plane waves propagating along  $\hat{z}$ , we may express the displacement as the vector

$$\mathbf{g}(z, t) = \hat{x}g_x(z, t) + \hat{y}g_y(z, t) + \hat{z}g_z(z, t) \quad (1.8)$$

where  $\hat{x}$ ,  $\hat{y}$ , and  $\hat{z}$  are unit vectors along the  $x$ -,  $y$ -, and  $z$ -axes, respectively. In the case of waves with transverse polarization, the vector  $\mathbf{g}(z, t)$  has only  $x$ - and  $y$ -components.

It is important to realize, however, that the concept of polarization applies only to waves that have at least two independent "polarization directions." For sound waves in air, for example, the displacement is along the propagation direction. These waves are called *longitudinal waves*; we do not ordinarily say that these

waves are longitudinally polarized. Rather, we reserve the term “polarization” to describe waves for which there are at least two alternative polarization directions. In the case of sound waves in solids, there are one longitudinal and two transverse polarization directions available. Accordingly, in this case we may have longitudinally polarized waves or two different transversely polarized waves or a superposition of all three polarizations.

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# 2

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## *Array Processing in Exploration Seismology*

### 2.1 INTRODUCTION

Seismic signal processing represents an extremely important area of application in array processing, and it is an area in which development of array processing procedures has been underway for some time. Seismic data acquisition as practiced today is inherently multidimensional in character; that is, the seismic experiment attempts to record the multidimensional character of acoustic wave fields which have both temporal and spatial characteristics. As a result, much of seismic signal processing is concerned with utilizing the full multidimensional character of the recorded wave field, and so, in one way or another, may be classified as array processing. While there are important exceptions to this rule, it is generally true, and therefore many of the procedures which have been developed for seismic data acquisition and processing will be discussed here.

Because seismic exploration holds a great fascination, but is not widely understood, we shall endeavor to provide necessary background information to create a framework and a context for these discussions. In particular, we shall examine wave propagation in a nonhomogeneous medium, the foundation for the seismic method, and from this we shall consider the origin of some of the models which form the basis for many of our array processing procedures.

The ultimate goal of the exploration process is to reconstruct as accurately as possible an image of the subsurface in which both structure and physical properties are delineated. We shall therefore enter into a brief discussion of the ultimate objectives of seismic exploration, that is, a discussion of hydrocarbon traps and trapping mechanisms which we hope to identify when our processed data is interpreted. The drill bit becomes the final arbiter in the determination of our success or failure in this process.

The exploration process may be broken down into three large categories once a prospective area has been identified. These are: data acquisition, data processing, and interpretation. We shall devote most of our discussion to the first two, the latter belonging to that gray area between art and science where only understanding, past experience, and logical inference can make the difference between success and failure. It is quite possible that some day this process will be well-explained or enhanced by techniques of artificial intelligence, unknown or in development today, but for now we leave this realm to the skill of the interpreter and shall only try to suggest what he looks for.

Finally, we shall consider many of the techniques which have been developed or are being developed to provide the pieces of the puzzle which must be fitted together by the interpreter.

The field of exploration seismology offers especially fertile ground to the signal processor since most areas of inquiry in signal processing seem to have some application to the problems in exploration seismology. These may range from the popular areas of spectral estimation to parameter estimation or multidimensional filtering, to image processing or antenna design. These would include the areas of adaptive signal processing, noise cancellation, time-delay estimation, and many other active areas of inquiry in signal processing. Indeed, the concept of holography as applied to acoustic wave fields, and even tomography, have been considered in exploration seismology. Thus it seems safe to predict that the areas of pattern recognition and artificial intelligence will find more and more application in exploration seismology as time goes on.

The signal processing procedures that have been developed for exploration seismology have one goal—to solve the geophysical inverse problem in which an acoustic signal is generated at the surface of the earth and is propagated through the earth, undergoing reflection where changes in acoustic impedance are encountered. The reflected signal is propagated back to the surface of the earth, suffering transmission losses, frequency-selective attenuation, and dispersion as it propagates. It is further contaminated by various types of noise both random and coherent, then modified by the response of the recording system, and mixed with near-field effects of the source, including surface and near-surface modes of propagation as well as a variety of other signal contaminants. From this multidimensional recorded data set, obtained by successive reflections within the medium of interest, we must somehow, in the final analysis, correctly infer the structure of that medium. It is interesting to note that today work is underway to provide a direct solution to this inverse imaging process. Although we still have a long way to go, encouraging results are

being reported. It is quite possible that within the next few decades we will see significant changes in the ways we process seismic exploration data.

Looking beyond what has been accomplished in the past and what is being done at present, it seems safe to speculate that whereas we have been content before to attempt a reconstruction of subsurface structure of the earth (which is more or less equivalent to saying that we have been content to infer the velocity profiles of the earth), the future seems to hold a greater potential and a greater challenge. Where past processing methods have been largely based on attempting to determine times of arrival of events from which velocity estimates, and thereby structure in depth of the earth can be inferred, the future tells us that the recorded wave field is carrying much more information than simple time of arrival, and that contained within these additional parameters, relating to the amplitude and phase characteristics of the recorded signal, is possibly information sufficient to make at least some judgment concerning the actual lithologic environment within the earth (rock type, porosity, saturation, etc.). Although it is true today that we can only poorly understand the mechanisms of attenuation and dispersion that operate within the earth, we are seeing increased activity in a relatively new field of petrophysics or rock properties intended to answer the question of how the various parameters of lithology express themselves in the seismic signature. With success, we may eventually be able to read the full seismic message that is sent to us by the propagating medium and infer with some sense of completeness a full knowledge of its structure. When we add these new parameters and variables to the seismic problem, then that which was already multidimensional becomes even more so and emphasizes the need to extend many of our mathematical tools and processes to higher-dimensional settings, so that they will be available as we increase our demand for these tools in order to deal with the increasing complexities of the seismic inverse problem.

In summary, exploration seismology as practiced today is indeed a multidimensional problem employing multidimensional processes in its (approximate) solution. Looking ahead to the future, we can predict that more sophisticated multidimensional processing, based on a large number of variables or parameters, will become increasingly important and will result in much more information eventually being derived from the seismic experiment. Understanding that our discussions can never be complete, we now go on to consider the concepts and methods which are a part of array processing in exploration seismology.

## 2.2 THE SEISMIC EXPERIMENT

Before proceeding with a detailed discussion of the various array processing procedures that are applied to seismic data in any of the many forms in which it may be displayed, it may be of value to review the procedures that are in use today to generate multidimensional data sets and to give some justification for these procedures. Let us recall that our primary objective is to unravel the complexities of structure and even the physical characteristics of that structure where possible,

contained within limited regions of the interior of the earth which may have the potential for trapping commercial quantities of hydrocarbons. Mathematical physics tells us that if we wish to determine the structure of the interior of a solid by making measurements on its surface, then if we know the *Green's function* for the medium, and if we can conduct measurement at all points on the surface obtained by impinging the object with an infinitely broad frequency source located outside the body, we may in fact completely and accurately infer the interior structure of the body [210]. The seismic problem is very much like this; we have a region in the earth which we suspect to contain commercial quantities of hydrocarbon and our problem is to determine its structure and probable physical properties while being restricted to make our measurements on the surface of the body. In the seismic problem, however, we have many restrictions that render the mathematical physics solution inapplicable. In particular, we cannot make measurements on an entire surface surrounding the region of interest. This means in fact that we must image the interior of the object from some finite aperture on the surface that does not include an entire surface surrounding the region of interest, and further, we are able to make measurements at only a finite number of points within this limited aperture. In addition, we do not know the appropriate Green's function, although we may approximate it in various ways, and finally, in the seismic experiment the source is actually interior to the region of interest, which means that we cannot neglect the volume integral whose data are unavailable to us. As a result, there is no simple direct solution to the seismic inversion problem, to date, which parallels the elegant solution to the mathematical physics problem alluded to, although progress is being made in this direction. Instead, we must resort to a wide variety of methods, each designed to fill in one piece of a very complex puzzle, with the hope that taken together they may yield sufficient information to at least justify or preclude the possibility of drilling. Indeed, we are only partially successful in solving this inverse problem, as is evident from the large number of dry holes drilled every year in the search for commercial quantities of hydrocarbons (over 33,000 dry holes were drilled in the United States in 1981 [14]).

Let us now consider the *seismic imaging problem*. To begin with, we must have a source of illumination for the subsurface of the earth, that is, some signal which can be propagated into the earth and will return to us carrying information about the subsurface structure. Since we are constrained to remain on the surface of the earth and since our targets typically occupy very limited regions in the near surface ("near surface" may mean the first few tens of thousands feet of the earth's crust), it follows that we will be able to record only signals that have been reflected, diffracted, or refracted to us from our surface source of illumination. That is, in the absence of a bore hole penetrating deeply into the earth, we will not receive transmitted signals which have passed through the medium directly without reflection. To be successful, then, it is important that the earth will, in fact, return a signal to us which is propagated into it from the surface.

The earth tends to act very much like an elastic medium for the propagation of acoustic energy. Since the velocity of acoustic wave propagation tends to increase