

STATICS

Das / Kassimali / Sami

ENGINEERING MECHANICS

STATICS

Braja M. Das Aslam Kassimali Sedat Sami

Department of Civil Engineering and Mechanics Southern Illinois University at Carbondale

IRWIN

Burr Ridge, Illinois Boston, Massachusetts Sydney, Australia

© RICHARD D. IRWIN, INC., 1994

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the publisher.

© Copyright 1987, Wayne Coble, Coble Studios, Inc.

Associate editor: Marketing manager: Kelley Butcher Robb Linsky

Production manager:

Ann Cassady Laurie Entringer

Designer: Art manager:

Kim Meriwether

Compositor:

CRWaldman Graphic Communications

Typeface:

10/12 Times Roman

Printer:

R. R. Donnelley & Sons Company

Library of Congress Cataloging-in-Publication Data

Das, Braja M., 1941-

Engineering mechanics : statics / Braja M. Das, Aslam Kassimali, Sedat Sami.

p. cm.

Includes index.

ISBN 0-256-11452-8 (5.25" Version) ISBN 0-256-11453-6 (3.5" Version)

II. SEE THE PARTY OF THE PARTY

1. Mechanics, Applied. 2. Statics. I. Kassimali, Aslam.

II. Sami, Sedat. III. Title.

TA350.D36 1993

620.1'03--dc20

91-42975

Printed in the United States of America

1 2 3 4 5 6 7 8 9 0 DOC 0 9 8 7 6 5 4 3

PREFACE

Engineering mechanics, concerned with the study of equilibrium and motion of rigid and elastic bodies, is regarded as essential to the basic education of an engineer. Since the problems confronted by today's engineers are seldom restricted to one's own specialization, it is imperative that the engineering student become thoroughly grounded in the fundamental principles of mechanics so necessary for the solution of many problems. A major objective of the authors has been to present, in a coherent and systematic fashion and by emphasizing the useful application, a fundamental treatment of the principles of mechanics. It was particularly important to illustrate the application of these principles to problems encountered in various fields of engineering.

We have paid special attention to the degree of clarity that should be within the grasp of an average sophomore with prior knowledge of algebra, geometry, trigonometry, and calculus. The examples offered in the book are representative applications of the fundamental principles developed previously. The problems have been designed with the goals of familiarizing the student with real-life problems and developing in them an appreciation for their own powers of analysis and the effective use of mathematical modeling.

This volume, the subject of which is statics—the equilibrium of bodies-is divided into 10 chapters and an appendix. The organization of subject matter may be considered somewhat unconventional. The concept of coplanar forces and the equilibrium of two-dimensional rigid bodies (Chapter 3) is covered in a separate chapter from that dealing with the three-dimensional force systems and the equilibrium of rigid bodies subjected to such forces (Chapter 4). Traditionally, the two- and three-dimensional cases have been covered together using vector notations. However, the authors believe that too much reliance on vector notations for two-dimensional problems encourages a "plug-andchug" attitude among students and deprives them of the intuitive sense necessary to understand some important basic concepts such as the moment of a force, couple, and equilibrium. Furthermore, the majority of upper-level engineering courses, of which the mechanics courses are prerequisite,

deal with two-dimensional analysis using the scalar formulation. In Chapter 3, the basic concepts are explained in detail using both the scalar (intuitive) as well as the vector approaches. The chapter contains a large number of solved examples as well as unsolved problems, with the goal of preparing the students for upper-level courses in the areas of mechanics of materials, and analysis and design of structures and machines. In Chapter 4, however, the vector approach is emphasized for solving three-dimensional problems. Distributed forces (Chapter 5) are covered before the analysis of trusses, frames, and machines (Chapter 6), thus enabling Chapter 6 to include examples and problems dealing with the analysis of structures subjected to distributed loads. The International System of Units (SI) and the U.S. Customary System of Units (USCS) have both been used throughout the problems at the end of each chapter.

This book can be covered in a three-semester-hour course. However, at the discretion of the instructor, certain sections of the first nine chapters and all of Chapter 10—indicated with an asterisk (*)—could be omitted without loss of continuity.

Each chapter has a brief chapter outline and introduction, many illustrations, and a summary section. To illustrate the application of methods and equations developed in the text, there is an abundance of worked examples in each chapter (over 140 total). Example solutions are detailed and clearly explained for maximum understanding. Homework problems appear at the end of each chapter. These problem sets represent a wide range of applications and progress from simple to more complex. There are approximately 900 homework problems in all.

The solutions to all of the examples and homework problems have been double checked by two independent accuracy checkers. We have taken every effort to provide you with an error-free book. Any remaining errors can be brought to the attention of the authors for correction in future editions. Special thanks to James Matthews, Jeff Filler, and Charles Atz for their diligent efforts at finding errors.

The Instructor's Manual, available from the publisher to

adopters of the text, contains complete typeset solutions to the homework problems. Transparency masters of important figures and examples are also available from the publisher.

It is almost impossible to list and give proper credit to all those who have aided in the preparation of this book. However, the authors are especially indebted to Janice Das and Maureen Kassimali for their tireless efforts in preparing the manuscript for publication. Thanks are due to our former editor, Bill Stenquist, and Kelley Butcher, associate editor, of Richard D. Irwin, Inc., for their constant help and support during the period of the development of the manuscript. In addition, the manuscript was class tested at Valparaiso University by Professor Kassim Tarhini. We would like to thank him and his students for trying the manuscript in the classroom and for providing valuable feedback to help us shape the final draft of the text. The authors are indebted to:

Philip J. Guichelaar Western Michigan University Kenneth Oster University of Missouri at Rolla Paul C. Chan New Jersey Institute of Technology James R. Matthews University of New Mexico Richard B. Reimer University of California at Berkeley James F. Devine University of South Florida George G. Adams Northeastern University John C. McWhorter Mississippi State University

Donald A. Grant University of Maine at Orono William W. Predebon Michigan Technological University Robert J. Schultz. Oregon State University Richard J. Leuba North Carolina State University William L. Bingham North Carolina State University Kenneth R. Johnson Indiana University, Purdue University at Ft. Wayne John W. Bird University of Nevada at Reno

Gholam H. Nazari North Dakota State University Ali E. Engin The Ohio State University Roberto A. Osegueda University of Texas at El Paso M. A. M. Torkamani University of Pittsburgh D. Joanne Wilson University of Wisconsin at Platteville Iury L. Maytin Clarkson University Ghassan Tarakji

San Francisco State

University

J. Edward Anderson
Boston University
Han-Chin Wu
University of Iowa
W. F. Swinson
Oakridge National Lab
Thalia Anagnos
San Jose State University
Carl Vilmann
Michigan Technological
University
David Oglesby
University of Missouri
at Rolla

for their helpful comments, suggestions, and critical reviews of the manuscript.

Braja M. Das Aslam Kassimali Sedat Sami

SYMBOLS

respectively Scalar components of the reactions at A in the x, y, $A_{\rm r}, A_{\rm v}, A_{\rm r}$ and z directions, respectively. Acceleration vector Acceleration, distance, $\frac{I_x + I_y}{2}$ В Width b Distance, coefficient of rolling resistance Flange width b_f CConstant, compressive axial force $\mathbf{D}_{x}, \mathbf{D}_{y}, \mathbf{D}_{z}$ Reactions at D in the x, y, and z directions, respectively. D_x, D_y, D_z Scalar components of the reactions at D in the x, y, and z directions, respectively Distance, moment arm Force vector Force $\mathbf{F}_{v}, \mathbf{F}_{v}, \mathbf{F}_{r}$ Components of F in the x, y, and z directions, respectively Scalar components of F in the x, y, and z F_x, F_y, F_z directions, respectively

Reactions at A in the x, y, and z directions,

A

 $\mathbf{A}_{x}, \mathbf{A}_{y}, \mathbf{A}_{z}$

 \mathbf{F}'

F'

Force vector

Force

Area

Acceleration due to gravity Η Height, distance Height 1 Moment of inertia Moment of inertia about an axis passing through I_{CG} the center of gravity I_m, I_n Moment of inertia with respect to the m and n axes, respectively I_{mn} Product of inertia with respect to the m and n axes I_{o} Principal moment of inertia $\mathbf{I}_{x}, \mathbf{I}_{y}, \mathbf{I}_{z}$ Moment of inertia with respect to the x, y, and zaxes, respectively Centroidal moment of inertia with respect to the $\mathbf{I}_{\mathbf{x}'}, \mathbf{I}_{\mathbf{y}'}, \mathbf{I}_{\mathbf{x}'}$ centroidal axes, x', y', and z', respectively I_{xy} Product of inertia with respect to the x and y axes Product of inertia with respect to the centroidal $I_{x'y'}$ axes x' and y'Unit vector in the x direction J_{o} Polar moment of inertia j Unit vector in the y direction k Unit vector in the z direction k Spring constant, radius of gyration

χi

Location of the centroid; universal constant of

gravitation

 k_O

Polar radius of gyration

xii Symbols

Flange thickness

Web thickness Centroidal polar radius of gyration tw. $k_{O'}$ V Shear force (vector) k_x, k_y, k_z Radius of gyration with respect to the x, y, and zaxes, respectively Potential energy, volume, shear force L Length, distance, lead W Force vector Length, distance Force, weight, work W Component of length L in the x, y, and z L_x, L_y, L_z Distributed load directions, respectively Coordinate of a point M Moment vector x'Coordinate of a point M moment \bar{x} Centroidal coordinate Reaction moment at support A; moment about M_{Λ} \overline{x}_{el} Centroidal coordinate of an element point A \mathbf{M}_{ab} Moment vector about axis ab Coordinate of a point, moment arm y \mathbf{M}_{κ} Resultant couple (vector) Centroidal coordinate m Mass Centroidal coordinate of an element \bar{y}_{el} Normal reaction vector N z Coordinate of a point Ν Normal reaction Centroidal coordinate unit vector n Centroidal coordinate of an element 0 Origin Angle, direction angle of a vector with respect to the x axis O'Location of the centroid Angle, direction angle of a vector with respect P Force vector to the y axis Force, load Angle, direction angle of a vector with respect Pitch, constant (specific distance), hydrostatic to the z axis, specific weight pressure ΔL Elemental length O Force vector δS Virtual displacement Force, weight δW Virtual work R Force vector, resultant force vector δx , δy , δz Components of δS in the x, y, and z directions, R Force, resultant force, reaction, radius of respectively Mohr's circle Coefficient of kinetic friction μ_{k} Distance, radius Coefficient of static friction μ_{s} r_c Radius of the earth φ Angle, angle of friction \mathbf{r}_G Position vector (for centroid of a body) Φ_{L} Angle of kinetic friction S Force vector, displacement vector Φ_s Angle of static friction Force, cable length, surface area S ρ Density T Force vector, torsional moment vector Normal stress TForce, cable tension, torsional moment, tensile Shear stress axial force Angle, angle of revolution Thickness

CONTENTS

2.5

List of Symbols xi

1.1

1.2

Introduction 2

Historical Background 2

	1.3	Basic Concepts 3
	1.4	Fundamental Principles 4
	1.5	Scalars and Vectors 6
	1.6	Units of Measurement 8
	1.7	Accuracy of Numerical Calculations 15
	1.8	Process of Problem Solving 15
	1.9	Summary 15
		Key Terms 16
		Problems 16
2	FOR	CE AND EQUILIBRIUM OF PARTICLES 18
2	FOR (2.1	CE AND EQUILIBRIUM OF PARTICLES 18 Introduction 19
2	2.1	· - · · ·
2	2.1	Introduction 19
2	2.1 (Con	Introduction 19 current Forces in Two Dimensions)
2	2.1 (Con 2.2	Introduction 19 current Forces in Two Dimensions) Resultant of Two Forces 19 Resultant of Several Concurrent Forces on a

1 INTRODUCTION TO RIGID-BODY MECHANICS 1

2.6	Unit Vectors 29			
2.7	Resultant of Several Forces from Their Rectangular Components 31			
2.8	Equilibrium of Particles 33			
2.9	Free-Body Diagrams 35			
2.10	Common Types of Connections 38			
(Cone	current Forces in Three Dimensions)			
2.11	Rectangular Components of a Force in Space 40			
2.12	_			
2.13	Representation of a Force Using Unit Vectors in Three Dimensions 45			
2.14	Representation of a Force Vector with Known Magnitude and Line of Action 47			
2.15				
2.16	Equilibrium of a Particle in Three Dimensions 51			
2.17	Summary 52			
	Key Terms 53			
	Problems 54			

Rectangular Components of a Force 27

viii Contents

3	3 EQUILIBRIUM OF RIGID BODIES			(Equ	ilibrium in Three Dimensions)
	IN TV	VO DIMENSIONS 78		4.7	Equilibrium of Rigid Bodies 194
	3.1	Introduction 79		4.8	Types of Supports and Connections 194
	3.2	Rigid Bodies 79		4.9	Statically Determinate Structures 197
	3.3	Forces on Rigid Bodies and the Principle of		4.10	Free-Body Diagrams 201
		Transmissibility 80		4.11	Procedure for Analysis of Reactions 201
	(Copl	anar Force Systems)		4.12	Summary 209
	3.4	Rigid Bodies in Two Dimensions 82			Key Terms 211
	3.5	Moment of a Force—Scalar Approach 83			Problems 211
	3.6	Cross Product of Vectors 87			
	3.7	Moments as Vectors 90	5	-	ΓER OF GRAVITY, CENTROID,
	3.8	Varignon's Theorem 93			DISTRIBUTED FORCE 227
	3.9	Procedure for Determining Moments 94		5.1	Introduction 228
	3.10	Couples 100		5.2	Centroid—Definition 230
	3.11	Resolution of a Force into a Force-Couple System 106			Dimensional Problems—Areas and Lines)
	3.12	Resultants of Nonconcurrent Coplanar Force		5.3	Centroid of an Area by Integration 233
		Systems 110		5.4	Centroid of a Line by Integration 238
	(Equil	librium in Two Dimensions)		5.5	Centroid of Composite Areas 239
	3.13	Equilibrium of Rigid Bodies 119		5.6	Centroid of Composite Lines 243
	3.14			*5.7	Theorems of Pappus and Guldinus 244
	3.14	Equilibrium Equations in Scalar Form 119 Equilibrium of Rigid Bodies Subjected to Forces at		*5.8	Distributed Load on Beams 248
	5.15	Two and Three Points 123		*5.9	Hydrostatic Force on Submerged Surfaces 250
	3.16	Types of Supports and Connections 125		(Three	e-Dimensional Problems—Volumes)
	3.17	Statically Determinate Structures 127		5.10	Centroid by Integration 255
	3.18	Free-Body Diagrams 134		5.11	Centroid of Composite Volumes 259
	3.19	Procedure for Analysis of Reactions 138		5.12	Summary 263
	3.20	Summary 146			Key Terms 264
		Key Terms 148			Problems 265
		Problems 149			
			6		YSIS OF STATICALLY DETERMINATE CTURES 286
4	FOU	IBRIUM OF RIGID BODIES		6.1	Introduction 287
7		SEE DIMENSIONS		6.2	Internal Forces at Connections 287
		Introduction 164		6.3	Trusses 294
				6.4	Assumptions for the Analysis of Trusses 296
	(Three-Dimensional Force Systems)			6.5	Arrangement of Members of Simple Plane
	4.2	Moment of a Force about a Point 164			Trusses 298
	4.3	Dot Product of Vectors 170		6.6	Statically Determinate Plane Trusses 300
	4.4	Moment of a Force about an Axis 173		6.7	Analysis of Plane Trusses by the Method of
		Couples 177			Joints 303
	4.6	Resultants of Nonconcurrent Three-Dimensional Force Systems 181		6.8	Analysis of Plane Trusses by the Method of Sections 315

J. st

Contents ix

	*6.9	Space Trusses 324	9.3	Radius of Gyration of an Area 470
	6.10	Frames and Machines 330	9.4	,
	6.11	Summary 351	9.5	
		Key Terms 352		of an Area 475
		Problems 352	9.6	Moment of Inertia of Composite Areas 479
			*9.7	7 Product of Inertia of an Area 485
7	DIST	RIBUTED LOAD—ANALYSIS OF BEAMS	*9.8	Product of Inertia of Composite Areas 489
		CABLES 369	*9.9	Principal Moment of Inertia of an Area 492
	7.1	Introduction 370	*9.1	0 Mohr's Circle for Moment of Inertia of an Area 496
	(Bea	ms)	9.11	Moment of Inertia and Radius of Gyration of
	7.2	Types of Beams 370		Masses 500
,	7.3	Internal Forces 372	9.12	Mass Moment of Inertia by Integration 503
•	7.4	Types of Loads on a Beam 373	9.13	Mass Moment of Inertia of Composite Bodies 508
•	7.5	Shear and Moment Diagrams 373	9.14	Summary 511
	7.6	Relations between Distributed Load, Shear Force,		Key Terms 513
		and Bending Moment 378		Problems 514
(Cabl	les)		
*	*7.7	Cable Carrying Concentrated Loads 384	10 WOI	RK AND ENERGY 527
*	⁴ 7.8	Cable Carrying Distributed Loads 389		1 Introduction 528
*	7.9	Cable Subjected to Its Own Weight 394	*10.2	2 Basic Concept of Work 528
7	.10	Summary 397		The Principle of Virtual Work 532
		Key Terms 398		4 Procedure for Analysis 534
		Problems 398		5 Potential Energy 538
			*10.6	5 Equilibrium—Principle of Stationary Potential
		FION 411		Energy 542
	. 1	Introduction 412	*10.7	7 Stability of Equilibrium—Principle of Minimum
		Mechanics of Dry Friction 412		Potential Energy 545
		Analysis of Some Dry Friction Problems 415	*10.8	3 Summary 548
		Wedges 424		Key Terms 549
8.		Square-Threaded Screws 428		Problems 549
8.		Belt Friction 431		
*{	3.7	Frictional Resistance on Thrust Bearings—Disc	Appe	endix 555
yl. C		Friction 436	A	SI Prefixes 555
		Journal Bearings 438	В	Conversion Factors 556
		Rolling Resistance 440	C	Specific Weight of Common Materials 557
8.		Summary 441	D	Mathematical Expressions 558
		Key Terms 443	E	Properties of Areas and Homogeneous Bodies 561
	J	Problems 443		- B Dodies - 301
9 MC	MOMENT OF INERTIA 462		Answ	ers to Selected Problems 568
9.1		Introduction 463		TO TOURS TOURS
9.2		Moment of Inertia of an Area 465	Index	576

Index 576

INTRODUCTION TO RIGID-BODY MECHANICS

- 1.1 Introduction
- 1.2 Historical Background
- 1.3 Basic Concepts
- 1.4 Fundamental Principles
- 1.5 Scalars and Vectors
- 1.6 Units of Measurement
- 1.7 Accuracy of Numerical Computations
- 1.8 Process of Problem Solving
- 1.9 Summary Key Terms Problems

OUTLINE

1.1 INTRODUCTION

Mechanics is the study of the effects of forces acting on bodies. It provides the fundamental principles by which the mechanical interaction and the motion of bodies are related, described, and understood. The application of engineering mechanics principles to physical situations has led to the solution of countless problems, such as the design of high-rise buildings, bridges, dams, bicycles, airplanes, and various types of machinery for manufacturing processes. The design and development of artificial satellites would not have been possible without the basic understanding of the fundamental principles of engineering mechanics.

Engineering mechanics can be divided into three broad categories:

- 1. Mechanics of rigid bodies.
- 2. Mechanics of deformable bodies.
- 3. Mechanics of fluids.

The study of the *mechanics of rigid bodies* assumes that the body under consideration is rigid and does not deform even when subjected to large forces. *Mechanics of deformable bodies* relates to real solids, which are not rigid and will deform to a certain degree, even under the application of small forces. *Mechanics of fluids* relates to the behavior of liquids, which are virtually incompressible, and gases, which are compressible. This text enumerates the principles of rigid bodies.

Rigid-body mechanics can in turn be divided into two parts: statics and dynamics. Volume 1 of this text deals with *statics*, in which we study the mechanical interaction of rigid bodies at rest or in motion with constant velocity. *Dynamics*, the study of rigid bodies in motion, is the subject of Volume 2.

This introductory chapter contains the definitions of such basic terms as *space*, *time*, *force*, *particle*, *rigid body*, and *mass*, as well as vector and scalar quantities, which are essential to the study of rigid-body mechanics. The laws of motion and gravitation are listed. A detailed discussion of the two fundamental systems of units of measurement is also presented.

1.2 HISTORICAL BACKGROUND

Mechanics is probably the oldest branch of physical science. Principles of mechanics were used to build the pyramids in ancient Egypt. Archimedes (287–212 B.C.) derived relationships for the equilibrium of levers. Stevinus (1548–1620) studied the principles of inclined planes. He also employed the principles of parallelograms of forces for addition of vectors, which we will describe in Section 1.5. Other ancient works address the principles of the pulley and wrench. Galileo Galilei (1564–1642) contributed immensely to the principles of dynamics by conducting experiments with the pendulum and with falling bodies. The most important contributions to the development of modern



Archimedes (287–212 B.C.)
Greek mathematician and inventor
(By permission of Oxford University Press.)

engineering mechanics, however, came from the scientific works of Sir Isaac Newton (1642–1727). He formulated the laws of motion and the law of gravitational attraction between bodies, which we will describe in Section 1.4. After publication of the laws of motion during the latter part of the 18th century, there came rapid development of the principles of engineering mechanics due to the work of Varignon, Euler, d'Alembert, Laplace, and others.

1.3 BASIC CONCEPTS

Certain fundamental definitions are essential to the study of statics and dynamics of rigid bodies. These will be specified in this section.

Space

300

Space is a geometric region occupied by a body or bodies. This region may be one, two, or three dimensional and will be defined in terms of linear and angular measurements relative to a specified coordinate system.

Time

Time is an absolute quantity that measures the succession of events. It is a basic quantity in dynamics. In statics, however, time is not directly of concern.

Force

Force is the action of one body on another body. The interaction can occur by direct contact, such as a person pushing or pulling a box. It can also occur through a distance by which the bodies are physically separated. Examples of this type of force are gravitational, magnetic, and electrical. Force is completely characterized by its magnitude, direction, and point of application. It is a *vector* quantity, which we will describe in Section 1.5.

Concentrated Force

A concentrated force is one assumed to act at a point on a body.

Particle

A *particle* is an object whose deformations and rotations are either negligible or not of interest. A particle is an object whose motion can, for our analysis, be represented quite adequately by specifying the position, velocity, or acceleration of a single representative point in that body. For the purposes of mechanics, a particle is *not* a body of negligible dimension, but rather a body whose dimensions are of no current interest.



Galileo Galilei (1564–1642)
Italian physicist and mathematician who wrote Dialogues Concerning Two New Sciences; with this book, he began the formal study of dynamics and mechanics of materials (Courtesy of the Library of Congress.)



Jean le Rond d'Alembert (1717–1783)
French physicist who contributed d'Alembert's principle to the study of rigid bodies (Courtesy of the French Embassy, Press and Information Division.)

Rigid Body

A *rigid body* is an object whose size and shape are assumed to remain unchanged under the influence of external forces. The relationship between those forces and the changes in position and orientation that they induce in rigid bodies is the subject of rigid-body mechanics. Since all real objects are deformable to some extent under the influence of forces, we may treat an object as a rigid body only when such deformations are not of interest to us in achieving our desired purposes. Thus, among the set of real objects (whose motions can be described by deformation, translation, and rotation), rigid bodies are those objects whose deformations are either negligible or not of interest.



Sir Isaac Newton (1642–1727) English mathematician who formulated the laws of motion and gravity (Deutsches Museum, München.)

Mass

Mass is a quantity used to measure the translational inertia of a body and represents the resistance of matter to a change in velocity. The mass of a body characterizes the mutual gravitational attraction with another body. Two bodies having the same mass will be attracted by the earth in a similar manner.

1.4 FUNDAMENTAL PRINCIPLES

Newton's Laws of Motion

In 1687, Sir Isaac Newton published his treatise, *The Principia*, in which the fundamental principles describing the motion of a particle were developed. These principles, known as *Newton's laws of motion*, are essential to the study of rigid-body mechanics. These laws of motion can be expressed in the following manner.

FIRST LAW

A particle remains at rest or continues to move along a straight line with a constant speed if it is not subjected to an unbalanced force.

SECOND LAW

A particle acted upon by an unbalanced force experiences an acceleration that is directly proportional to the force and has the same direction as the force.

The second law is mathematically expressed in a consistent set of units as

 $\mathbf{F} = m\mathbf{a} \tag{1.1}$

where F is the force acting on the particle, m is the mass of the particle, and a is the acceleration. Boldface for force and acceleration indicates that these are vector quantities (see Section 1.5).

The forces of action and reaction between interacting bodies have the same line of action, are equal in magnitude, and are opposite in direction.

THIRD LAW

Force from the

Force

(b)

from the cord to the

chandelier

chandelier

to the cord

This law is basic to our understanding of force. It states that forces always occur in pairs. As an example, consider a chandelier hanging by a cord from a ceiling (Figure 1.1a). The chandelier is exerting its own weight on the cord. This force has a magnitude of F and is directed downward (Figure 1.1b). To countereffect this downward force is the equal upward force of the cord acting on the chandelier (i.e., F), thus preventing the chandelier from falling.

Newton's Law of Gravitation

Newton also proposed a law that mathematically describes the gravitational force of attraction between two particles. Expressed in a mathematical form,

that mathematically describes the gravitational particles. Expressed in a mathematical form,
$$F = G \frac{m_1 m_2}{r^2}$$
(1.2) Figure 1.1

F =gravitational force of attraction

 m_1, m_2 = masses of the two particles under consideration (Figure 1.2)

r = distance between the centers of the two particles (Figure 1.2)

G = universal constant of gravitation, which has a magnitude of $6.673 \times 10^{-11} \,\mathrm{m}^3/\mathrm{kg}\cdot\mathrm{s}^2$

For a small body (which can be considered a particle) of mass m near the surface of the earth, being attracted by the earth of mass M, the force of attraction is

$$F = G\frac{Mm}{r_e^2} = \left(\frac{GM}{r_e^2}\right)m$$

where r_e = radius of the earth. This equation can be rewritten as

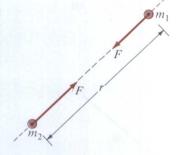


Figure 1.2

$$F = mg ag{1.3}$$

where $g = GM/r_e^2$ = acceleration due to gravity. The magnitude of the acceleration due to gravity, g, varies slightly from place to place on the earth's surface (ranging from 9.78 m/s² to 9.82 m/s²); however, the average value is 9.81 m/s² (or 32.2 ft/sec²). The gravitational attraction of the earth on a body of mass m is called the weight, W, of the body. Thus

Weight of a body
$$(W)$$
 = Mass of the body (m) × Acceleration due to gravity (g) (1.4)

Although Equations (1.1) and (1.3) are in a similar form, it is important not to confuse Newton's second law of motion with Newton's law of gravitation. In Equation (1.3) the acceleration due to gravity, g, is a constant. Objects do not have an acceleration of g unless they are in free-fall.

1.5 SCALARS AND VECTORS

In mechanics problems one will encounter both scalar and vector quantities. *Scalar* quantities have magnitude only. Examples of scalar quantities are time, temperature, mass, and volume. Mathematical operations involving scalar quantities follow the usual laws of algebra.

Vector quantities have *magnitude* and *direction* and obey the rules of vector algebra. Examples of vector quantities are force, velocity, acceleration, displacement, moment, and momentum.

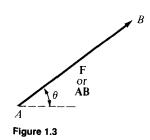
An example of a force vector is shown in Figure 1.3. The force has a magnitude of F, and it is directed at an angle θ with respect to the horizontal. The magnitude is represented by the length of the line. The arrow defines the sense, or direction. Point A is called the *tail* of the vector, and point B is the *tip* of the vector. Vectors in the text will be shown by boldface letters; thus, the vector shown in Figure 1.3 can be written as either F or AB. When writing longhand, it is usually shown as \overrightarrow{F} (or \overrightarrow{F}) or \overrightarrow{AB} (or \overrightarrow{AB}). The magnitude of the vector in the text will be shown in lightface italic (i.e., F or AB). In longhand, it is shown as \overrightarrow{F} (or |F|) or $|\overrightarrow{AB}|$ (or |AB|).

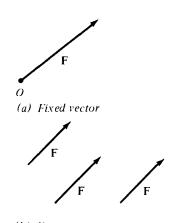
Terminology for Vectors

In statics, various types of vectors are encountered, such as fixed vector, free vector, sliding vector, equal vectors, negative vector, concurrent vectors, collinear vectors, and coplanar vectors. Each is briefly described here.

A fixed vector has a unique point of application in space. Figure 1.4a shows a fixed force vector, **F**, whose point of application is O. The action of a force on a deformable body is specified by a fixed vector at the point of application of the force.

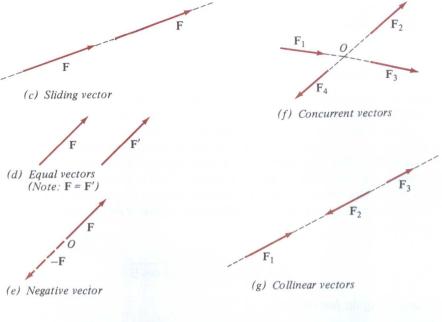
A free vector can act anywhere in space provided its direction and magnitude are retained; it is not uniquely associated with any given point or line in space (Figure 1.4b). An example of a free vector is the velocity of a car moving along a straight path, as shown in Figure 1.5. Assuming the car is rigid, the velocity, \mathbf{v} , of the car can be described accurately whether it acts at point A, B, C, D, \ldots

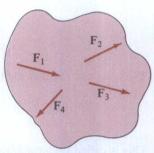




(b) Free vector

Figure 1.4





(h) Coplanar vectors

Figure 1.4 (continued)

A sliding vector can be applied at any point along the line of action as shown in Figure 1.4c. An example of a sliding vector is shown in Figure 1.6. If we are interested in the resulting motion of the car, then the force \mathbf{F} applied at point O or at point O' will produce the same consequence. Hence, \mathbf{F} is a sliding vector.

Equal vectors have the same magnitude and direction, as shown in Figure 1.4d. Note that F = F'.

Figure 1.4e shows a *negative vector*, $-\mathbf{F}$. This vector has the same magnitude as the vector \mathbf{F} , but it is in a direction opposite to that of vector \mathbf{F} .

Concurrent vectors are those whose lines of action pass through the same point. Figure 1.4f shows the vectors \mathbf{F}_1 , \mathbf{F}_2 , \mathbf{F}_3 , and \mathbf{F}_4 whose lines of action pass through the common point O. These are concurrent vectors.

Collinear vectors have the same line of action. In Figure 1.4g, the vectors \mathbf{F}_1 , \mathbf{F}_2 , and \mathbf{F}_3 have the same line of action and, hence, are collinear.

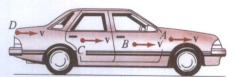


Figure 1.5





Figure 1.6