QUANTUM PHYSICS

ROLF G. WINTER

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College of William and Mary

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Preface

This book is a result of my thirty years' war in quantum physics. Through learning, teaching, and research, I have collected, mostly from the brains of others, ways of dealing with some quantal topics that seem to be useful to advanced undergraduates, and so I have gone down the usual route of increasingly voluminous handouts to publication.

My purpose is to introduce the basic structure and some applications of nonrelativistic quantum physics. The constraints of a course that I taught recently are reflected in the book. Most of my students had completed general physics in their first year and courses dealing with classical wave phenomena and qualitative modern physics in their second year. Only an elementary exposure to classical mechanics and electromagnetic theory is assumed. I have tried to select applications that both illustrate the general theory and also teach something about several fields of physics. Such an approach has intrinsic virtue, and my students having to choose their senior research area at the end of the year provided motivation. I have avoided topics that require much use of the phrase, "it can be shown that," and have concentrated on those that permit the calculation of something interesting from first principles. Formal matters are not stressed; starting the study of quantum physics is a hard business, and it is enough if the general structure of the theory begins to be absorbed osmotically.

Chapter 1 provides reminders of, but does not repeat, basic modern physics background, and it develops the Schrödinger equation. Chapter 2 solves the Schrödinger equation for some systems that can be handled in rectangular Cartesian coordinates. Chapter 3 contains the essential formal tools of the theory. I have tied it to the examples of Chapter 2, but the nature of the material guarantees that it will cause pain. The road then leads down from the glacial heights of Chapter 3 to pleasant and fertile valleys that both develop general methods and present applications in shorter chapters. The applications appear along with the methods. For example, calculations about the hydrogen atom, the deuteron, and quark-antiquark bound states are not concentrated in special sections, but are scattered throughout.

For a one-semester course I suggest either going straight through as far as time permits, or going through Section 3.8 without treating Sections 2.6, 2.9, and 3.4 in detail, and then making selections from later chapters.

For a two-semester course the entire book is either about right or somewhat long; there is a high density of things that should be chewed slowly and carefully. Some of the following sections might be omitted: 2.6, 3.4 except for the statement of the theorem (3.4-1), 3.9, 3.11, 5.2, 9.2, 9.3, 10.2, 10.4, 11.3, 12.3. These sections are more difficult than most, or a little off the main road of the argument, or both. Results of a few of these sections are mentioned subsequently, but it is not necessary to work through them to proceed.

At the risk of frightening some and insulting others, I will express the hope that occasionally this book will also be of use to graduate students, not for mathematical techniques, but for physical discussions. Not all advanced students are comfortable with questions about detecting particles in classically forbidden regions, about the meaning of the time-energy uncertainty principle, and so forth. I have put in some topics of this kind that seem to be considered, erroneously, too subtle for elementary courses and too trivial for advanced courses.

Books that try to be too self-contained, like overly solicitous parents, do a rotten job of raising the young, and numerous references to other texts and to journal articles are provided. It is important for undergraduates to learn that readable articles in *Physical Review Letters* do exist.

Most problems are imbedded in the text and should at least be read as they occur. To reduce the canonical I-don't-know-how-to-start-on-the-problems agony, there are worked-out examples, and many of the problems contain suggestions and parts of the answers. The long Chapters 1, 2, and 3 have additional problems at their ends to provide options and inspiration for review.

The notation is generally standard, and rococo symbols are avoided. A consequence is that sometimes the same symbol recurs with different uses in different places. The danger of confusion is negligible and the alternative of unfamiliar or highly decorated symbols can make easy things look hard.

Footnotes are numbered sequentially in each chapter. Equations and expressions that need labels are numbered sequentially in each section. Thus (3.2-1) is the first equation of the second section of the third chapter. Within each section, only the sequential number is used for reference. For example, in Section 3.2 the first equation is just called (1), while it is called by its full name (3.2-1) whenever it is referred to in another section.

I should like to thank those who helped me with this book. There are ways of looking at the subject that I trace to courses I took long ago, particularly with H. C. Corben, S. DeBenedetti, and F. Seitz. The William and Mary physics department has lots of people who know lots of things, and who like to talk; many of my colleagues made important contributions. H. C. von Baeyer and A. Sher were especially helpful and suggested many improvements. The students who used the preliminary versions took a lively interest in the project, caught many mistakes, and taught me how to explain things better. Mrs. S. Stout was indispensable while the manuscript was being prepared. Her ability to divine what I had meant to write was uncanny. The staff and the reviewers of Wadsworth Publishing Company deserve much of the credit for the final result. The following people reviewed the manuscript: Ronald A. Brown, SUNY, Oswego; Roland H. Good, Pennsylvania State University; B. J. Graham, U.S. Naval Academy; Arthur Hobson, University of Arkansas; Larry D. Johnson, Northeast Louisiana University; Mortimer N. Moore, California State University at Northridge; Vector J. Stenger, University of Hawaii, Manoa Campus; James V. Tucci, University of Bridgeport; Michael W. Webb, Alfred University.

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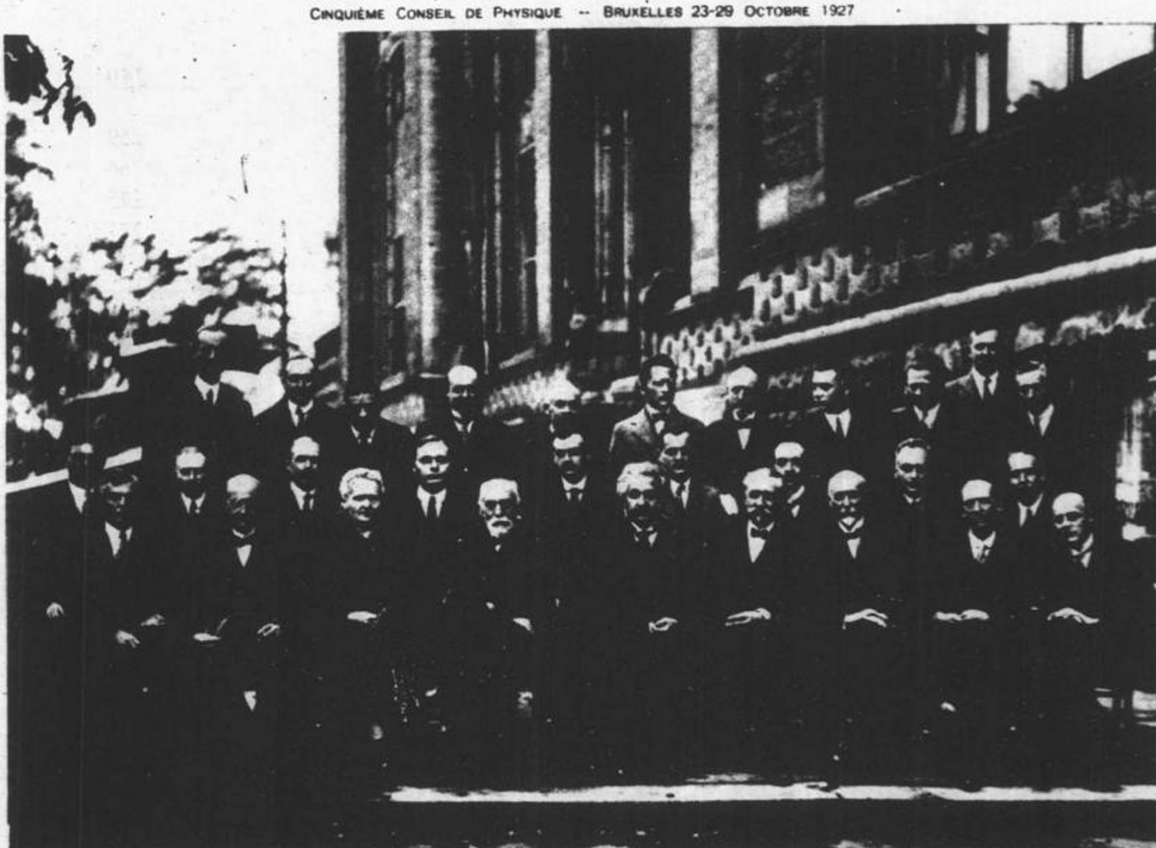
INSTITUT INTERNATIONAL DE PHYSIQUE SOLVAY

CINQUIÈME CONSEIL DE PHYSIQUE — BRUXELLES 23-29 OCTOBRE 1927



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The fifth Solvay Conference on Physics, Brussels, October 1927. Every few years, the Solvay Conferences bring together small groups of physicists to discuss major unsolved problems. The 1927 meeting was pivotal in clarifying the content of the new quantum physics. The history of the conferences is described by Jagdish Mehra, The Solvay Conferences on Physics *(Dortrecht, Holland: R. Reidel Publishing Company, 1975). [Reproduced by permission of Instituts Internationaux de Physique et de Chimie (Solvay), Brussels, Belgium]*

CONVERSION FACTORS AND PHYSICAL CONSTANTS

See *Rev. Mod. Phys.* 48, 51 (1976) for more accurate values and uncertainties.

Angström = Å = 10^{-8} cm = 10^{-10} m

Fermi or fentometer = f = 10^{-13} cm = 10^{-15} m

Elementary charge = e = 1.6022×10^{-19} coulomb = 4.8032×10^{-10} esu

Electron volt = eV = 10^{-6} MeV = 1.6022×10^{-19} joule = 1.6022×10^{-12} erg

Avogadro's number = 6.0221×10^{23}/mole

Speed of light = $c = 2.9979 \times 10^8$ m/sec

Atomic mass unit = $M(C^{12})/12 = 1.6606 \times 10^{-27}$ kg = 931.50 MeV/c^2

Planck's constant = $h = 6.6262 \times 10^{-34}$ joule sec

$\hbar = h/2\pi = 1.0546 \times 10^{-34}$ joule sec = 6.5822×10^{-16} eV sec

$\hbar c = 1.9733 \times 10^{-5}$ eV cm

Boltzmann's constant = $k_B = 1.3807 \times 10^{-23}$ joule/°K

$= 8.6174 \times 10^{-5}$ eV/°K

Electron e^-: mass = $M_e = 9.1095 \times 10^{-31}$ kg = 0.51100 MeV/c^2,
spin = $\frac{1}{2}$

Proton p^+: mass = 938.28 MeV/c^2 = 1836.15 M_e, spin = $\frac{1}{2}$

Neutron n^0: mass = 939.57 MeV/c^2, spin = $\frac{1}{2}$, mean life τ = 918 sec

Deuteron d^+: mass = 1875.63 MeV/c^2, spin = 1

Muon $\mu^\pm$: mass = 105.66 MeV/c^2, spin = $\frac{1}{2}$, $\tau = 2.20 \times 10^{-6}$ sec

Charged pion $\pi^\pm$: mass = 139.57 MeV/c^2, spin = 0, $\tau = 2.60 \times 10^{-8}$ sec

Neutral pion π^0: mass = 134.96 MeV/c^2, spin = 0, $\tau = 8. \times 10^{-17}$ sec

Fine structure constant = $\alpha = \dfrac{e^2}{(4\pi\varepsilon_0)\hbar c} = \dfrac{1}{137.036}$

Bohr radius = $a = (4\pi\varepsilon_0)\hbar^2/M_e e^2 = 0.52918$ Å

$\lambda\!\!\!^{-}_e = \hbar/M_e c = \alpha a = 386.16$ f

$r_e = e^2/(4\pi\varepsilon_0 M_e c^2) = \alpha \lambda\!\!\!^{-}_e = \alpha^2 a = 2.8179$ f

Rydberg energy = $M_e e^4/2\hbar^2(4\pi\varepsilon_0)^2 = M_e c^2\alpha^2/2 = 13.606$ eV

Coulomb's law: $F = q_1 q_2/4\pi\varepsilon_0 r^2$, $\varepsilon_0 = 8.8542 \times 10^{-12}$ coul^2/nt-m^2

Magnetic field at center of circular loop: $B = \mu_0 I/2r$,
$\mu_0 = 4\pi \times 10^{-7}$ webers/amp-m = $(\varepsilon_0 c^2)^{-1}$, 1 weber/m^2 = 10^4 gauss

VARIOUS RESULTS WITH REFERENCES TO THE SECTIONS WHERE THEY ARE DEVELOPED

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2M(E - V)}}, \qquad E = \hbar\omega \tag{1.2}$$

$$H\Psi = i\hbar\frac{\partial\Psi}{\partial t}, \qquad H = -\frac{\hbar^2}{2M}\nabla^2 + V \tag{1.4}$$

$$\rho = |\Psi|^2, \qquad \vec{j} = \frac{\hbar}{2Mi}[\Psi^*(\vec{\nabla}\Psi) - (\vec{\nabla}\Psi^*)\Psi] \tag{1.5}$$

$$\text{If } \Psi(\vec{r}, t) = \psi(\vec{r})e^{-iEt/\hbar}, \qquad H\psi = E\psi \tag{2.1}$$

$$a \times b \times c \text{ box, } \infty \text{ walls: } E_{n_x n_y n_z} = \frac{\pi^2\hbar^2}{2M}\left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2}\right),$$

$$\psi = \sqrt{\frac{8}{abc}}\sin\left(\frac{\pi n_x x}{a}\right)\sin\left(\frac{\pi n_y y}{b}\right)\sin\left(\frac{\pi n_z z}{c}\right) \tag{2.4}$$

$$V = \frac{Kx^2}{2}: \; E_n = (n + \tfrac{1}{2})\hbar\sqrt{\frac{K}{M}}, \qquad \beta^2 = \frac{\sqrt{KM}}{\hbar},$$

$$\psi_0 = \sqrt{\frac{\beta}{\sqrt{\pi}}}e^{-\beta^2x^2/2}, \qquad \psi_1 = \sqrt{\frac{\beta}{2\sqrt{\pi}}}2\beta x e^{-\beta^2x^2/2} \tag{2.7}$$

$$V = gx, x \geq 0; V = \infty, x < 0: E_n = |\xi_n|\left(\frac{g^2\hbar^2}{2M}\right)^{1/3}, \qquad Ai(\xi_n) = 0,$$

$$\xi_1 = -2.3381, \qquad \xi_2 = -4.0879, \qquad \xi_3 = -5.5206 \tag{2.9}$$

$$\langle\Omega\rangle = \int\Psi^*\Omega\Psi\,d\tau, \qquad \langle\vec{p}\rangle = \int\Psi^*\frac{\hbar}{i}\vec{\nabla}\Psi\,d\tau \tag{3.2}$$

$$\text{If } \langle\Omega\rangle = \langle\Omega\rangle^* \text{ for all } \Psi, \int\Psi_1^*(\Omega\Psi_2)\,d\tau = \int(\Omega\Psi_1)^*\Psi_2\,d\tau \tag{3.3}$$

$$xp_x - p_x x = [x, p_x] = i\hbar \tag{3.3}$$

$$\text{Uncertainty principle: } \Delta P\Delta Q \geq \tfrac{1}{2}|\langle i[P, Q]\rangle| \tag{3.4}$$

$$\Psi = \sum_n C_n\Psi_n, \qquad C_n = \int\Psi_n^*\Psi\,d\tau \tag{3.6}$$

$$\sum_n |C_n|^2 = 1, \qquad \langle\Omega\rangle = \sum_n |C_n|^2\omega_n \quad \text{if} \quad \Omega\Psi_n = \omega_n\Psi_n \tag{3.7}$$

$$\Psi_n \rightarrow |n\rangle, \qquad \Psi_m^* \rightarrow \langle m|, \qquad \int\Psi_m^*\Omega\Psi_n\,d\tau \rightarrow \langle m|\Omega|n\rangle \tag{3.8}$$

$$\frac{d}{dt}\langle\Omega\rangle = \left\langle\frac{i}{\hbar}[H, \Omega]\right\rangle + \left\langle\frac{\partial\Omega}{\partial t}\right\rangle \tag{3.10}$$

(continued inside back cover)

Reduced mass $= \mu = \dfrac{M_1 M_2}{M_1 + M_2}$ (4.1)

$\Psi(2, 1, t) = +\Psi(1, 2, t)$ for bosons, $= -\Psi(1, 2, t)$ for fermions (4.2)

Fermi energy $= \dfrac{\hbar^2}{2M}(3\pi^2 N)^{2/3}$, N = no. spin $\frac{1}{2}$ particles/vol (5.1)

$[J_x, J_y] = i\hbar J_z, \quad [J_z, J_x] = i\hbar J_y, \quad [J_y, J_z] = i\hbar J_x$ (6.1)

$J_\pm = J_x \pm iJ_y, \quad J_\pm \psi_{jm} = N_{jm}^\pm \psi_{jm\pm 1}, \; N_{jm}^\pm = \hbar\sqrt{j(j+1) - m(m \pm 1)}$ (6.1)

$L^2 Y_{lm} = \hbar^2 l(l+1) Y_{lm}, \quad L_z Y_{lm} = \hbar m Y_{lm},$

$$Y_{00} = \frac{1}{\sqrt{4\pi}}, \quad Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta, \quad Y_{1\pm1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\varphi},$$

$$Y_{20} = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1), \quad Y_{2\pm1} = \mp\sqrt{\frac{15}{8\pi}}\cos\theta\sin\theta e^{\pm i\varphi},$$

$$Y_{2\pm2} = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{\pm 2i\varphi} \tag{6.2}$$

$$u(r) = rR(r): \; \frac{-\hbar^2}{2\mu}\frac{d^2u}{dr^2} + \left[V(r) + \frac{\hbar^2 l(l+1)}{2\mu r^2}\right]u = Eu,$$

$$V(r) \sim r^q, \quad q > -2: R(r) \sim r^l \tag{7.1}$$

$$V = \frac{e^2}{4\pi\varepsilon_0 r}: \; E_n = \frac{-\mu e^4}{2\hbar^2(4\pi\varepsilon_0)^2 n^2}, \quad a = \frac{(4\pi\varepsilon_0)\hbar^2}{\mu e^2},$$

$$\psi_{100} = \frac{2e^{-r/a}}{a^{3/2}} Y_{00}, \quad \psi_{200} = \frac{1}{\sqrt{2a^3}}\left(1 - \frac{r}{2a}\right)e^{-r/2a} Y_{00},$$

$$\psi_{21m} = \frac{1}{\sqrt{24a^3}}\frac{r}{a}e^{-r/2a} Y_{1m} \tag{7.3}$$

$$\chi_+ = \begin{pmatrix}1\\0\end{pmatrix}, \quad \chi_- = \begin{pmatrix}0\\1\end{pmatrix}, \quad \vec{s} = \frac{\hbar}{2}\vec{\sigma},$$

$$\sigma_x = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}, \quad \sigma_y = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix}, \quad \sigma_z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix} \tag{8.2}$$

$$\zeta_{00}(1,2) = \frac{1}{\sqrt{2}}[\chi_+(1)\chi_-(2) - \chi_-(1)\chi_+(2)], \quad \zeta_{1+1}(1,2) = \chi_+(1)\chi_+(2)$$

$$\zeta_{10}(1,2) = \frac{1}{\sqrt{2}}[\chi_+(1)\chi_-(2) + \chi_-(1)\chi_+(2)], \quad \zeta_{1-1}(1,2) = \chi_-(1)\chi_-(2) \tag{9.1}$$

$$E_n = E_{0n} + \langle n|H'|n\rangle + \sum_{m\neq n} \frac{\langle n|H'|m\rangle\langle m|H'|n\rangle}{E_{0n} - E_{0m}} + \cdots \tag{10.1}$$

Scattering: $\psi \to A\left[e^{ikz} + \frac{f(\theta,\varphi)e^{ikr}}{r}\right], \quad \frac{d\sigma}{d\Omega}(\theta,\varphi) = |f(\theta,\varphi)|^2$ (11.1)

Born approximation: $f(\theta) = \frac{-2\mu}{\hbar^2 K}\int_0^\infty V(r)\sin(Kr)r\,dr,$

$K = 2k\sin(\theta/2)$ (11.2)

Electric dipole radiation: $\frac{1}{\tau} = \frac{1}{137.04}\frac{4\omega_{12}^3}{3c^2}|\langle 1|\vec{r}|2\rangle|^2$ (12.2)

Golden Rule: $W = \frac{2\pi}{\hbar}\rho(E_f)|H'_{fi}|^2$ (12.3)

Contents

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Exhortations

The study of quantum physics leads one to pull together, sort, and clarify much of the physics that one has learned before, and it will pay to do a little reviewing at this time. Make sure that the basic ingredients of Newtonian mechanics are clear; many students discover in a quantum course that their classical concepts of kinetic, potential, and total energy are a little fuzzy. Review particularly your introduction to "modern physics" as in

K. W. Ford, *Classical and Modern Physics* (Lexington, Mass.: Xerox College Publishing, 1972), Part VII

or similar books. We shall recapitulate many of the arguments there, but look now at the experiments and observations that led to the development of quantum ideas. Be sure that you have a feeling for magnitudes. What is the size of atoms? Of nuclei? What is the thermal energy at room temperature? At liquid helium temperature? What are the characteristic energies of chemistry, of nuclear physics, and of particle physics? What are the "classical radius of the electron," the Compton wavelength, the Bohr radius, and the fine structure constant?

Look over introductions to wave phenomena in a general physics book and in slightly more specialized texts such as

F. S. Crawford, *Waves, Berkeley Physics Course*, Vol. 3
(New York: McGraw-Hill Book Company, 1968),
A. P. French, *Vibrations and Waves*
(New York: W. W. Norton & Co., Inc., 1971),
J. R. Pierce, *Almost All About Waves*
(Cambridge, Mass: M.I.T. Press, 1974).

The basic nomenclature, and topics such as interference phenomena, the motion of wave packets, and so forth depend rather little on what is waving. If you know something about sound or other classical waves, you already know much of what follows.

As we proceed, keep on reviewing. A glance at previous encounters with our topics may prevent your getting lost in the underbrush. For example, the treatment of angular momentum will involve some serious mathematics, but many of the conclusions are described and partly justified in basic "modern physics" books.

Look at other quantum texts and find at least one that you will use regularly as collateral reading. The following range from a little more elementary to a little more advanced, depending on the particular topic, than this book:

E. E. Anderson, *Modern Physics and Quantum Mechanics*
(Philadelphia: W. B. Saunders Co., 1971),
R. Eisberg and R. Resnick, *Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles*
(New York: John Wiley & Sons, Inc., 1974),
R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics*, Vol. III, *Quantum Mechanics*
(Reading, Mass.: Addison-Wesley Publishing Co., Inc., 1965),
A. P. French and E. F. Taylor, *An Introduction to Quantum Physics*
(New York: W. W. Norton & Co., Inc., 1978),
S. Gasiorowicz, *Quantum Physics*
(New York: John Wiley & Sons, Inc., 1974),
D. Park, *Introduction to the Quantum Theory*, 2d ed.
(New York: McGraw-Hill Book Company, 1974),
J. L. Powell and B. Crasemann, *Quantum Mechanics*
(Reading, Mass.: Addison-Wesley Publishing Co., Inc., 1961),
D. S. Saxon, *Elementary Quantum Mechanics*
(San Francisco: Holden-Day, Inc., 1968),
E. H. Wichman, *Quantum Physics, Berkeley Physics Course*, Vol. 4
(New York: McGraw-Hill Book Company, 1971),
K. Ziock, *Basic Quantum Mechanics*
(New York: John Wiley & Sons, Inc., 1969).

The easier parts of more advanced texts will be within your reach soon, and they are useful for extending some of our discussions. Such texts include

E. Merzbacher, *Quantum Mechanics*, 2d ed.
(New York: John Wiley & Sons, Inc., 1970),
L. I. Schiff, *Quantum Mechanics*, 3d ed.
(New York: McGraw-Hill Book Company, 1968),
A. Messiah, *Quantum Mechanics*, trans. G. M. Temmer
(Amsterdam: North-Holland Publishing Company, 1962), particularly Vol. 1.

Keep reminding yourself that quantum physics deals with real things. Many of the experiments done in advanced undergraduate laboratories involve our topics. Look at books designed to help with the laboratories, particularly

A. C. Melissinos, *Experiments in Modern Physics*
(New York: Academic Press, Inc., 1966);

the discussion of the underlying physics there complements our approach. Find out about research being done around you and try to connect it with your studies.

A knowledge of only the customary ingredients of freshman and sophomore mathematics is assumed. Through a cavalier nonrigorous use of physicists' mathematics, as distinguished from mathematicians' mathematics, specialized topics are developed as needed. These topics include Fourier series, Fourier integrals, tricks for dealing with differential equations, a few messy integrals, and some properties of matrices. If you find that the material is mathematically demanding, think about the reason. I believe that the difficulty has nothing to do with the particular techniques used, but comes from the fact that in the past you have been able to complete most calculations in five lines. Now you will have to do computations involving some pages of algebra, and the hard new thing that must be learned is calculational craftsmanship. You will need to organize your work so that you and others can verify it, to give short names to recurring long expressions, to check units and physical reasonableness at each step, and to reach for tables of formulas at the right moment, but these are obvious details. The key to it all is the realization that calculating, like carpentry, is indeed a craft, with tricks and habits that make the difference between success and failure.

Be critical. Keep asking yourself whether you really understand, and how you would explain it all to someone else. Devise better ways to present this material and make up other examples, for two reasons. First, that's the way to learn this kind of thing. Second, some of you will be writing physics books in a few years, and you should start practicing now.