Boundary-Layer Theory

BY

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Foreword

To the First English Edition

Boundary-layer theory is the cornerstone of our knowledge of the flow of air and other fluids of small viscosity under circumstances of interest in many engineering applications. Thus many complex problems in aerodynamics have been clarified by a study of the flow within the boundary layer and its effect on the general flow around the body. Such problems include the variations of minimum drag and maximum lift of airplane wings with Reynolds number, wind-tunnel turbulence, and other parameters. Even in those cases where a complete mathematical analysis is at present impracticable, the boundary-layer concept has been extraordinarily fruitful and useful.

The development of boundary-layer theory during its first fifty years is a fascinating illustration of the birth of a new concept, its slow growth for many years in the hands of its creator and his associates, its belated acceptance by others, and the subsequent almost exponential rise in the number of contributors to its further development.

The first decade following the classical paper of Prandtl in 1904 brought forth fewer than 10 papers by Prandtl and his students, a rate of about one paper per year. During the past year over 100 papers were published on various aspects of boundary-layer theory and related experiments. The name of H. Schlichting first appears in 1930 with his doctoral thesis on the subject of wake flow. Shortly thereafter Schlichting devoted major effort to the problem of the stability of laminar boundary-layer flow.

My own interest in the experimental aspects of boundary-layer flow began in the late twenties. With the appearance of Schlichting's papers intensive attempts were made to find the amplified disturbances predicted by the theory. For 10 years the experimental results not only failed to confirm this theory but supported the idea that transition resulted from the presence of turbulence in the free air stream as described in a theory set forth by G. I. Taylor. Then on a well-remembered day in August, 1940, the predicted waves were seen in the flow near a flat plate in a wind tunnel of very low turbulence. The theory of stability described in the papers of Tollmien and Schlichting was soon confirmed quantitatively as well as qualitatively.

German periodicals available in the United States after the war referred to a series of lectures by Schlichting on boundary-layer theory which had been published in 1942. This document of 279 pages with 116 figures was not available for some time. An English translation was given limited distribution as NACA Technical Memorandum No. 1217 in 1949. These lectures were completely rewritten to include material previously classified, confidential, or secret from Germany and other countries.

The result was the book of 483 pages and 295 figures published in 1951 in the German language. When this book became known to research workers and educators in the United States, there was an immediate request from several quarters for an English translation, since no comparable book was available in the English language.

The technical content of the present English edition is described in the author's preface. The emphasis is on the fundamental physical ideas rather than on mathematical refinement. Methods of theoretical analysis are set forth along with such experimental data as are pertinent to define the regions of applicability of the theoretical results or to give physical insight into the phenomena.

Aeronautical engineers and research scientists owe a debt of gratitude to Professor Schlichting for this timely review of the present state of boundary-layer theory.

Washington D. C., December 13, 1954 Hugh L. Dryden

Author's Preface to the Sixth (English) Edition

The first English edition of this book appeared in 1955 and the second in 1960. The latter was not a literal translation of the previous German edition of 1958, but contained a considerable amount of new material which had appeared up to 1960. Both English editions were very well received, particularly in the English-speaking countries, and there is little doubt that Dr. J. Kestin's success with the translation contributed to it in no small measure.

It could be foreseen in 1963 that the German edition of 1958 would become exhausted by the end of 1964. Therefore, in 1963 I decided to prepare a thoroughly revised German edition which appeared subsequently early in 1965. During the seven years elapsed between 1958 and 1965, the field of boundary-layer research continued to grow at a rapid pace. I have tried to include all important new developments and thus to bring the book up to date once more.

The two Publishers, Messrs. McGraw-Hill in New York and Messrs. G. Braun in Karlsruhe, once again suggested that a new English edition should be prepared in order to encompass these new developments, and I gladly accepted their proposals.

I owe a great debt of gratitude to Professor J. Kestin who once more undertook the arduous task of preparing the translation. The present English edition is a more-or-less literal translation of the fifth (German) edition of 1965.

I hope that this new English edition will again be found useful by engineers working in industry and in research establishments and by students of mechanical engineering and machine design as well as by physicists and chemists who have to deal with problems of fluid flow.

Goettingen, August 1966 H. Schlichting

Author's Preface to the Fifth (German) Edition

Since the publication of the third (German) edition of this book in 1958, the field of boundary layer research has again expanded enormously. The English translation which was prepared by Professor J. Kestin and published in 1960 by the McGraw-Hill Book Company of New York was largely based on the third (German) edition of 1958. Since it can be expected that the latest German edition as well as the new English edition will become exhausted in a short period of time, I have decided to prepare a thoroughly revised version.

As in previous editions, I have attempted to include in the book the most important advances from among the many which have been made in the meantime in the general field of boundary-layer research, without, however, altering its basic structure. I trust that the principal aim of the book with its emphasis on the presentation of theoretical considerations in a form which is accessible to practising engineers has once again been achieved. The subdivision of the book into four parts (A. Fundamental laws of motion for a viscous fluid; B. Laminar boundary layers; C. Transition; D. Turbulent boundary layers) has been retained unchanged. However, the ordering of chapters within the part on laminar boundary layers has been modified in order to achieve greater clarity. This necessitated the displacement of the chapters on non-steady boundary layers and on boundary-layer control towards the end of this part. The part on turbulent boundary layers was extended by an inclusion of a new chapter, thus increasing the extent of the book to a total of 25 chapters.

As far as *additional material* is concerned, I should like to make special mention of some of it here. In the sections on laminar boundary layers much new material has been added to those dealing with axially symmetrical and three-dimensional boundary layers as well as with non-steady boundary layers. The chapter on thermal boundary layers (Chap. XII) has been expanded to a particularly large extent, and the same applies to the one on laminar boundary layers in compressible flow (Chap. XIII). In the part on transition, I have once more extended the chapter on the applications of the stability theory (Chap. XVII). The considerations regarding the effect of compressibility on turbulent boundary layers have been augmented to such an extent that it became convenient to encompass them all in a separate, new chapter (Chap. XXIII).

Along with this new material, I feel that I ought to mention the topics which I specifically omitted to include. I do not discuss the effect of chemical reactions on flow processes in boundary layers as they occur in the presence of hypersonic flow, because it appears to me that this field is still in a state of flux. The same applies to boundary layers in magneto-fluid-dynamics. I still thought that I ought to refrain from giving an exposition of the statistical theory of turbulence in this new edition, as in the previous ones, because nowadays there are available other, good presentations in book form.

Once again, the lists of references have been expanded considerably in several chapters. The number of illustrations increased by about 30, and the number of pages increased by 130, to use round figures. In spite of this, I hope that the original

character of the book has been retained, and that it still can provide the reader with a *bird's-eye view* of this important branch of the physics of fluids.

It was my good fortune that I enjoyed the effective assistance of several of my professional colleagues during the work on the manuscript. In view of my other duties, it would have been impossible for me to complete this new edition without their help. Professor K. Gersten thoroughly re-worked Chapter XII (Thermal boundary layers in laminar flow) and XV (Non-steady boundary layers) and enriched them with many new additions. Dr. F. Riegels inserted fresh parts into Chapter XIII (Boundary layers in compressible flow), particularly those dealing with the Illingworth-Stewartson transformation. Mr. J. C. Rotta carefully revised the section on turbulent boundary layers, provided numerous supplements, the new Chapter XXIII on turbulent boundary layers in compressible flow being the most important among them. Mr. Steinheuer and Dr. Pechau helped with the revision of several chapters in Parts A and B. The valuable assistance received from all those gentlemen is hereby gratefully acknowledged.

I express my thanks to Messrs. Rotta, Steinheuer and Tanner as well as to my daughter Heike for their help with the reading of proofs and to my secretaries, the Misses Ursula Scheede and Christa Behrens, for the careful preparation of the clean copy of the typescript. Last, but not least, thanks are due to Verlag Braun for their willingness to accede to my wishes and for the pleasing appearance of the book.

Goettingen, October 1964 H. Schlichting

Translator's Preface to the Sixth (English) Edition

The present is the third edition in the English language of Professor H. Schlichting's "Grenzschicht-Theorie". Once again, the new edition was prepared in close cooperation with the Author whom I visited in Goettingen and in Braunschweig in order to finalize the contents and the wording. I wish to thank Professor Schlichting for his hospitality and Messrs. McGraw-Hill for having made this visit possible.

This time the English version differs from its German predecessor only in matters of small detail, except for Chapter III on the derivation of the Navier-Stokes equations which has been completely revised.

I owe a deep debt of gratitude to Mr. J. Steinheuer of the Deutsche Forschungsanstalt fuer Luft- und Raumfahrt of Braunschweig who twice meticulously read the proofs and eradicated many mistakes. In the task of proof-reading I was also helped by Professor B. Caswell of Brown University. Last, but not least, I express my appreciation to Frl. Lühr, Frau Höbbelmann, and Miss Kortum for the assistance they rendered in typing and checking the manuscript. Both publishers, Messrs. G. Braun of Karlsruhe and Messrs. McGraw-Hill of New York, spared no trouble in meeting our wishes regarding the production of the book.

Braunschweig, August 1966. J. Kestin

From Author's Preface to the First (German) Edition

Since about the beginning of the current century modern research in the field of fluid dynamics has achieved great successes and has been able to provide a theoretical clarification of observed phenomena which the science of classical hydrodynamics of the preceding century failed to do. Essentially three branches of fluid dynamics have become particularly well developed during the last fifty years; they include boundary-layer theory, gas dynamics, and aerofoil theory. The present book is concerned with the branch known as boundary-layer theory. This is the oldest branch of modern fluid dynamics; it was founded by L. Prandtl in 1904 when he succeeded in showing how flows involving fluids of very small viscosity, in particular, water and air, the most important ones from the point of view of applications, can be made amenable to mathematical analysis. This was achieved by taking the effects of friction into account only in regions where they are essential, namely in the thin boundary layer which exists in the immediate neighbourhood of a solid body. This concept made it possible to clarify many phenomena which occur in flows and which had previously been incomprehensible. Most important of all, it has become possible to subject problems connected with the occurrence of drag to a theoretical analysis. The science of aeronautical engineering was making rapid progress and was soon able to utilize these theoretical results in practical applications. It did, furthermore, pose many problems which could be solved with the aid of the new boundary-layer theory. Aeronautical engineers have long since made the concept of a boundary layer one of everyday use and it is now unthinkable to do without it. In other fields of machine design in which problems of flow occur, in particular in the design of turbomachinery, the theory of boundary layers made much slower progress, but in modern times these new concepts have come to the fore in such applications as well.

The present book has been written principally for engineers. It is the outcome of a course of lectures which the Author delivered in the Winter Semester of 1941/42 for the scientific workers of the Aeronautical Research Institute in Braunschweig. The subject matter has been utilized after the war in many special lectures held at the Engineering University in Braunschweig for students of mechanical engineering and physics. Dr. H. Hahnemann prepared a set of lecture notes after the first series of lectures had been given. These were read and amplified by the Author. They were subsequently published in mimeographed form by the Office for Scientific Documentation (Zentrale für wissenschaftliches Berichtswesen) and distributed to a limited circle of interested scientific workers.

Several years after the war the author decided completely to re-edit this older compilation and to publish it in the form of a book. The time seemed particularly propitious because it appeared ripe for the publication of a comprehensive book, and because the results of the research work carried out during the last ten to twenty years rounded off the whole field.

The book is divided into four main parts. The first part contains two introductory chapters in which the fundamentals of boundary-layer theory are expounded without the use of mathematics and then proceeds to prepare the mathematical and physical justification for the theory of laminar boundary layers, and includes the theory of thermal boundary layers. The third part is concerned with the pheno-

menon of transition from laminar to turbulent flow (origin of turbulence), and the fourth part is devoted to turbulent flows. It is now possible to take the view that the theory of laminar boundary layers is complete in its main outline. The physical relations have been completely clarified; the methods of calculation have been largely worked out and have, in many cases, been simplified to such an extent that they should present no difficulties to engineers. In discussing turbulent flows use has been made essentially only of the semi-empirical theories which derive from Prandtl's mixing length. It is true that according to present views these theories possess a number of shortcomings but nothing superior has so far been devised to take their place, nothing, that is, which is useful to the engineer. No account of the statistical theories of turbulence has been included because they have not yet attained any practical significance for engineers.

As intimated in the title, the emphasis has been laid on the theoretical treatment of problems. An attempt has been made to bring these considerations into a form which can be easily grasped by engineers. Only a small number of results has been quoted from among the very voluminous experimental material. They have been chosen for their suitability to give a clear, physical insight into the phenomena and to provide direct verification of the theory presented. Some examples have been chosen, namely those associated with turbulent flow, because they constitute the foundation of the semi-empirical theory. An attempt was made to demonstrate that essential progress is not made through an accumulation of extensive experimental results but rather through a small number of fundamental experiments backed by theoretical considerations.

Braunschweig, October 1950 H. Schlichting

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