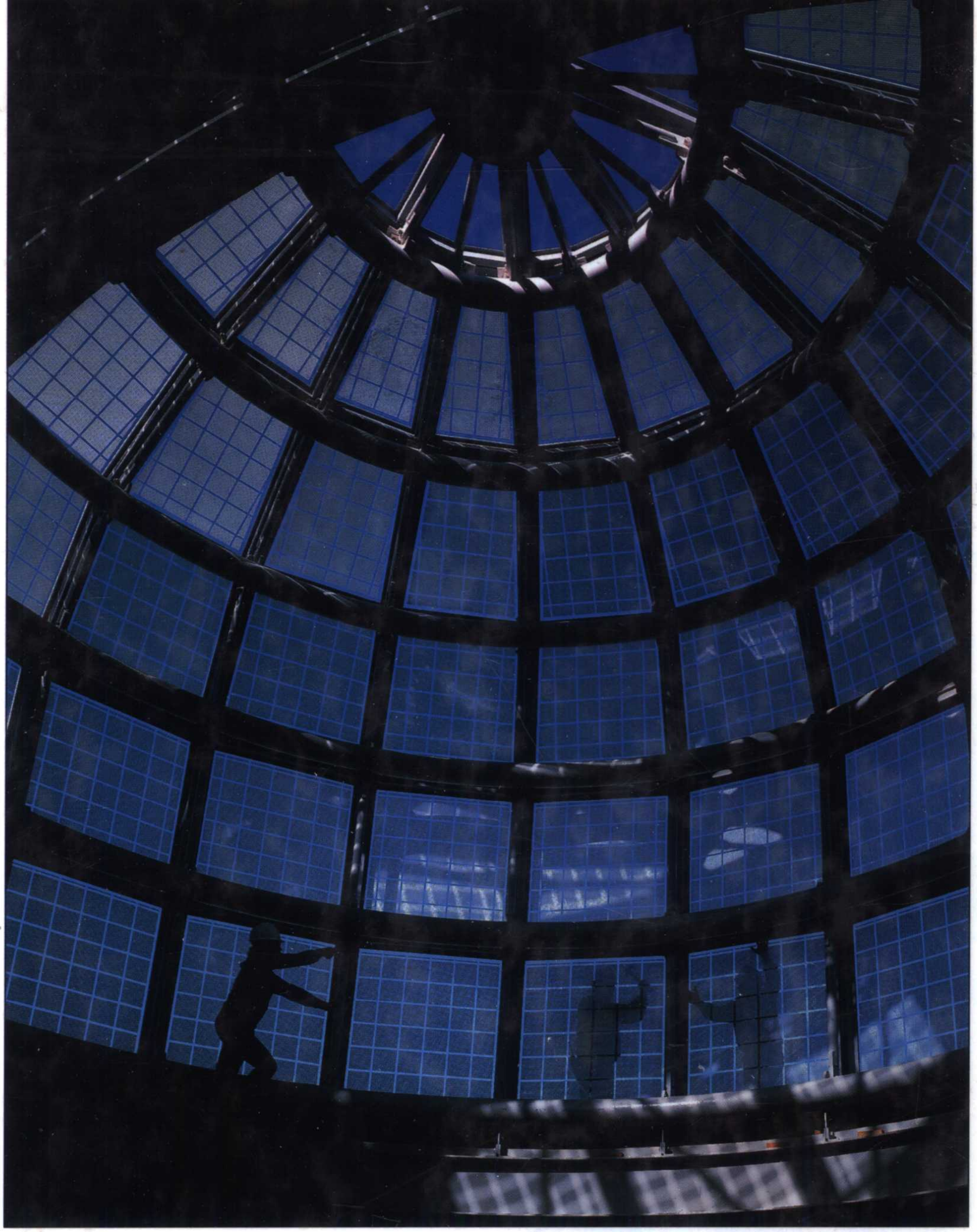

ENGINEERING MECHANICS

STATICS

Das / Kassimali / Sami

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STATICS

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PREFACE

Engineering mechanics, concerned with the study of equilibrium and motion of rigid and elastic bodies, is regarded as essential to the basic education of an engineer. Since the problems confronted by today's engineers are seldom restricted to one's own specialization, it is imperative that the engineering student become thoroughly grounded in the fundamental principles of mechanics so necessary for the solution of many problems. A major objective of the authors has been to present, in a coherent and systematic fashion and by emphasizing the useful application, a fundamental treatment of the principles of mechanics. It was particularly important to illustrate the application of these principles to problems encountered in various fields of engineering.

We have paid special attention to the degree of clarity that should be within the grasp of an average sophomore with prior knowledge of algebra, geometry, trigonometry, and calculus. The examples offered in the book are representative applications of the fundamental principles developed previously. The problems have been designed with the goals of familiarizing the student with real-life problems and developing in them an appreciation for their own powers of analysis and the effective use of mathematical modeling.

This volume, the subject of which is *statics*—the equilibrium of bodies—is divided into 10 chapters and an appendix. The organization of subject matter may be considered somewhat unconventional. The concept of coplanar forces and the equilibrium of two-dimensional rigid bodies (Chapter 3) is covered in a separate chapter from that dealing with the three-dimensional force systems and the equilibrium of rigid bodies subjected to such forces (Chapter 4). Traditionally, the two- and three-dimensional cases have been covered together using vector notations. However, the authors believe that too much reliance on vector notations for two-dimensional problems encourages a "plug-and-chug" attitude among students and deprives them of the intuitive sense necessary to understand some important basic concepts such as the moment of a force, couple, and equilibrium. Furthermore, the majority of upper-level engineering courses, of which the mechanics courses are prerequisite, deal with two-dimensional analysis using the scalar formulation. In Chapter 3, the basic concepts are explained in detail using both the scalar (intuitive) as well as the vector approaches. The chapter contains a large number of solved examples as well as unsolved problems, with the goal of preparing the students for upper-level courses in the areas of mechanics of materials, and analysis and design of structures and machines. In Chapter 4, however, the vector approach is emphasized for solving three-dimensional problems. Distributed forces (Chapter 5) are covered before the analysis of trusses, frames, and machines (Chapter 6), thus enabling Chapter 6 to include examples and problems dealing with the analysis of structures subjected to distributed loads. The International System of Units (SI) and the U.S. Customary System of Units (USCS) have both been used throughout the problems at the end of each chapter.

This book can be covered in a three-semester-hour course. However, at the discretion of the instructor, certain sections of the first nine chapters and all of Chapter 10—indicated with an asterisk (*)—could be omitted without loss of continuity.

Each chapter has a brief chapter outline and introduction, many illustrations, and a summary section. To illustrate the application of methods and equations developed in the text, there is an abundance of worked examples in each chapter (over 140 total). Example solutions are detailed and clearly explained for maximum understanding. Homework problems appear at the end of each chapter. These problem sets represent a wide range of applications and progress from simple to more complex. There are approximately 900 homework problems in all.

The solutions to all of the examples and homework problems have been double checked by two independent accuracy checkers. We have taken every effort to provide you with an error-free book. Any remaining errors can be brought to the attention of the authors for correction in future editions. Special thanks to James Matthews, Jeff Filler, and Charles Atz for their diligent efforts at finding errors.

The Instructor's Manual, available from the publisher to adopters of the text, contains complete typeset solutions to the homework problems. Transparency masters of important figures and examples are also available from the publisher.

It is almost impossible to list and give proper credit to all those who have aided in the preparation of this book. However, the authors are especially indebted to Janice Das and Maureen Kassimali for their tireless efforts in preparing the manuscript for publication. Thanks are due to our former editor, Bill Stenquist, and Kelley Butcher, associate editor, of Richard D. Irwin, Inc., for their constant help and support during the period of the development of the manuscript. In addition, the manuscript was class tested at Valparaiso University by Professor Kassim Tarhini. We would like to thank him and his students for trying the manuscript in the classroom and for providing valuable feedback to help us shape the final draft of the text. The authors are indebted to:

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Braja M. Das
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SYMBOLS

A Area

$\mathbf{A}_x, \mathbf{A}_y, \mathbf{A}_z$ Reactions at A in the x, y, and z directions, respectively

A_x, A_y, A_z Scalar components of the reactions at A in the x, y, and z directions, respectively.

$\mathbf{a}$ Acceleration vector

a Acceleration, distance, $\frac{I_x + I_y}{2}$

B Width

b Distance, coefficient of rolling resistance

b_f Flange width

C Constant, compressive axial force

$\mathbf{D}_x, \mathbf{D}_y, \mathbf{D}_z$ Reactions at D in the x, y, and z directions, respectively.

D_x, D_y, D_z Scalar components of the reactions at D in the x, y, and z directions, respectively

d Distance, moment arm

$\mathbf{F}$ Force vector

F Force

$\mathbf{F}_x, \mathbf{F}_y, \mathbf{F}_z$ Components of F in the x, y, and z directions, respectively

F_x, F_y, F_z Scalar components of F in the x, y, and z directions, respectively

$\mathbf{F}'$ Force vector

F' Force

G Location of the centroid; universal constant of gravitation

g Acceleration due to gravity

H Height, distance

h Height

I Moment of inertia

I_{CG} Moment of inertia about an axis passing through the center of gravity

I_m, I_n Moment of inertia with respect to the m and n axes, respectively

I_{mn} Product of inertia with respect to the m and n axes

I_p Principal moment of inertia

$\mathbf{I}_x, \mathbf{I}_y, \mathbf{I}_z$ Moment of inertia with respect to the x, y, and z axes, respectively

$\mathbf{I}_{x'}, \mathbf{I}_{y'}, \mathbf{I}_{z'}$ Centroidal moment of inertia with respect to the centroidal axes, x', y', and z', respectively

I_{xy} Product of inertia with respect to the x and y axes

$I_{x'y'}$ Product of inertia with respect to the centroidal axes x' and y'

$\mathbf{i}$ Unit vector in the x direction

J_O Polar moment of inertia

$\mathbf{j}$ Unit vector in the y direction

$\mathbf{k}$ Unit vector in the z direction

k Spring constant, radius of gyration

k_O Polar radius of gyration

$k_{O'}$	Centroidal polar radius of gyration
k_x, k_y, k_z	Radius of gyration with respect to the x, y, and z axes, respectively
L	Length, distance, lead
l	Length, distance
L_x, L_y, L_z	Component of length L in the x, y, and z directions, respectively
$\mathbf{M}$	Moment vector
M	moment
M_A	Reaction moment at support A; moment about point A
$\mathbf{M}_{ab}$	Moment vector about axis ab
$\mathbf{M}_R$	Resultant couple (vector)
m	Mass
$\mathbf{N}$	Normal reaction vector
N	Normal reaction
$\mathbf{n}$	unit vector
O	Origin
O'	Location of the centroid
$\mathbf{P}$	Force vector
P	Force, load
p	Pitch, constant (specific distance), hydrostatic pressure
$\mathbf{Q}$	Force vector
Q	Force, weight
$\mathbf{R}$	Force vector, resultant force vector
R	Force, resultant force, reaction, radius of Mohr's circle
$\mathbf{r}$	Distance, radius
r_e	Radius of the earth
$\mathbf{r}_G$	Position vector (for centroid of a body)
$\mathbf{S}$	Force vector, displacement vector
S	Force, cable length, surface area
$\mathbf{T}$	Force vector, torsional moment vector
T	Force, cable tension, torsional moment, tensile axial force
t	Thickness
t_f	Flange thickness
t_w	Web thickness
$\mathbf{V}$	Shear force (vector)
V	Potential energy, volume, shear force
$\mathbf{W}$	Force vector
W	Force, weight, work
w	Distributed load
x	Coordinate of a point
x'	Coordinate of a point
$\bar{x}$	Centroidal coordinate
$\bar{x}_{el}$	Centroidal coordinate of an element
y	Coordinate of a point, moment arm
$\bar{y}$	Centroidal coordinate
$\bar{y}_{el}$	Centroidal coordinate of an element
z	Coordinate of a point
$\bar{z}$	Centroidal coordinate
$\bar{z}_{el}$	Centroidal coordinate of an element
α	Angle, direction angle of a vector with respect to the x axis
β	Angle, direction angle of a vector with respect to the y axis
γ	Angle, direction angle of a vector with respect to the z axis, specific weight
ΔL	Elemental length
$\delta\mathbf{S}$	Virtual displacement
δW	Virtual work
$\delta\mathbf{x}$, $\delta\mathbf{y}$, $\delta\mathbf{z}$	Components of $\delta\mathbf{S}$ in the x, y, and z directions, respectively
μ_k	Coefficient of kinetic friction
μ_s	Coefficient of static friction
ϕ	Angle, angle of friction
ϕ_k	Angle of kinetic friction
ϕ_s	Angle of static friction
ρ	Density
σ	Normal stress
τ	Shear stress
θ	Angle, angle of revolution

CONTENTS

CHAPTER

1

INTRODUCTION TO RIGID–BODY MECHANICS

OUTLINE

1.1 INTRODUCTION

Mechanics is the study of the effects of forces acting on bodies. It provides the fundamental principles by which the mechanical interaction and the motion of bodies are related, described, and understood. The application of engineering mechanics principles to physical situations has led to the solution of countless problems, such as the design of high-rise buildings, bridges, dams, bicycles, airplanes, and various types of machinery for manufacturing processes. The design and development of artificial satellites would not have been possible without the basic understanding of the fundamental principles of engineering mechanics.

Engineering mechanics can be divided into three broad categories:

1. Mechanics of rigid bodies.
2. Mechanics of deformable bodies.
3. Mechanics of fluids.

The study of the *mechanics of rigid bodies* assumes that the body under consideration is rigid and does not deform even when subjected to large forces. *Mechanics of deformable bodies* relates to real solids, which are not rigid and will deform to a certain degree, even under the application of small forces. *Mechanics of fluids* relates to the behavior of liquids, which are virtually incompressible, and gases, which are compressible. This text enumerates the principles of rigid bodies.

Rigid-body mechanics can in turn be divided into two parts: statics and dynamics. Volume 1 of this text deals with *statics*, in which we study the mechanical interaction of rigid bodies at rest or in motion with constant velocity. *Dynamics*, the study of rigid bodies in motion, is the subject of Volume 2.

This introductory chapter contains the definitions of such basic terms as *space, time, force, particle, rigid body*, and *mass*, as well as vector and scalar quantities, which are essential to the study of rigid-body mechanics. The laws of motion and gravitation are listed. A detailed discussion of the two fundamental systems of units of measurement is also presented.

Archimedes (287–212 B.C.)
Greek mathematician and inventor
(By permission of Oxford University Press.)

1.2 HISTORICAL BACKGROUND

Mechanics is probably the oldest branch of physical science. Principles of mechanics were used to build the pyramids in ancient Egypt. Archimedes (287–212 B.C.) derived relationships for the equilibrium of levers. Stevinus (1548–1620) studied the principles of inclined planes. He also employed the principles of *parallelograms of forces* for addition of vectors, which we will describe in Section 1.5. Other ancient works address the principles of the pulley and wrench. Galileo Galilei (1564–1642) contributed immensely to the principles of dynamics by conducting experiments with the pendulum and with falling bodies. The most important contributions to the development of modern engineering mechanics, however, came from the scientific works of Sir Isaac Newton (1642–1727). He formulated the laws of motion and the law of gravitational attraction between bodies, which we will describe in Section 1.4. After publication of the laws of motion during the latter part of the 18th century, there came rapid development of the principles of engineering mechanics due to the work of Varignon, Euler, d'Alembert, Laplace, and others.

Galileo Galilei (1564–1642)
Italian physicist and mathematician who wrote *Dialogues Concerning Two New Sciences*; with this book, he began the formal study of dynamics and mechanics of materials
(Courtesy of the Library of Congress.)

1.3 BASIC CONCEPTS

Certain fundamental definitions are essential to the study of statics and dynamics of rigid bodies. These will be specified in this section.

Space

Space is a geometric region occupied by a body or bodies. This region may be one, two, or three dimensional and will be defined in terms of linear and angular measurements relative to a specified coordinate system.

Time

Time is an absolute quantity that measures the succession of events. It is a basic quantity in dynamics. In statics, however, time is not directly of concern.

Force

Force is the action of one body on another body. The interaction can occur by direct contact, such as a person pushing or pulling a box. It can also occur through a distance by which the bodies are physically separated. Examples of this type of force are gravitational, magnetic, and electrical. Force is completely characterized by its magnitude, direction, and point of application. It is a *vector* quantity, which we will describe in Section 1.5.

Jean le Rond d'Alembert (1717–1783)
French physicist who contributed d'Alembert's principle to the study of rigid bodies
(Courtesy of the French Embassy, Press and Information Division.)

Concentrated Force

A *concentrated force* is one assumed to act at a point on a body.

Particle

A ***particle*** is an object whose deformations and rotations are either negligible or not of interest. A particle is an object whose motion can, for our analysis, be represented quite adequately by specifying the position, velocity, or acceleration of a single representative point in that body. For the purposes of mechanics, a particle is *not* a body of negligible dimension, but rather a body whose dimensions are of no current interest.

Rigid Body

A ***rigid body*** is an object whose size and shape are assumed to remain unchanged under the influence of external forces. The relationship between those forces and the changes in position and orientation that they induce in rigid bodies is the subject of rigid-body mechanics. Since all real objects are deformable to some extent under the influence of forces, we may treat an object as a rigid body only when such deformations are not of interest to us in achieving our desired purposes. Thus, among the set of real objects (whose motions can be described by deformation, translation, and rotation), rigid bodies are those objects whose deformations are either negligible or not of interest.

Sir Isaac Newton (1642–1727)
English mathematician who formulated the laws of motion and gravity
(Deutsches Museum, München.)

Mass

Mass is a quantity used to measure the translational inertia of a body and represents the resistance of matter to a change in velocity. The mass of a body characterizes the mutual gravitational attraction with another body. Two bodies having the same mass will be attracted by the earth in a similar manner.

1.4 FUNDAMENTAL PRINCIPLES

Newton's Laws of Motion

In 1687, Sir Isaac Newton published his treatise, *The Principia*, in which the fundamental principles describing the motion of a particle were developed. These principles, known as *Newton's laws of motion*, are essential to the study of rigid-body mechanics. These laws of motion can be expressed in the following manner.

FIRST LAW

A particle remains at rest or continues to move along a straight line with a constant speed if it is not subjected to an unbalanced force.

SECOND LAW

A particle acted upon by an unbalanced force experiences an acceleration that is directly proportional to the force and has the same direction as the force.

The second law is mathematically expressed in a consistent set of units as

$$\mathbf{F} = m\mathbf{a} \tag{1.1}$$

where **F** is the force acting on the particle, m is the mass of the particle, and **a** is the acceleration. Boldface for force and acceleration indicates that these are vector quantities (see Section 1.5).

THIRD LAW

The forces of action and reaction between interacting bodies have the same line of action, are equal in magnitude, and are opposite in direction.

This law is basic to our understanding of force. It states that forces always occur in pairs. As an example, consider a chandelier hanging by a cord from a ceiling (Figure 1.1a). The chandelier is exerting its own weight *on the cord*. This force has a magnitude of F and is directed *downward* (Figure 1.1b). To countereffect this downward force is the equal upward force of the cord acting on the chandelier (i.e., F), thus preventing the chandelier from falling.

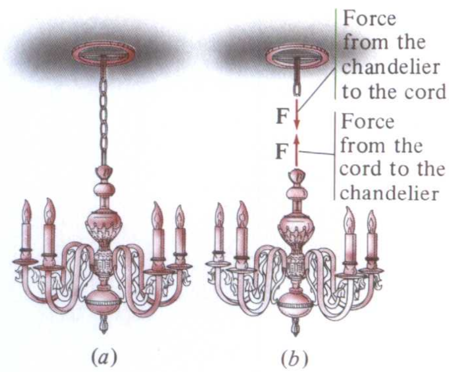


Figure 1.1

Newton's Law of Gravitation

Newton also proposed a law that mathematically describes the gravitational force of attraction between two particles. Expressed in a mathematical form,

$$F = G\frac{m_1 m_2}{r^2} \tag{1.2}$$

where F = gravitational force of attraction
m_1, m_2 = masses of the two particles under consideration (Figure 1.2)
r = distance between the centers of the two particles (Figure 1.2)
G = ***universal constant of gravitation***, which has a magnitude of $6.673 \times 10^{-11}\ \text{m}^3/\text{kg} \cdot \text{s}^2$

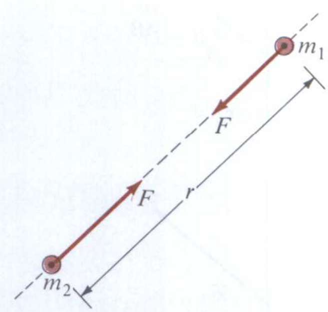


Figure 1.2

For a small body (which can be considered a particle) of mass m near the surface of the earth, being attracted by the earth of mass M, the force of attraction is

$$F = G\frac{Mm}{r_e^2} = \left(\frac{GM}{r_e^2}\right)m$$

where r_e = radius of the earth. This equation can be rewritten as

$$F = mg \tag{1.3}$$

where $g = GM/r_e^2$ = acceleration due to gravity. The magnitude of the acceleration due to gravity, g, varies slightly from place to place on the earth's surface (ranging from $9.78\ \text{m/s}^2$ to $9.82\ \text{m/s}^2$); however, the average value is

9.81 m/s^2 (or 32.2 ft/sec^2). The gravitational attraction of the earth on a body of mass m is called the *weight*, W, of the body. Thus

$$\text{Weight of a body } (W) = \text{Mass of the body } (m) \times \text{Acceleration due to gravity } (g) \tag{1.4}$$

Although Equations (1.1) and (1.3) are in a similar form, it is important not to confuse Newton's second law of motion with Newton's law of gravitation. In Equation (1.3) the acceleration due to gravity, g, is a constant. Objects do not have an acceleration of g unless they are in free-fall.

1.5 SCALARS AND VECTORS

In mechanics problems one will encounter both scalar and vector quantities. ***Scalar*** quantities have magnitude only. Examples of scalar quantities are time, temperature, mass, and volume. Mathematical operations involving scalar quantities follow the usual laws of algebra.

Vector quantities have *magnitude* and *direction* and obey the rules of vector algebra. Examples of vector quantities are force, velocity, acceleration, displacement, moment, and momentum.

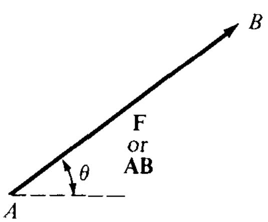


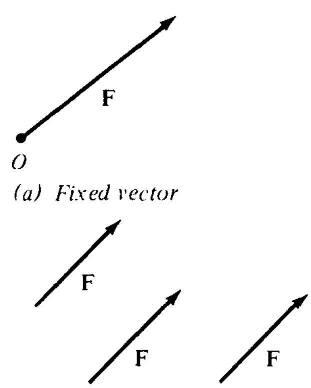
Figure 1.3

An example of a force vector is shown in Figure 1.3. The force has a magnitude of F, and it is directed at an angle θ with respect to the horizontal. The magnitude is represented by the length of the line. The arrow defines the sense, or direction. Point A is called the *tail* of the vector, and point B is the *tip* of the vector. Vectors in the text will be shown by boldface letters; thus, the vector shown in Figure 1.3 can be written as either **F** or **AB**. When writing longhand, it is usually shown as $\overrightarrow{F}$ (or $\underline{F}$) or $\overrightarrow{AB}$ (or $\underline{AB}$). The magnitude of the vector in the text will be shown in lightface italic (i.e., F or AB). In longhand, it is shown as $|\overrightarrow{F}|$ (or $|\underline{F}|$) or $|\overrightarrow{AB}|$ (or $|\underline{AB}|$).

Terminology for Vectors

In statics, various types of vectors are encountered, such as fixed vector, free vector, sliding vector, equal vectors, negative vector, concurrent vectors, collinear vectors, and coplanar vectors. Each is briefly described here.

A *fixed vector* has a unique point of application in space. Figure 1.4*a* shows a fixed force vector, **F**, whose point of application is O. The action of a force on a *deformable body* is specified by a fixed vector at the point of application of the force.

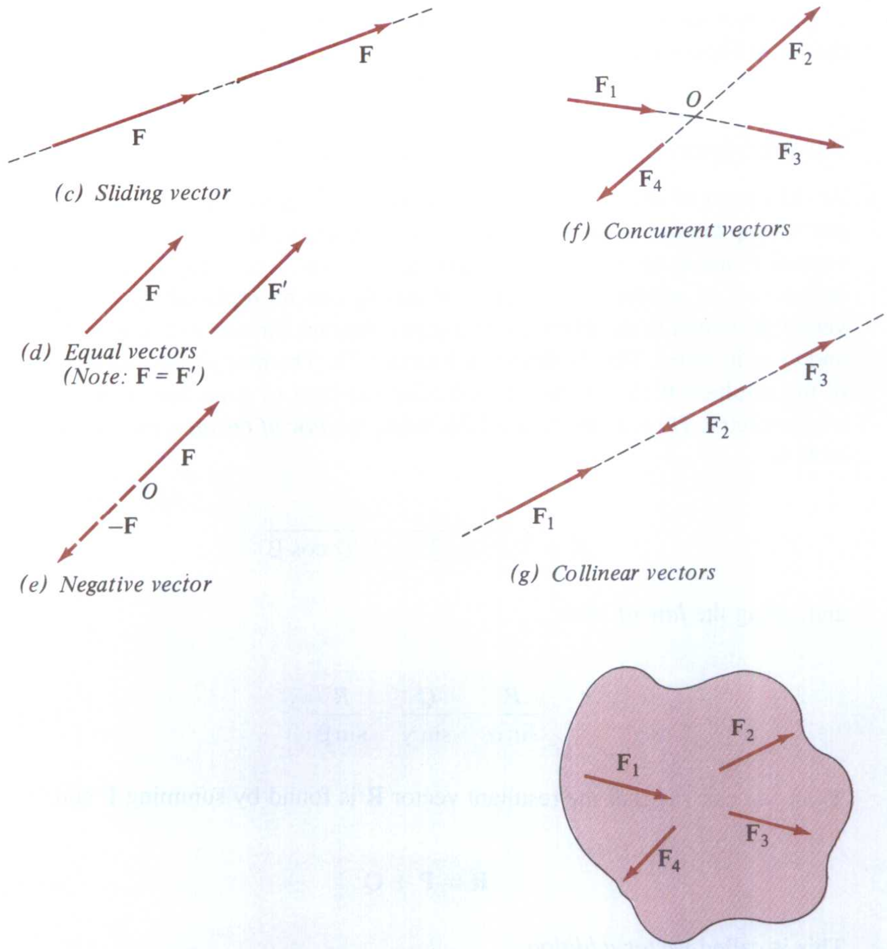


(a) Fixed vector

(b) Free vector

Figure 1.4

A *free vector* can act anywhere in space provided its direction and magnitude are retained; it is not uniquely associated with any given point or line in space (Figure 1.4*b*). An example of a free vector is the velocity of a car moving along a straight path, as shown in Figure 1.5. Assuming the car is rigid, the velocity, **v**, of the car can be described accurately whether it acts at point $A, B, C, D, \ldots$.



(c) Sliding vector

(d) Equal vectors (Note: $\mathbf{F} = \mathbf{F}'$)

(e) Negative vector

(f) Concurrent vectors

(g) Collinear vectors

(h) Coplanar vectors

Figure 1.4 *(continued)*

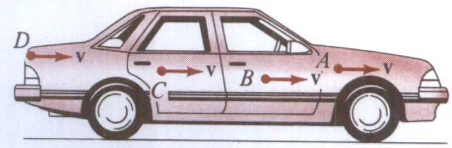


Figure 1.5

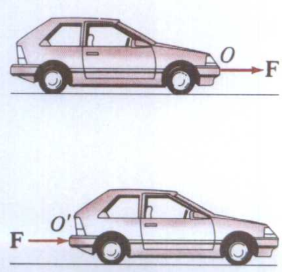


Figure 1.6

A *sliding vector* can be applied at any point along the line of action as shown in Figure 1.4*c*. An example of a sliding vector is shown in Figure 1.6. If we are interested in the resulting motion of the car, then the force **F** applied at point O or at point O' will produce the same consequence. Hence, **F** is a sliding vector.

Equal vectors have the same magnitude and direction, as shown in Figure 1.4*d*. Note that $F = F'$.

Figure 1.4*e* shows a *negative vector*, $-\mathbf{F}$. This vector has the same magnitude as the vector **F**, but it is in a direction opposite to that of vector **F**.

Concurrent vectors are those whose lines of action pass through the same point. Figure 1.4*f* shows the vectors $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_4$ whose lines of action pass through the common point O. These are concurrent vectors.

Collinear vectors have the same line of action. In Figure 1.4*g*, the vectors $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ have the same line of action and, hence, are *collinear*.