Volume I Foundations

THE QUANTUM THEORY OF FIELDS

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Preface To Volume I

Why another book on quantum field theory? Today the student of quantum field theory can choose from among a score of excellent books, several of them quite up-to-date. Another book will be worth while only if it offers something new in content or perspective.

As to content, although this book contains a good amount of new material, I suppose the most distinctive thing about it is its generality; I have tried throughout to discuss matters in a context that is as general as possible. This is in part because quantum field theory has found applications far removed from the scene of its old successes, quantum electrodynamics, but even more because I think that this generality will help to keep the important points from being submerged in the technicalities of specific theories. Of course, specific examples are frequently used to illustrate general points, examples that are chosen from contemporary particle physics or nuclear physics as well as from quantum electrodynamics.

It is, however, the perspective of this book, rather than its content, that provided my chief motivation in writing it. I aim to present quantum field theory in a manner that will give the reader the clearest possible idea of *why* this theory takes the form it does, and why in this form it does such a good job of describing the real world.

The traditional approach, since the first papers of Heisenberg and Pauli on general quantum field theory, has been to take the existence of fields for granted, relying for justification on our experience with electromagnetism, and 'quantize' them — that is, apply to various simple field theories the rules of canonical quantization or path integration. Some of this traditional approach will be found here in the historical introduction presented in Chapter 1. This is certainly a way of getting rapidly into the subject, but it seems to me that it leaves the reflective reader with too many unanswered questions. Why should we believe in the rules of canonical quantization or path integration? Why should we adopt the simple field equations and Lagrangians that are found in the literature? For that matter, why have fields at all? It does not seem satisfactory to me to appeal to experience; after all, our purpose in theoretical physics is not just to describe the world as we find it, but to explain — in terms of a few fundamental principles — why the world is the way it is.

The point of view of this book is that quantum field theory is the way it is because (aside from theories like string theory that have an infinite number of particle types) it is the only way to reconcile the principles of quantum mechanics (including the cluster decomposition property) with those of special relativity. This is a point of view I have held for many years, but it is also one that has become newly appropriate. We have learned in recent years to think of our successful quantum field theories, including quantum electrodynamics, as 'effective field theories,' low-energy approximations to a deeper theory that may not even be a field theory, but something different like a string theory. On this basis, the reason that quantum field theories describe physics at accessible energies is that *any* relativistic quantum theory will look at sufficiently low energy like a quantum field theory. It is therefore important to understand the rationale for quantum field theory in terms of the principles of relativity and quantum mechanics. Also, we think differently now about some of the problems of quantum field theories, such as non-renormalizability and 'triviality,' that used to bother us when we thought of these theories as truly fundamental, and the discussions here will reflect these changes. This is intended to be a book on quantum field theory for the era of effective field theories.

The most immediate and certain consequences of relativity and quantum mechanics are the properties of particle states, so here particles come first — they are introduced in Chapter 2 as ingredients in the representation of the inhomogeneous Lorentz group in the Hilbert space of quantum mechanics. Chapter 3 provides a framework for addressing the fundamental dynamical question: given a state that in the distant past looks like a certain collection of free particles, what will it look like in the future? Knowing the generator of time-translations, the Hamiltonian, we can answer this question through the perturbative expansion for the array of transition amplitudes known as the S-matrix. In Chapter 4 the principle of cluster decomposition is invoked to describe how the generator of time-translations, the Hamiltonian, is to be constructed from creation and annihilation operators. Then in Chapter 5 we return to Lorentz invariance, and show that it requires these creation and annihilation operators to be grouped together in causal quantum fields. As a spin-off, we deduce the CPT theorem and the connection between spin and statistics. The formalism is used in Chapter 6 to derive the Feynman rules for calculating the S-matrix.

It is not until Chapter 7 that we come to Lagrangians and the canonical formalism. The rationale here for introducing them is not that they have proved useful elsewhere in physics (never a very satisfying explanation) but rather that this formalism makes it easy to choose interaction Hamiltonians for which the S-matrix satisfies various assumed symmetries. In particular, the Lorentz invariance of the Lagrangian density ensures the existence of a set of ten operators that satisfy the algebra of the Poincaré group and, as we show in Chapter 3, this is the key condition that we need to prove the Lorentz invariance of the S-matrix. Quantum electrodynamics finally appears in Chapter 8. Path integration is introduced in Chapter 9, and used to justify some of the hand-waving in Chapter 8 regarding the Feynman rules for quantum electrodynamics. This is a somewhat later introduction of path integrals than is fashionable these days, but it seems to me that although path integration is by far the best way of rapidly deriving Feynman rules from a given Lagrangian, it rather obscures the quantum mechanical reasons underlying these calculations.

Volume I concludes with a series of chapters, 10–14, that provide an introduction to the calculation of radiative corrections, involving loop graphs, in general field theories. Here too the arrangement is a bit unusual; we start with a chapter on non-perturbative methods, in part because the results we obtain help us to understand the necessity for field and mass renormalization, without regard to whether the theory contains infinities or not. Chapter 11 presents the classic one-loop calculations of quantum electrodynamics, both as an opportunity to explain useful calculational techniques (Feynman parameters, Wick rotation, dimensional and Pauli–Villars regularization), and also as a concrete example of renormalization in action. The experience gained in Chapter 11 is extended to all orders and general theories in Chapter 12, which also describes the modern view of non-renormalizability that is appropriate to effective field theories. Chapter 13 is a digression on the special problems raised by massless particles of low energy or parallel momenta. The Dirac equation for an electron in an external electromagnetic field, which historically appeared almost at the very start of relativistic quantum mechanics, is not seen here until Chapter 14, on bound state problems, because this equation should not be viewed (as Dirac did) as a relativistic version of the Schrödinger equation, but rather as an approximation to a true relativistic quantum theory, the quantum field theory of photons and electrons. This chapter ends with a treatment of the Lamb shift, bringing the confrontation of theory and experiment up to date.

The reader may feel that some of the topics treated here, especially in Chapter 3, could more properly have been left to textbooks on nuclear or elementary particle physics. So they might, but in my experience these topics are usually either not covered or covered poorly, using specific dynamical models rather than the general principles of symmetry and quantum mechanics. I have met string theorists who have never heard of the relation between time-reversal invariance and final-state phase shifts, and nuclear theorists who do not understand why resonances are governed by the Breit–Wigner formula. So in the early chapters I have tried to err on the side of inclusion rather than exclusion.

Volume II will deal with the advances that have revived quantum field theory in recent years: non-Abelian gauge theories, the renormalization group, broken symmetries, anomalies, instantons, and so on.

I have tried to give citations both to the classic papers in the quantum theory of fields and to useful references on topics that are mentioned but not presented in detail in this book. I did not always know who was responsible for material presented here, and the mere absence of a citation should not be taken as a claim that the material presented here is original. But some of it is. I hope that I have improved on the original literature or standard textbook treatments in several places, as for instance in the proof that symmetry operators are either unitary or antiunitary; the discussion of superselection rules; the analysis of particle degeneracy associated with unconventional representations of inversions; the use of the cluster decomposition principle; the derivation of the reduction formula; the derivation of the external field approximation; and even the calculation of the Lamb shift.

I have also supplied problems for each chapter except the first. Some of these problems aim simply at providing exercise in the use of techniques described in the chapter; others are intended to suggest extensions of the results of the chapter to a wider class of theories.

In teaching quantum field theory, I have found that each of the two volumes of this book provides enough material for a one-year course for graduate students. I intended that this book should be accessible to students who are familiar with non-relativistic quantum mechanics and classical electrodynamics. I assume a basic knowledge of complex analysis and matrix algebra, but topics in group theory and topology are explained where they are introduced.

This is not a book for the student who wants immediately to begin calculating Feynman graphs in the standard model of weak, electromagnetic, and strong interactions. Nor is this a book for those who seek a higher level of mathematical rigor. Indeed, there are parts of this book whose lack of rigor will bring tears to the eyes of the mathematically inclined reader. Rather, I hope it will suit the physicists and physics students who want to understand *why* quantum field theory is the way it is, so that they will be ready for whatever new developments in physics may take us beyond our present understandings.

* * *

Much of the material in this book I learned from my interactions over

the years with numerous other physicists, far too many to name here. But I must acknowledge my special intellectual debt to Sidney Coleman, and to my colleagues at the University of Texas: Arno Bohm, Luis Boya, Phil Candelas, Bryce DeWitt, Cecile DeWitt-Morette, Jacques Distler, Willy Fischler, Josh Feinberg, Joaquim Gomis, Vadim Kaplunovsky, Joe Polchinski, and Paul Shapiro. I owe thanks for help in the preparation of the historical introduction to Gerry Holton, Arthur Miller, and Sam Schweber. Thanks are also due to Alyce Wilson, who prepared the illustrations and typed the LaTeX input files until I learned how to do it, and to Terry Riley for finding countless books and articles. For finding various errors in the first printing of this volume, I am greatly indebted to numerous students and colleagues, especially Kevin Cahill. I am grateful to Maureen Storey and Alison Woollatt of Cambridge University Press for helping to ready this book for publication, and especially to my editor, Rufus Neal, for his friendly good advice.

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Notation

Latin indices i, j, k, and so on generally run over the three spatial coordinate labels, usually taken as 1, 2, 3.

Greek indices μ, ν, etc. generally run over the four spacetime coordinate labels 1, 2, 3, 0, with x^0 the time coordinate.

Repeated indices are generally summed, unless otherwise indicated.

The spacetime metric $\eta_{\mu\nu}$ is diagonal, with elements $\eta_{11} = \eta_{22} = \eta_{33} = 1$, $\eta_{00} = -1$.

The d'Alembertian is defined as $\Box \equiv \eta^{\mu\nu}\partial^2/\partial x^\mu \partial x^\nu = \nabla^2 - \partial^2/\partial t^2$, where ∇^2 is the Laplacian $\partial^2/\partial x^i \partial x^i$.

The 'Levi–Civita tensor' $\epsilon^{\mu\nu\rho\sigma}$ is defined as the totally antisymmetric quantity with $\epsilon^{0123} = +1$.

Spatial three-vectors are indicated by letters in boldface.

A hat over any vector indicates the corresponding unit vector: Thus, $\hat{\mathbf{v}} \equiv \mathbf{v}/|\mathbf{v}|$.

A dot over any quantity denotes the time-derivative of that quantity.

Dirac matrices γ_μ are defined so that $\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\eta_{\mu\nu}$. Also, $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$, and $\beta = i\gamma^0$.

The step function $\theta(s)$ has the value $+1$ for $s > 0$ and 0 for $s < 0$.

The complex conjugate, transpose, and Hermitian adjoint of a matrix or vector A are denoted A^*, A^{T}, and $A^\dagger = A^{*\mathrm{T}}$, respectively. The Hermitian adjoint of an operator O is denoted $O^\dagger$, except where an asterisk is used to emphasize that a vector or matrix of operators is not transposed. +H.c. or +c.c. at the end of an equation indicates the addition of the Hermitian adjoint or complex conjugate of the foregoing terms. A bar on a Dirac spinor u is defined by $\bar{u} = u^\dagger \beta$.

Except in Chapter 1, we use units with $\hbar$ and the speed of light taken to be unity. Throughout $-e$ is the rationalized charge of the electron, so that the fine structure constant is $\alpha = e^2/4\pi \simeq 1/137$.

Numbers in parenthesis at the end of quoted numerical data give the uncertainty in the last digits of the quoted figure. Where not otherwise indicated, experimental data are taken from 'Review of Particle Properties,' *Phys. Rev.* **D50**, 1173 (1994).

Contents

Sections marked with an asterisk are somewhat out of the book's main line of development and may be omitted in a first reading.