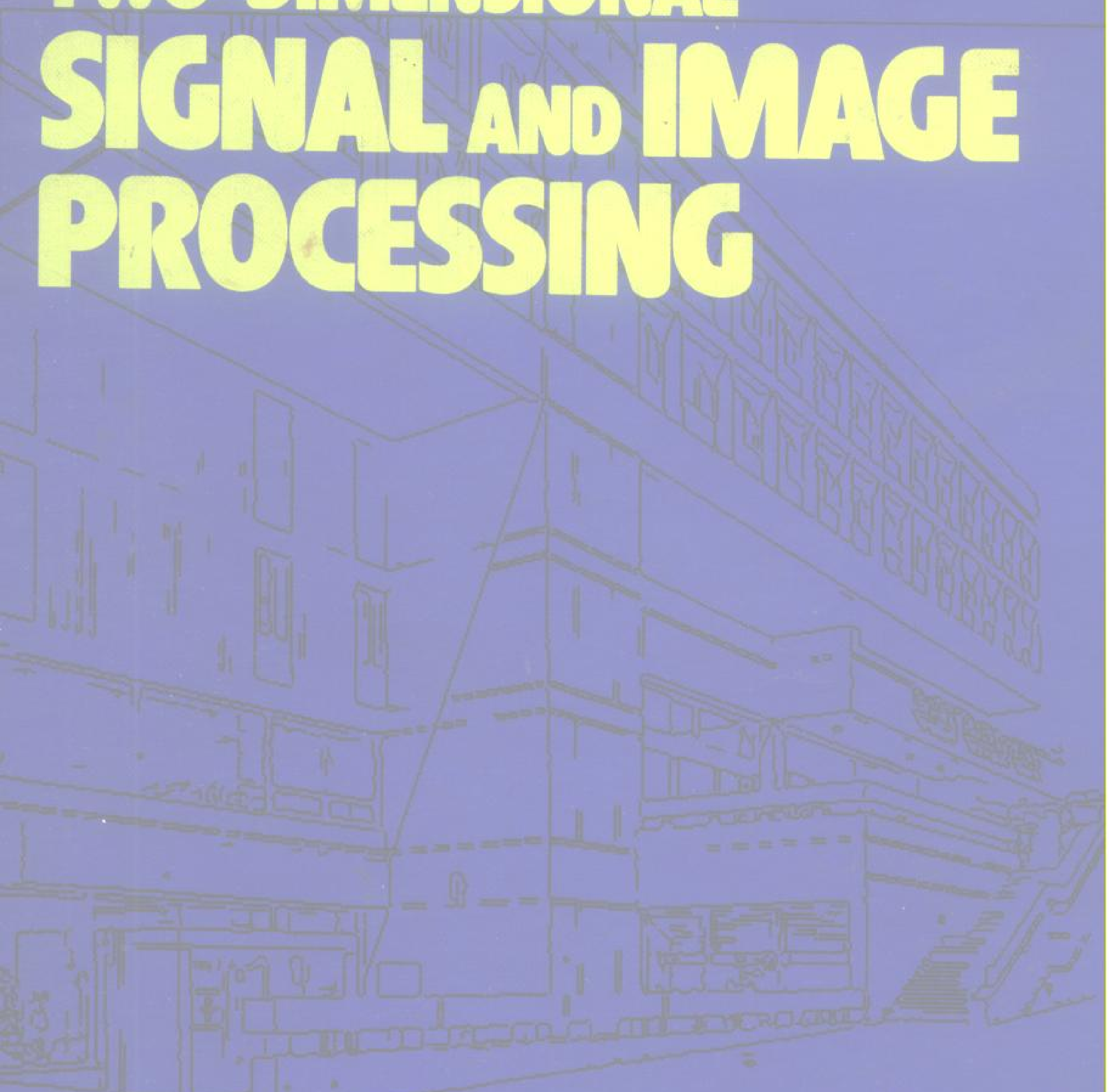
TWO-DIMENSIONAL SIGNAL AND IMAGE PROCESSING



JAE S. LIM

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TWO-DIMENSIONAL SIGNAL AND IMAGE PROCESSING

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TO
KYUHO and TAEHO

Preface

This book has grown out of the author's teaching and research activities in the field of two-dimensional signal and image processing. It is designed as a text for an upper-class undergraduate level or a graduate level course. The notes on which this book is based have been used since 1982 for a one-semester course in the Department of Electrical Engineering and Computer Science at M.I.T. and for a continuing education course at industries including Texas Instruments and Bell Laboratories.

In writing this book, the author has assumed that readers have prior exposure to fundamentals of one-dimensional digital signal processing, which are readily available in a variety of excellent text and reference books. Many two-dimensional signal processing theories are developed in the book by extension and generalization of one-dimensional signal processing theories.

This book consists of ten chapters. The first six chapters are devoted to fundamentals of two-dimensional digital signal processing. Chapter 1 is on signals, systems, and Fourier transform, which are the most basic concepts in signal processing and serve as a foundation for all other chapters. Chapter 2 is on z-transform representation and related topics including the difference equation and stability. Chapter 3 is on the discrete Fourier series, discrete Fourier transform, and fast Fourier transform. The chapter also covers the cosine and discrete cosine transforms which are closely related to Fourier and discrete Fourier transforms. Chapter 4 is on the design and implementation of finite impulse response filters. Chapter 5 is on the design and implementation of infinite impulse response filters. Chapter 6 is on random signals and spectral estimation. Throughout the first six chapters, the notation used and the theories developed are for two-dimensional signals and systems. Essentially all the results extend to more general multidimensional signals and systems in a straightforward manner.

The remaining four chapters are devoted to fundamentals of digital image processing. Chapter 7 is on the basics of image processing. Chapter 8 is on image enhancement including topics on contrast enhancement, noise smoothing, and use of color. The chapter also covers related topics on edge detection, image interpolation, and motion-compensated image processing. Chapter 9 is on image restoration and treats restoration of images degraded by both signal-independent and signal-dependent degradation. Chapter 10 is on image coding and related topics.

One goal of this book is to provide a single-volume text for a course that covers both two-dimensional signal processing and image processing. In a one-semester course at M.I.T., the author covered most topics in the book by treating some topics in reasonable depth and others with less emphasis. The book can also be used as a text for a course in which the primary emphasis is on either two-dimensional signal processing or image processing. A typical course with emphasis on two-dimensional signal processing, for example, would cover topics in Chapters 1 through 6 with reasonable depth and some selected topics from Chapters 7 and 9. A typical course with emphasis on image processing would cover topics in Chapters 1 and 3, Section 6.1, and Chapters 7 through 10. This book can also be used for a two-semester course, the first semester on two-dimensional signal processing and the second semester on image processing.

Many problems are included at the end of each chapter. These problems are, of course, intended to help the reader understand the basic concepts through drill and practice. The problems also extend some concepts presented previously and develop some new concepts.

The author is indebted to many students, friends, and colleagues for their assistance, support, and suggestions. The author was very fortunate to learn digital signal processing and image processing from Professor Alan Oppenheim, Professor Russell Mersereau, and Professor William Schreiber. Thrasyvoulos Pappas, Srinivasa Prasanna, Mike McIlrath, Matthew Bace, Roz Wright Picard, Dennis Martinez, and Giovanni Aliberti produced many figures. Many students and friends used the lecture notes from which this book originated and provided valuable comments and suggestions. Many friends and colleagues read drafts of this book, and their comments and suggestions have been incorporated. The book was edited by Beth Parkhurst and Patricia Johnson. Phyllis Eiro, Leslie Melcer, and Cindy LeBlanc typed many versions of the manuscript.

The author acknowledges the support of M.I.T. which provided an environment in which many ideas were developed and a major portion of the work was accomplished. The author is also grateful to the Woods Hole Oceanographic Institution and the Naval Postgraduate School where the author spent most of his sabbatical year completing the manuscript.

Jae S. Lim

Introduction

The fields of two-dimensional digital signal processing and digital image processing have maintained tremendous vitality over the past two decades and there is every indication that this trend will continue. Advances in hardware technology provide the capability in signal processing chips and microprocessors which were previously associated with mainframe computers. These advances allow sophisticated signal processing and image processing algorithms to be implemented in real time at a substantially reduced cost. New applications continue to be found and existing applications continue to expand in such diverse areas as communications, consumer electronics, medicine, defense, robotics, and geophysics. Along with advances in hardware technology and expansion in applications, new algorithms are developed and existing algorithms are better understood, which in turn lead to further expansion in applications and provide a strong incentive for further advances in hardware technology.

At a conceptual level, there is a great deal of similarity between one-dimensional signal processing and two-dimensional signal processing. In one-dimensional signal processing, the concepts discussed are filtering, Fourier transform, discrete Fourier transform, fast Fourier transform algorithms, and so on. In two-dimensional signal processing, we again are concerned with the same concepts. As a consequence, the general concepts that we develop in two-dimensional signal processing can be viewed as straightforward extensions of the results in one-dimensional signal processing.

At a more detailed level, however, considerable differences exist between one-dimensional and two-dimensional signal processing. For example, one major difference is the amount of data involved in typical applications. In speech processing, an important one-dimensional signal processing application, speech is typically sampled at a 10-kHz rate and we have 10,000 data points to process in a second. However, in video processing, where processing an image frame is an important two-dimensional signal processing application, we may have 30 frames per second, with each frame consisting of 500 × 500 pixels (picture elements). In this case, we would have 7.5 million data points to process per second, which is orders of magnitude greater than the case of speech processing. Due to this difference in data rate requirements, the computational efficiency of a signal processing algorithm plays a much more important role in two-dimensional signal processing, and advances in hardware technology will have a much greater impact on two-dimensional signal processing applications.

Another major difference comes from the fact that the mathematics used for one-dimensional signal processing is often simpler than that used for two-dimensional signal processing. For example, many one-dimensional systems are described by differential equations, while many two-dimensional systems are described by partial differential equations. It is generally much easier to solve differential equations than partial differential equations. Another example is the absence of the fundamental theorem of algebra for two-dimensional polynomials. For one-dimensional polynomials, the fundamental theorem of algebra states that any one-dimensional polynomial can be factored as a product of lower-order polynomials. This difference has a major impact on many results in signal processing. For example, an important structure for realizing a one-dimensional digital filter is the cascade structure. In the cascade structure, the z-transform of the digital filter's impulse response is factored as a product of lower-order polynomials and the realizations of these lower-order factors are cascaded. The z-transform of a two-dimensional digital filter's impulse response cannot, in general, be factored as a product of lower-order polynomials and the cascade structure therefore is not a general structure for a two-dimensional digital filter realization. Another consequence of the nonfactorability of a two-dimensional polynomial is the difficulty associated with issues related to system stability. In a one-dimensional system, the pole locations can be determined easily, and an unstable system can be stabilized without affecting the magnitude response by simple manipulation of pole locations. In a two-dimensional system, because poles are surfaces rather than points and there is no fundamental theorem of algebra, it is extremely difficult to determine the pole locations. As a result, checking the stability of a two-dimensional system and stabilizing an unstable two-dimensional system without affecting the magnitude response are extremely difficult.

As we have seen, there is considerable similarity and at the same time considerable difference between one-dimensional and two-dimensional signal processing. We will study the results in two-dimensional signal processing that are simple extensions of one-dimensional signal processing. Our discussion will rely heavily on the reader's knowledge of one-dimensional signal processing theories. We will also study, with much greater emphasis, the results in two-dimensional signal processing that are significantly different from those in one-dimensional signal processing. We will study what the differences are, where they come from, and what impacts they have on two-dimensional signal processing applications. Since we will study the similarities and differences of one-dimensional and two-dimensional signal processing and since one-dimensional signal processing is a special case of two-dimensional signal processing, this book will help us understand not only two-dimensional signal processing theories but also one-dimensional signal processing theories at a much deeper level.

An important application of two-dimensional signal processing theories is image processing. Image processing is closely tied to human vision, which is one of the most important means by which humans perceive the outside world. As a result, image processing has a large number of existing and potential applications and will play an increasingly important role in our everyday life.

Digital image processing can be classified broadly into four areas: image enhancement, restoration, coding, and understanding. In image enhancement, images either are processed for human viewers, as in television, or preprocessed to aid machine performance, as in object identification by machine. In image restoration, an image has been degraded in some manner and the objective is to reduce or eliminate the effect of degradation. Typical degradations that occur in practice include image blurring, additive random noise, quantization noise, multiplicative noise, and geometric distortion. The objective in image coding is to represent an image with as few bits as possible, preserving a certain level of image quality and intelligibility acceptable for a given application. Image coding can be used in reducing the bandwidth of a communication channel when an image is transmitted and in reducing the amount of required storage when an image needs to be retrieved at a future time. We study image enhancement, restoration, and coding in the latter part of the book.

The objective of image understanding is to symbolically represent the contents of an image. Applications of image understanding include computer vision and robotics. Image understanding differs from the other three areas in one major respect. In image enhancement, restoration, and coding, both the input and the output are images, and signal processing has been the backbone of many successful systems in these areas. In image understanding, the input is an image, but the output is symbolic representation of the contents of the image. Successful development of systems in this area involves not only signal processing but also other disciplines such as artificial intelligence. In a typical image understanding system, signal processing is used for such lower-level processing tasks as reduction of degradation and extraction of edges or other image features, and artificial intelligence is used for such higher-level processing tasks as symbol manipulation and knowledge base management. We treat some of the lower-level processing techniques useful in image understanding as part of our general discussion of image enhancement, restoration, and coding. A complete treatment of image understanding is outside the scope of this book.

Two-dimensional signal processing and image processing cover a large number of topics and areas, and a selection of topics was necessary due to space limitation. In addition, there are a variety of ways to present the material. The main objective of this book is to provide fundamentals of two-dimensional signal processing and image processing in a tutorial manner. We have selected the topics and chosen the style of presentation with this objective in mind. We hope that the fundamentals of two-dimensional signal processing and image processing covered in this book will form a foundation for additional reading of other books and articles in the field, application of theoretical results to real-world problems, and advancement of the field through research and development.

Contents

1

Signals, Systems, and the Fourier Transform

1.0 INTRODUCTION

Most signals can be classified into three broad groups. One group, which consists of *analog* or *continuous-space* signals, is continuous in both space* and amplitude. In practice, a majority of signals falls into this group. Examples of analog signals include image, seismic, radar, and speech signals. Signals in the second group, *discrete-space* signals, are discrete in space and continuous in amplitude. A common way to generate discrete-space signals is by sampling analog signals. Signals in the third group, *digital* or *discrete* signals, are discrete in both space and amplitude. One way in which digital signals are created is by amplitude quantization of discrete-space signals. Discrete-space signals and digital signals are also referred to as *sequences*.

Digital systems and computers use only digital signals, which are discrete in both space and amplitude. The development of signal processing concepts based on digital signals, however, requires a detailed treatment of amplitude quantization, which is extremely difficult and tedious. Many useful insights would be lost in such a treatment because of its mathematical complexity. For this reason, most digital signal processing concepts have been developed based on discrete-space signals. Experience shows that theories based on discrete-space signals are often applicable to digital signals.

A system maps an input signal to an output signal. A major element in studying signal processing is the analysis, design, and implementation of a system that transforms an input signal to a more desirable output signal for a given application. When developing theoretical results about systems, we often impose

*Although we refer to "space," an analog signal can instead have a variable in time, as in the case of speech processing.

the constraints of linearity and shift invariance. Although these constraints are very restrictive, the theoretical results thus obtained apply in practice at least approximately to many systems. We will discuss signals and systems in Sections 1.1 and 1.2, respectively.

The Fourier transform representation of signals and systems plays a central role in both one-dimensional (1-D) and two-dimensional (2-D) signal processing. In Sections 1.3 and 1.4, the Fourier transform representation including some aspects that are specific to image processing applications is discussed. In Section 1.5, we discuss digital processing of analog signals. Many of the theoretical results, such as the 2-D sampling theorem summarized in that section, can be derived from the Fourier transform results.

Many of the theoretical results discussed in this chapter can be viewed as straightforward extensions of the one-dimensional case. Some, however, are unique to two-dimensional signal processing. Very naturally, we will place considerably more emphasis on these. We will now begin our journey with the discussion of signals.

1.1 SIGNALS

The signals we consider are discrete-space signals. A 2-D discrete-space signal (sequence) will be denoted by a function whose two arguments are integers. For example, $x(n_1, n_2)$ represents a sequence which is defined for all integer values of n_1 and n_2. Note that $x(n_1, n_2)$ for a noninteger n_1 or n_2 is not zero, but is undefined. The notation $x(n_1, n_2)$ may refer either to the discrete-space function x or to the value of the function x at a specific (n_1, n_2). The distinction between these two will be evident from the context.

An example of a 2-D sequence $x(n_1, n_2)$ is sketched in Figure 1.1. In the figure, the height at (n_1, n_2) represents the amplitude at (n_1, n_2). It is often tedious to sketch a 2-D sequence in the three-dimensional (3-D) perspective plot as shown

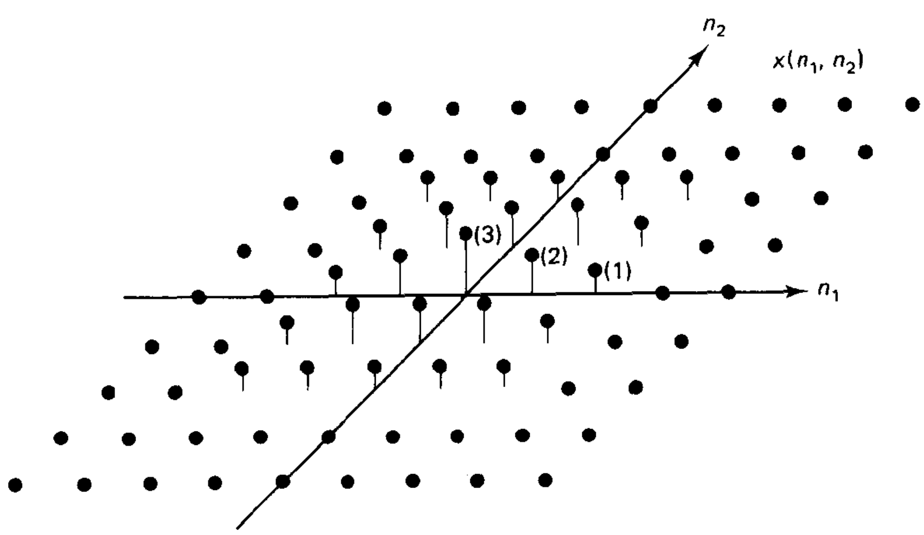

Figure 1.1 2-D sequence $x(n_1, n_2)$.