

# LASER HANDBOOK

VOLUME 4

*edited by*

M.L. STITCH

*and*

M. BASS



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*Rockwell Hanford Operations  
Richland, Washington, USA*

*and*

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## *Preface*

Volume 4 of the Laser Handbook, like the preceding volumes, presents articles on laser technology and applications by authors active in the field from academic, industrial and government laboratory organizations.

The articles are again expository monographs which serve as detailed discussions for active workers and other scientists and engineers. The treatments are also designed as in-depth introductions to the latest concepts for graduate students. In fact, one of us (MB) described the type of article we desired to prospective authors as "the article that I would want a student to read before starting research, and one that the student would continue to use as a reference when writing a Ph.D. thesis."

The articles can no longer be called "short expository monographs" as was done in Volume 3. In order to emphasize the "expository" aspect we did not press the authors to limit the size of their contributions, so that what was originally planned as Volume 4 now appears as Volumes 4 and 5.

The first two chapters on laser technology cover two vastly different kinds of lasers which are both tunable primarily in the infra-red.

The first chapter gives a detailed presentation on the stimulated synchrotron emission from free electron lasers (FEL). It is balanced between theory, covering both quantum and classical FEL theory, and experiment, discussing work at a number of establishments including Stanford, Orsay, Frascati, Novosibirsk, Los Alamos, TRW, Mathematical Sciences-Boeing, Brookhaven and Livermore.

The second chapter, on color center lasers, by the co-inventor of the first CW, tunable color center laser, covers both the physics and engineering of these narrow linewidth devices and also discusses soliton laser action with pulses in the tens of femtoseconds.

The remaining three chapters cover laser applications that range from speculative to technological achievements that have created a thriving new product.

The third chapter, on the multioscillator ring laser gyroscope (RLG), is the survivor of what was meant to be three chapters on three different approaches to RLGs. For various reasons including fierce rivalry among a number of firms in the field leading to extra sensitivity on the part of legal departments, and military applications requiring appropriate clearances by federal agencies, two of the chapters were stillborn. The senior author of chapter 3 is to be admired for his perseverance in running this intimidating gauntlet as well as (with his colleagues) presenting us with an elegant theoretical discussion of one of the latest developments in this

triumph of the laser art—the RLG. They also describe a RLG which operates at the limit set by spontaneous emission noise and quantum-mechanical uncertainty.

The fourth and longest chapter, on nonlinear optical phase conjugation (NOPC), is in a way a reward for patience. A planned chapter in Volume 3 on adaptive optics using coherent optical adaptive techniques (COAT) fell through. Had it not, we might have felt constrained from including the related topic of NOPC and thus deprived the reader of this catholic treatment of what the author defines as real-time spatial and/or temporal information processing of electro-magnetic fields using nonlinear optical techniques. There are many useful and ingenious tables that help explain and relate the numerous developments in this field and tie them to the appropriate bibliography. The author covers NOPC phenomena with concentration on degenerate four-wave mixing and stimulated Brillouin scattering.

Although from a technology and engineering viewpoint NOPC is still in its infancy, more than 40% of the chapter is devoted to the topics of experimental demonstrations and selected applications of NOPC.

The last and shortest chapter is a treatment of an application whose potential is completely unrealized: bistable optical devices. Because as the author says these may be the “essential circuit elements for optical computing, communication and information processing” we decided to include this novel and interesting subject.

The authors are at leading institutions where the work described has been pursued. No particular significance should be attached to the fact that three of the institutions are industrial laboratories in the US, one a university in the US and one a national laboratory in Italy, other than they have individuals who do good work and it was convenient for the editors to deal with them.

For their aid and support in various phases of the preparation of this book we should like to thank: Dr. Pieter S.H. Bolman, Division Director of North-Holland Physics Publishing; Dr. Joost Kircz, Physics Editor; Ir. J. Soutberg, Desk Editor; our wives, Sharon Stitch and Judith Bass; and Drs. Willem H. Wimmers, retired from North-Holland. We also gratefully acknowledge the significant contribution of Dr. J.P. Stone, who prepared the Subject Index.

March, 1985

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# 1 Experimental and Theoretical Aspects of the Free-Electron Laser

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## Abstract

A review of the main problems relevant to the free-electron laser (FEL) is presented. The basic features of the spontaneous and stimulated emission mechanisms in undulator magnets are analysed. The Stanford oscillator experiment is described and the developed theoretical models are compared with the experimental results. The theory and experiments relevant to variable parameters FEL amplifiers and FEL storage ring devices are analysed in some detail. Finally a review is presented of FEL devices actually in progress.

## 1. Introduction

At the beginning of 1977 a new source of coherent radiation (Deacon et al. 1977) generated infrared light with wavelength  $\lambda = 3.417 \mu\text{m}$ , linewidth  $\Delta\lambda \approx 8 \text{ nm}$  and average power  $P \approx 0.36 \text{ W}$ . In that device, designed and realized by J.M.J. Madey and coworkers of Stanford University, the active medium was a beam of relativistic electrons. As a consequence of this very peculiar characteristic, this new radiation source was called "Free-Electron Laser" (FEL).

The feasibility of a laser device based on the emission of radiation from free electrons seems, at a first glance, not consistent with the fundamental physical laws. Indeed a free charge cannot emit or absorb electromagnetic radiation without violating the energy and linear momentum conservation principles. However, if a charge is travelling in an external field, the field itself absorbs part of the momentum and the emission process can take place. This external field can be generated by the bending magnetic system of a high energy electron accelerator [in this case we have synchrotron emission (Jackson 1975, p. 677)], or it may be the Coulomb field of an atomic nucleus [Bremsstrahlung (Jackson 1975, p. 708)] or the field of a free electromagnetic wave [Compton scattering (Jackson 1975, p. 679)]. Furthermore, this field can be generated by the charges induced on the surface of a metal diffraction grating [Smith-Purcell radiation (Smith and Purcell 1953)]. Finally a free charge can emit electromagnetic waves (Čerenkov light\*) if it is moving in a medium at a speed larger than the light velocity in that medium (which supports the momentum needed for fulfilling the conservation laws).

All these kinds of processes can be utilized, in principle, for the generation of "coherent" light. The Free-Electron Laser, which is based on "stimulated synchrotron emission", is just the last developed radiation source in the large family of the "free-electron" devices. The history of this class of electromagnetic (EM) wave generators started in the middle of the 1930s, with the development of the klystron (Slater 1963, p. 222), which allowed the production of high-power coherent centimetric EM waves. The design of such a device is due to W.W. Hansen and coworkers of Stanford University. They overcame the difficulties in developing high frequency tubes, arising from the fact that the dimensions of the triode become of the same order of magnitude as the radiation wavelength and that the electron flight time between cathode and anode is not negligible with respect to the EM oscillation period, by taking advantage just of these effects. Indeed, in a klystron the EM field is

\*For a comprehensive review, see Jelley (1958); analyses of stimulated Čerenkov radiation are provided by Gover and Yariv (1978) and Walsh (1982).

contained in conducting cavities, whose dimensions are of the order of the radiation wavelength. An electron beam (e-beam), accelerated by a d.c. voltage, is injected into the input cavity. The longitudinal component of a radio-frequency (r.f.) electric field produces a velocity modulation, which, after a drift space, is converted into density modulation (bunching). The bunched beam is successively injected into the output cavity, in which it excites an EM wave oscillation. If the klystron is operated as an amplifier, output and input cavities are not coupled. In the oscillator configuration part of the output r.f. signal is fed back to the input cavity in order to have regeneration. Furthermore, it is possible to realize a self-excited oscillation klystron by using a single cavity and by reflecting back the e-beam by using a negatively charged electrode (reflex klystron).

We have described in some detail the working principle of a klystron (velocity modulation  $\rightarrow$  bunching  $\rightarrow$  coherent emission) because, as we shall see later, it is similar — “*mutatis mutandis*” — to that of all other free-electron devices.

After the klystron, other microwave tubes were developed, like the magnetron (Slater 1963, p. 302) and the Travelling Wave Tube (TWT) (Slater 1963, p. 280). Let us consider this last device in more detail. In the TWT two elements allow the exchange of energy between electrons and r.f. field. Firstly we must slow down the EM wave phase velocity, by using a helix or a loaded wave guide, in such a way that the electrons, which are moving at the same speed as the r.f. field, remain in phase with the radiation. Secondly, the r.f. mode must have a longitudinal electric component (mode TM), which modulates the e-beam velocity and extracts energy from the bunched beam\*.

There is a practical limit in the minimum wavelength which is possible to generate with this kind of devices, connected with the system components (cavities, wave guides, etc.). There are two ways for overcoming these difficulties, which arise in working in the millimeter and submillimeter wavelength range. The first one is to change the geometry. Namely, if we cut a TWT along the medium plane we can have interaction between a beam running parallel to the one-side waveguide and the longitudinal electric field of the evanescent component of an EM wave trapped in the “cavity” made of the half waveguide (which can be simply a metallic diffraction grating) and a smooth reflecting mirror. This kind of device, called Orottron (Rusin and Bogomolov 1966, Mizuno et al. 1973, Wachtel 1979), in which the emission process is essentially a “stimulated Smith–Purcell effect”, can operate in the millimeter wavelength region with a cw power on the order of a fraction of watt. The other approach for generating short-wavelength radiation is to transfer the problems from the circuit to the e-beam. Namely, if we can make the e-beam periodic, rather than the circuit, it is possible to obtain synchronism between electrons and “fast” transverse electric (TE) EM waves, propagating along a smooth conducting wall waveguide at a phase velocity greater than the speed of light in the vacuum. In this connection the circuit construction problems are strongly reduced. Many “fast-wave”

\*Note that in a TWT, velocity modulation, bunching and emission are not separate steps as in the klystron, but continuously evolve during the interaction. We shall find this same feature in the FEL.

devices of this kind have been realized during the sixties and the seventies. The first one was the Ubitron (Phillips 1960) (Undulated Beam Interaction), in which the e-beam modulation is obtained with a special device, called Undulator Magnet (UM)\*, which generates a magnetic field with a spatial periodicity. The UM, which can be made of a periodic array of permanent magnets, forces the electrons to follow an undulated trajectory. In this configuration the electron velocity has a transverse component parallel to the electric field of the fast TE wave. In this way we can have exchange of energy (via velocity modulation and bunching) between electrons and EM field. Another device of this family is the Gyratron (Chow and Pantell 1962, Granatstein et al. 1975, Hirshfield and Granatstein 1977), in which the modulation is achieved by using a longitudinal magnetic field which forces the electrons to spiral at the cyclotron frequency. With this device it is possible, in the millimeter wavelength range, to generate many kW of cw radiation and several MW in the pulsed regime.

The extension of the tunability of free-electron devices beyond the millimeter region into the near infrared has been successfully obtained, as pointed out at the beginning of this Introduction, by Madey and collaborators of the Stanford University, with the "Free-Electron Laser", which can be considered as the relativistic version of the Ubitron.

The common feature of these three devices (Ubitron, Gyratron and FEL) is the presence of a magnetic field. From this point of view we can say that their working principle is based on "stimulated synchrotron radiation", or (which is equivalent) on "stimulated scattering of longitudinal photons (of a static magnetic field) into transverse ones". More generally, we can include in this class the "stimulated Compton scattering devices" (Pantell et al. 1968) and talk about:

"Stimulated scattering of photons (longitudinal or transverse) into transverse ones".

After the announcement of the first successful operation of the Stanford FEL a noticeable amount of work has been done in order to clarify the theoretical and experimental aspects of these devices. Namely, many conferences and schools have been devoted to FEL topics. First in Telluride, 1977, 1979 (Jacobs et al. 1978, 1980) then in Varenna, 1978 (Pellegrini 1981), Erice, 1980 (Martellucci and Chester 1983), and Sun Valley, 1981 (Jacobs et al. 1982), Bendor, 1982 (Deacon and Billardon 1983), Orcas Island, 1983 (Brau et al. 1984), Brookhaven, 1983 (Madey and Pellegrini 1984), Castelgandolfo, 1984 (Madey and Renieri 1985) and Como, 1984 (Bonifacio and Pellegrini 1985).

This chapter is mainly devoted to a review of the theoretical and experimental aspects of the "magnetic" FEL devices (like the Stanford one). For this reason we shall focus our attention on stimulated synchrotron radiation. However, for relativistic electrons, the physics of stimulated Compton scattering is very similar to that process, so that many results we found are valid for this case, too. Finally, let us

\*Some authors utilize the notation Wiggler Magnet.

point out that, inside the FEL device family, we can distinguish the two following different regimes:

(a) FEL Compton regime. In this configuration we can neglect the interaction between the electrons, and the process can be seen just as a single electron-photon scattering. For this reason it is called "Compton" regime also if we are dealing with a magnetic device (longitudinal photons). The operating wavelength can be very short (as we shall see in the following sections).

(b) FEL Raman regime. The interaction between the electrons cannot be neglected in this case. The EM wave excites a collective e-beam motion (plasma wave). The frequency of the emitted radiation is down-shifted with respect to the Compton case, because part of the energy is delivered to the plasma modes (Stokes wave). The analogy with the Raman effect is immediate (this is the reason of the name).

Strictly speaking, in this connection the electrons are not very "free". However, it is generally preferred to maintain the notation "FEL", which is related to the fact that the electrons are not bound in an atom or in a molecule. Furthermore, the operation of a Raman FEL at short wavelength is strongly limited by the Debye length in the plasma. Namely, for typical e-beam energy spread, the minimum wavelength is on the order of tenths of a millimeter.

The physics and the technology involved in Raman devices are very different from those for Compton ones, and, apart from very general considerations, it is not convenient to treat the two topics at the same time with the same formalism. In this article we shall deal with Compton devices, like the Stanford one. A collection of articles dedicated to the FEL Raman source can be found in the Proceedings edited by Jacobs et al. (1978, 1980) and by Martellucci and Chester (1983).

The plan of the paper is the following. In § 2 a brief survey is given of synchrotron radiation emission. §§ 3, 4 and 5 are dedicated to the analysis of spontaneous and stimulated synchrotron radiation emission in undulator magnets. In § 6 the Stanford device is described in some detail. §§ 7 and 8 are dedicated to the quantum and classical FEL theories. In § 9 we describe the main FEL devices in operation. Finally, § 10 is dedicated to conclusions and outlooks.

Table 1 lists the symbols used throughout the chapter. The cgs Gauss unit system is mainly utilized. However, in some places it was more practical to use other units (these are clearly indicated in the text).

## 2. Synchrotron radiation emission

Charged particles moving along a curved path emit synchrotron radiation. This phenomenon occurs spontaneously in nature. For example, electrons orbiting in galactic magnetic fields radiate visible light in a broad-band spectrum. The generation in the laboratory of this kind of radiation became possible during the second World War with the construction of the first high energy electron accelerators.

The history of the discovery of synchrotron radiation can be found, for example, in a letter of G.C. Baldwin (1975) to *Physics Today*, which was a direct witness of the first experiments concerning this effect. The radiation emission from accelerated

Table 1  
List of symbols used throughout this chapter

Physical constants	
$e$	electron charge
$m_0$	rest electron mass
$c$	velocity of light
$\hbar$	reduced Planck constant
$r_0$	$e^2/(m_0 c^2)$ classical electron radius
$\lambda_c$	$\hbar/(m_0 c)$ reduced Compton wavelength of the electron
$I_0 = ec/r_0$	Alfvén current
Undulator magnet parameters	
$\lambda_q$	spatial period
$L$	length
$N = L/\lambda_q$	number of periods
$B_0$	peak magnetic field
$\langle B^2 \rangle^{1/2}$	rms magnetic field $\begin{cases} = B_0/\sqrt{2} & \text{linear UM} \\ = B_0 & \text{helical UM} \end{cases}$
$K$	$e\langle B^2 \rangle^{1/2}\lambda_q/2\pi m_0 c^2$ undulator parameter
$h_{x,y}$	transverse magnetic field sextupolar terms
Electron beam parameters	
$E$	energy
$\gamma = E/m_0 c^2$	relativistic factor
$\hat{I}$	peak current
$\bar{I}$	average current
$\tau_b$	bunch length
$\sigma_e$	rms relative energy spread
$\sigma_u$	rms transverse dimension <sup>a</sup>
$\sigma_{u'}$	rms angular spread <sup>a</sup>
$E_u$	$2\pi\sigma_u\sigma_{u'}$ = rms emittance <sup>a</sup>
$\Sigma_E$	$2\pi\sigma_x\sigma_y$ = cross section
UM Radiation parameters	
$\lambda_0$	$\frac{\lambda_q}{2\gamma^2}(1+K^2)$ resonant wavelength
$\omega_0$	$2\pi c/\lambda_0$ resonant frequency
$\left(\frac{\Delta\omega}{\omega}\right)_0$	$\frac{\lambda_q}{2L} = \frac{1}{2N}$ homogeneous bandwidth
$\left(\frac{\Delta\omega}{\omega}\right)_e$	$2\sigma_e$ energy spread inhomogeneous bandwidth
$\left(\frac{\Delta\omega}{\omega}\right)_u$	$\sqrt{2 h_u } \frac{K}{1+K^2} \cdot \frac{\gamma E_u}{\lambda_q}$ emittance inhomogeneous bandwidth <sup>a</sup>
$\mu_i$	$= \left(\frac{\Delta\omega}{\omega}\right)_i / \left(\frac{\Delta\omega}{\omega}\right)_0$ normalized inhomogeneous bandwidth <sup>b</sup>
FEL parameters	
$\omega$	laser wavelength
$k$	$\omega/c$ laser wavenumber
$\nu$	$2\pi N(\omega_0 - \omega)/\omega_0$ normalized detuning from the resonant frequency
$\Sigma_L$	laser cross-section
$\mathcal{F}$	filling factor

<sup>a</sup>  $u = x, y$

<sup>b</sup>  $i = e, x, y$

charged particles was predicted long before experiments involving high energy electron accelerators were performed.

Namely, in 1897 Larmor derived the instantaneous total power  $P$  radiated by a single, nonrelativistic accelerated particle of charge  $e$  and mass  $m_0$ , in the framework of classical electrodynamics,

$$P = \frac{2}{3} \frac{e^2}{m_0^2 c^3} \left| \frac{d\mathbf{p}}{dt} \right|^2, \quad (2.1)$$

where  $c$  is the velocity of light and  $\mathbf{p}$  is the particle momentum. It is worthwhile to note that at the beginning of this century Schott (1907, 1912) attempted to explain the atomic spectra by using the generalization of eq. (2.1) for relativistic velocities which is due to Liénard (1898). Needless to say that the experimental data and, in particular, the discrete nature of the spectra were not accounted for by such a theory. Namely, it was necessary to await the Bohr theory of stationary energy states for the correct description of the atomic spectra.

The interest in this phenomenon was renewed, as said before, by the construction of the high energy electron accelerators. Indeed, in 1944 Iwanenko and Pomeranchuk, in searching for the maximum energy attainable in a Betatron (which is a circular induction accelerator; see, e.g. Bruck 1966) derived the relationship between the power  $P_0$  emitted by a relativistic charged particle moving in a magnetic field  $B$  and its energy  $E$ ,

$$P_0 = \frac{2}{3} \frac{e^4 c}{(m_0 c^2)^4} E^2 B^2. \quad (2.2)$$

From eq. (2.2) we see that the emitted power increases rapidly with the energy ( $E^2$ ), and decreases strongly with the mass ( $1/m_0^4$ ). As a consequence, synchrotron radiation is important only for high-energy electrons and positrons. However, recently a weak emission was observed from the 300 GeV proton beam in the SPS CERN Synchrotron in Geneva (Bossart et al. 1979). This light is now used as a proton beam diagnostic tool.

The spectral and directional properties of the synchrotron light were theoretically investigated by J. Schwinger (1946, 1949) and experimentally tested for the first time in 1948 (Elder et al. 1948) by analyzing the light emitted by the electrons in the 80 MeV General Electric Synchrotron.

A peculiar characteristic of this radiation is the very large emission band.

In fig. 1 we have plotted the spectral distribution as a function of  $\omega/\omega_c$ , where  $\omega$  is the emitted frequency and  $\omega_c$  is the so-called "critical frequency", which is given by

$$\omega_c = \frac{3}{2} \frac{c}{\rho} \left( \frac{E}{m_0 c^2} \right)^3, \quad (2.3)$$

where  $\rho$  is the trajectory bending radius,

$$\rho = E/eB. \quad (2.4)$$

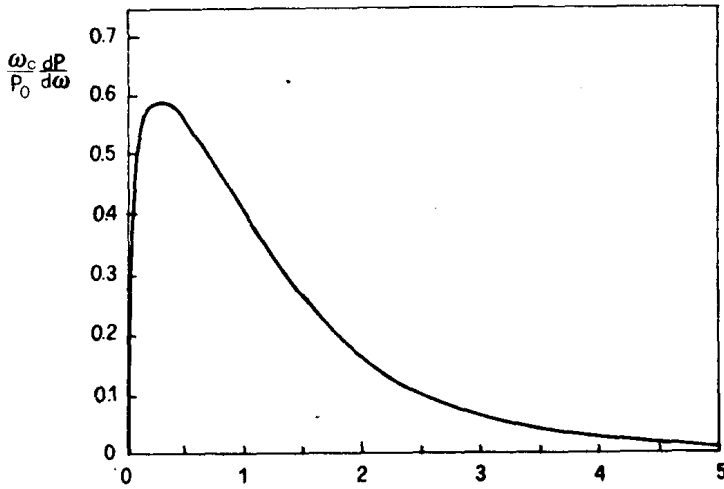


Fig. 1. Synchrotron radiation spectrum [eq. (2.5)].

The analytical expression for the spectrum displayed in fig. 1 was derived by Schwinger (1946, 1949):

$$\frac{dP}{d\omega} = \frac{P_0}{\omega_c} \frac{9\sqrt{3}}{8\pi} \frac{\omega}{\omega_c} \int_{\omega/\omega_c}^{\infty} K_{5/3}(u) du, \quad (2.5)$$

where  $K_{5/3}(u)$  is the modified Bessel function of order 5/3 (Kostroun 1980) and  $P_0$  is the total radiated power [eq. (2.2)].

From fig. 1 we see that there is radiation emission up to frequencies on the order of  $\omega_c$ . For highly relativistic electrons ( $E \gg m_0 c^2$ ) this frequency is very large (up to X or  $\gamma$  region of the spectrum), as can be derived from fig. 2, which shows  $\omega_c$  versus  $E$  for various bending magnetic fields.

The broadness of the spectral width (which is on the order of  $\omega_c$ , see fig. 1) is correlated with the shortness of the synchrotron light pulse. In order to clarify this point, in fig. 3 is sketched the geometry of the radiation emission for a particle moving in a circular orbit. The light is emitted around the motion direction in a cone of angle

$$\vartheta \sim m_0 c^2 / E \quad (2.6)$$

(relativistic angular contraction; Jackson 1975, p. 662). This angle, for highly relativistic electrons ( $E \gg m_0 c^2$ ), is very small. As a consequence, an observer placed on the orbit plane can see the light emitted by a small part  $l_e$  of the trajectory (see fig. 3), which is given by

$$l_e = 2\rho\vartheta. \quad (2.7)$$

The duration  $\delta t$  of the light pulse seen by the observer will be given by the difference



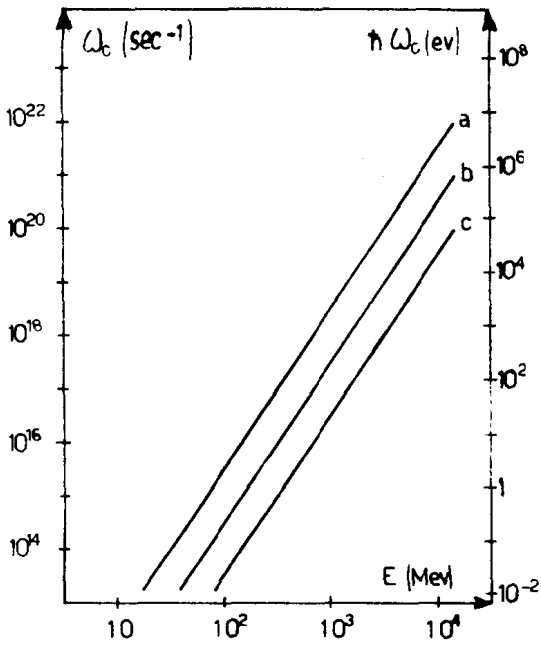


Fig. 2. Critical frequency versus electron energy [eq. (2.3)]: (a)  $\rho = 100$  m; (b)  $\rho = 10$  m; (c)  $\rho = 1$  m.

between the transit times of the electron along the arc  $l_e$  and the photons along the cord  $l_p$ , which is given by

$$l_p = 2\rho \sin \vartheta. \tag{2.8}$$

In conclusion, we have

$$\delta t = l_e/v - l_p/c, \tag{2.9}$$

where  $v$  is the particle velocity, which is related to the energy  $E$  by the well-known

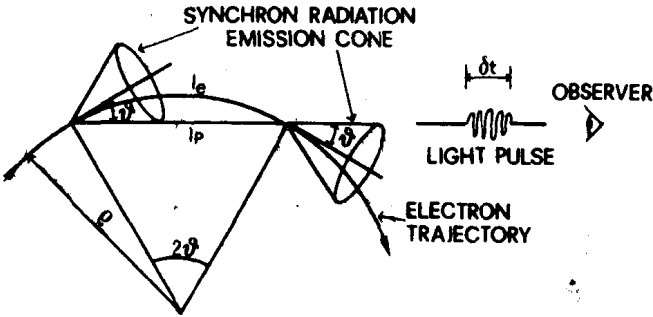


Fig. 3. Geometry of synchrotron radiation emission in constant magnetic field.