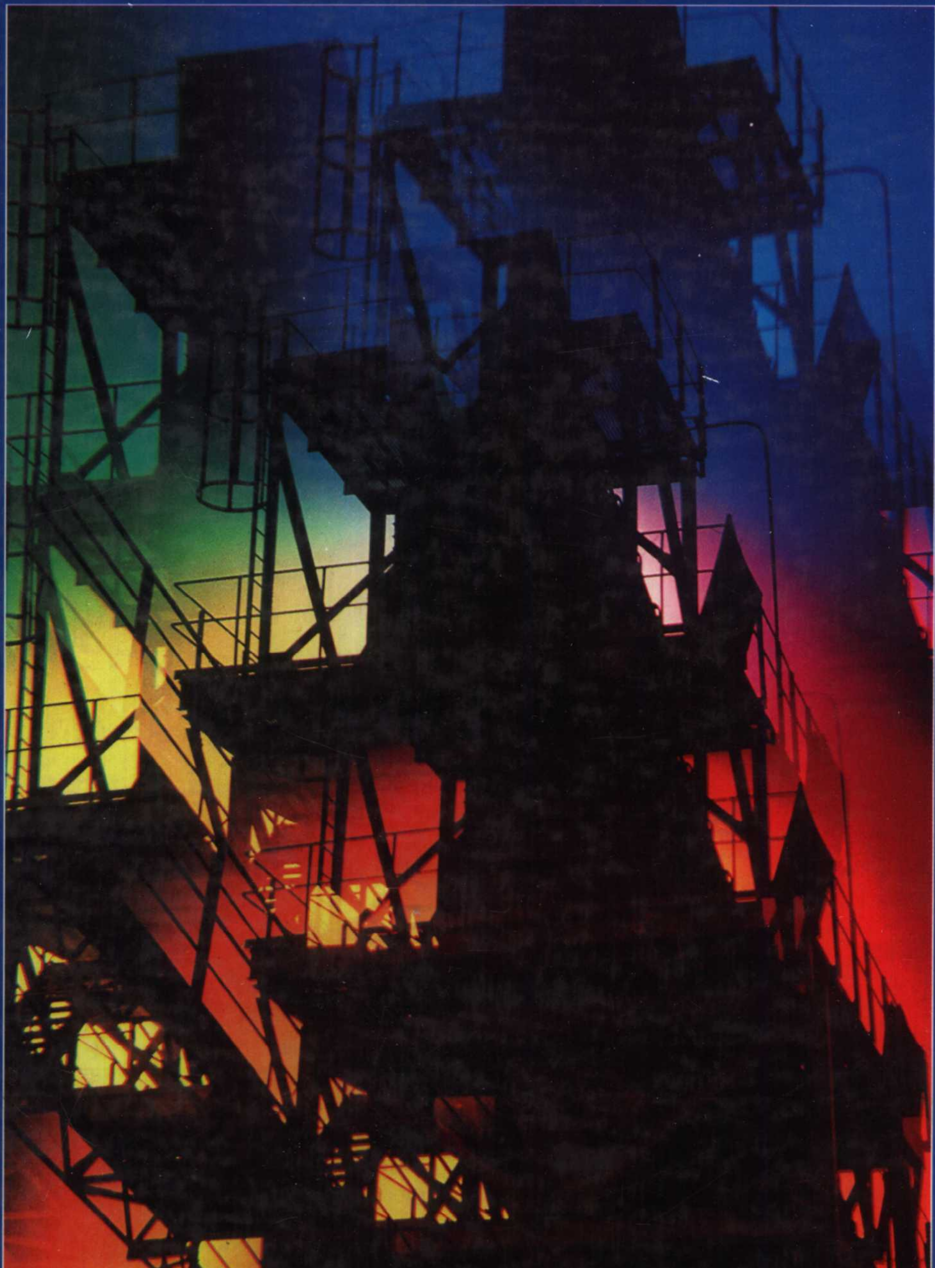
TECHNICAL MATHEMATICS WITH CALCULUS



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TECHNICAL MATHEMATICS WITH CALCULUS

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PREFACE

This text, with its performance-based format, has been developed to provide the essential elements of a technical mathematics sequence that includes calculus. It provides a comprehensive coverage of algebraic expressions, linear equations and inequalities, polynomial and transcendental functions, and the fundamental concepts of differential and integral calculus. There is sufficient material for a two semester sequence.

The subject material is related to specific skills needed by technologists. Our colleagues in technical fields and several reviewers have been helpful in suggesting topic areas for emphasis and in isolating skill deficiencies observed in their students. Numerous applications have been incorporated along with the related topic coverage. We have attempted to include only those mathematical concepts for which we can demonstrate relevance in a technical program. Although rigorous proofs have been avoided, we have attempted to provide motivation for various results by including intuitive discussions that lead to the results. For example, a motivational approach is used to lead to the form $y - k = a(x - h)^2$ for a quadratic function. We have also attempted to provide some sense of direction through the text by the narratives in the chapter introductions and in the review sections. Each review section includes a glossary of terms. One of our goals is that our students will not only be able to do mathematics but talk about it as well.

For the benefit of students with weak math backgrounds, the text includes a review of elementary concepts from arithmetic and basic algebra. However, it is assumed that the math background of the students does include basic algebra. This is the rationale for a somewhat cursory treatment of certain topics such as real number operations. Instructors whose students are better prepared may choose to bypass selected topics from the earlier chapters.

The use of calculators and computers is integrated throughout the text, and an appendix on BASIC is included, although the text can be used without a computer. We have included keystroke sequences for problem solutions on any calculator that uses algebraic logic and have provided BASIC programs for problem solutions by computer. We have used computer programs only where they are definitely beneficial. The syntax used in the BASIC programs conforms to minimum BASIC standards and should work on any computer. We have avoided the use of graphics since they are different on every computer. The programs contain minimal documentation, but they are well structured. Our purpose is to suggest that a student can use existing programs, modify those programs, and even write programs that do useful things without formal instruction in programming.

In addition to the integration of calculators and computers, another unique feature of the text is the emphasis on measurement and data handling. Such material is often treated in basic physics or chemistry texts and receives little reinforcement in mathematics classes. The material in the body of the text is supplemented by two appendixes on working with real data and data analysis.

The text gives considerable emphasis to the treatment of linear systems and to vectors, including some vector algebra. Separate chapters on each have been included. This emphasis is in response to a trend toward more sophisticated applications in engineering technology courses.

The text provides independent guidance for the student through numerous worked examples, answers to odd-numbered exercises, performance objectives inserted throughout the text, chapter practice tests keyed to the performance objectives, and a review and glossary at the end of each chapter. Exercises are paired so that for each odd-numbered exercise of a given type, there is a corresponding even-numbered exercise. Answers to even-numbered exercises, practice tests answers, and two forms of tests for each chapter are available to instructors.

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CONTENTS

WORKING WITH NUMBERS

1

OUTLINE

INTRODUCTION

In this chapter, we present a brief review of the real number system along with some techniques for operating on numbers with calculators and computers. These techniques will be applied throughout the text.

One of the products of technology is the low-cost computing system. Electronic calculators, hand-held computers, and desktop computers are in routine use by technologists as well as by engineers.

While we can usually depend on a machine to compute rapidly and accurately, we cannot expect it to verify the validity of the input data nor to interpret the results. That the computation process is invisible imposes added responsibility on human users of computing machines. We must always ask the question "Does the result make sense?" We must also understand the various forms in which the machine accepts numerical input and delivers numerical output.

SECTION 1.1 REVIEW OF THE REAL NUMBER SYSTEM

In this section, we review the various types of real numbers and the rules for performing arithmetic operations on those numbers. The reason for including this material is not to teach new concepts but to review basic concepts that will be assumed to be part of the student's background as new concepts are introduced.

The numbers with which we count, that is, the numbers 1, 2, 3, 4, and so forth, are called the *natural numbers*. We will denote the collection, or set, of natural numbers by N. If we include the number 0 along with the natural numbers, we get the set of *whole numbers*, which we will denote by W. Since N and W contain infinitely many numbers, we can't actually list every number. So we list enough numbers to establish the pattern of the numbers and put three dots (ellipses) to indicate that the pattern continues.

DEFINITION

Classification of Numbers

The natural numbers are the counting numbers

$$1, 2, 3, \ldots$$

The whole numbers are the counting numbers plus zero, that is,

$$0, 1, 2, 3, \ldots$$

The natural numbers are often called *positive integers*. For each natural number or positive integer a, there exists another number, denoted by $-a$ and read as "negative a," such that $a + (-a) = 0$. We call $-a$ the *additive inverse* of a or the opposite of a. For example, the additive inverse or opposite of 3 is -3 (negative 3) since $3 + (-3) = 0$. The set of *negative integers* consists of the additive inverses of all the positive integers. The set of negative integers includes $\ldots, -4, -3, -2, -1$.

DEFINITION

Integers

The set of integers, denoted by I, is the combination of the set of whole numbers and the set of negative integers, that is,

$$\ldots, -2, -1, 0, +1, +2, \ldots$$

The *rational numbers* can be defined in two different forms—fraction form and decimal form.

DEFINITION

Fraction Form of a Rational Number

A rational number is a number that can be written in the form

$$\frac{a}{b}$$

where a and b are integers and $b \neq 0$.

Note: The symbol $\neq$ means "is not equal to."

The set of rational numbers is denoted by Q. Some examples of rational numbers in fraction form are

$$\frac{2}{3} \qquad \frac{1}{5} \qquad -\frac{3}{4} \qquad -\frac{5}{2}$$

Note that the negative fractions were written with the minus sign in front of the fraction. The fractions could also have been written with the minus sign in the numerator (the top half of the fraction) or in the denominator (the bottom half of the fraction), that is,

$$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$$

For example,

$$-\frac{3}{4} = \frac{-3}{4} = \frac{3}{-4}$$

We see that an integer is also a rational number since any integer can be written as itself divided by 1. For example,

$$2 = \frac{2}{1} \qquad -5 = \frac{-5}{1} \qquad 0 = \frac{0}{1}$$

So 2, -5, and 0 are rational numbers.

DEFINITION

Decimal Form of a Rational Number

A rational number is a number that can be written as a decimal that either terminates or repeats.

A *terminating decimal* is one that has only a finite number of nonzero decimal places. For example, 0.125, −0.5, 226.29, −17.4256, and 8.333 are terminating decimals. A *repeating decimal* is one that continues indefinitely but that eventually begins to repeat the same digit or block of digits over and over without end. Examples of repeating decimals are 0.666 ..., 2.08333 ..., −0.8989 ..., and −5.4123123 .... The dots in these numbers indicate that the repeating digit or block of digits continues to repeat indefinitely.

Rational numbers have a unique property. For each rational number a, except 0, there exists a rational number

$$\frac{1}{a} \quad \text{such that} \quad a \times \frac{1}{a} = 1.$$

We call $^1/_a$ the *multiplicative inverse* of a or the *reciprocal* of a. For example,

$^1/_2$ is the multiplicative inverse of 2
$-^4/_3$ is the multiplicative inverse of $-^3/_4$
-5 is the multiplicative inverse of $-^1/_5$
$^1/_{0.25} = 4$ is the multiplicative inverse of 0.25

There does exist yet another set of numbers called the *irrational numbers*. These numbers are distinct from the rational numbers. When the irrational numbers are combined with the rational numbers, a bigger collection of numbers called the set of *real numbers* is obtained. The set of real numbers is denoted by R.

But what type of numbers are the irrational numbers? Since an irrational number is not rational, it cannot be written in the form

$$\frac{a}{b}$$

where a and b are integers. Likewise, it cannot be written as a terminating or a repeating decimal. If we were to attempt to write an irrational number in decimal form, the decimal expansion would continue forever and never begin to repeat the same block of digits. Some specific examples of irrational numbers are π (pi), $\sqrt{2}$ (the square root of 2), $-\sqrt{3}$ (the negative square root of 3), and $\sqrt{6}$. (Not all square roots are examples of irrational numbers.)

Numbers that are not real also exist. They are called *imaginary numbers* and *complex numbers*. Combinations of real numbers and imaginary numbers form the set of complex numbers, which we will consider later. For now, we will be concerned with the set of real numbers.

Figure 1.1 shows the relationship between the various types of real numbers that we have considered.

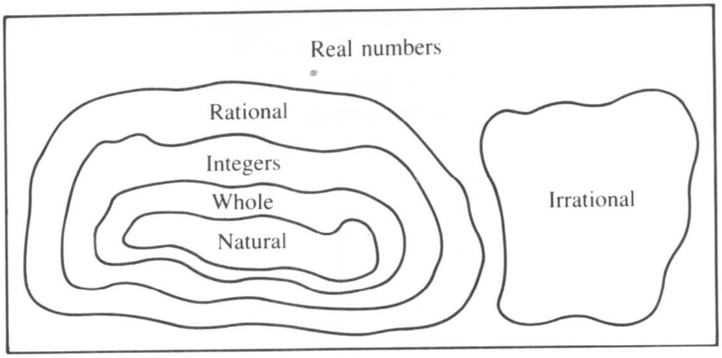

Figure 1.1

PERFORMANCE OBJECTIVE 1

You should now be able to recognize the various types of real numbers.

Check Classify each of the following real numbers into the smallest group to which it belongs.

$$-3 \qquad \frac{5}{4} \qquad \sqrt{5} \qquad 2 \qquad 1.73 \qquad 0$$

Answer Integer, rational number, irrational number, natural number, rational number, and whole number

We use a number line to represent the real numbers graphically. A number line is constructed according to the following rule.

RULE Constructing a Number Line

Step 1. Draw a straight line, usually in a horizontal or a vertical direction.

Step 2. Select an arbitrary point on the line, call this point the origin of the number line, and assign it the number 0.

Step 3. Decide on the length of one unit on the number line and the positive direction on the line.

Step 4. Assign the number 1 to the point located one unit in length from the origin in the positive direction. Likewise, the number 2 is assigned to the point located two units in length from the origin in the positive direction.

Step 5. Continue in this manner, assigning whole numbers to points on the number line.

Figure 1.2 shows examples of number lines that exhibit the set of whole numbers. As the illustrations in this figure show, horizontal number