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## Computation and Interpretation of Biological Statistics of Fish Populations

W. E. Ricker

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# **Computation and Interpretation of Biological Statistics of Fish Populations**

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## PREFACE

This Bulletin is the author's third that deals with the general field of biological statistics of fish populations. The earlier ones date from 1948 and 1958, respectively, and both are long out of print. The present work began as a revision of the 1958 text, but so many changes, additions, and deletions proved desirable that it has become in many respects a new work. Even so, the text does not attempt to include all the developments in this field in recent years. The general plan and arrangement of materials is similar to that of the 1958 Bulletin, but it proved impossible to indicate which passages are new and which are quotations from the earlier volumes. However, where Examples are repeated this has been indicated. Methods which seem conceptually similar are presented in the same chapter, proceeding from the simpler to the more complex as far as possible. Some attention is given also to the historical development of each topic, and this will be considered in somewhat greater detail elsewhere. The amount of space that each topic receives varies with its importance and with its availability. Procedures described in standard western journals are not, as a rule, given detailed development: usually only the formulae most useful for estimating population statistics are quoted, and a discussion of sampling error and of the conditions which make them usable. More extended treatment is given to methods taken from obscure sources and new developments or new aspects of existing methods. This plan does not give ideal balance, but it does perhaps make for maximum usefulness within a limited compass.

In selecting illustrative examples, no attempt has been made to give representation to effort in fishery research on a geographical basis: rather, examples close at hand have usually been selected. The examples from "borrowed" data involve risks of misinterpretation, and are used here to illustrate methodology rather than as a factual treatment of the situations concerned; although, at the same time, I have tried to be as realistic as possible. Some examples have been simplified for presentation here, and others have been invented, in order to keep the text within bounds. However, the practicing biologist quickly discovers that the situations he has to tackle tend to be more complex than those in any handbook, or else the conditions differ from any described to date and demand modifications of existing procedures. It can be taken as a general rule that experiments or observations which seem simple and straightforward will prove to have important complications when analyzed carefully — complications that stem from the complexity and variability of the living organism, and from the changes that take place in it, continuously, from birth to death. Two general precautions should always be taken. Firstly, divide up any body of data into different categories, for example by the size, age, sex, or history of the fish involved, and compare statistics calculated from the two or more subsets obtained in

each such category. Secondly, divide the data on the basis of time (successive hours, days, or seasons), and make similar comparisons.

From the point of view of fishery management, information from computations of the kinds described in this Bulletin provide only a part of the basic information upon which policy can be based. Sometimes, to be sure, they can provide the greater part of the necessary information. In other situations they have as yet given only equivocal answers to important questions. This is particularly true where several species are possible occupants of and competitors for an important environment, and their relative abundance may vary with the intensity of the fishery or with physical changes. The fishery administrator has also the problem (often not an easy one) of selecting an objective which his regulations are designed to serve, and this involves questions of economics and public policy not touched on here. However, there is no question that the increase of biological information has already improved, and will continue to improve, the precision and effectiveness of fishery management.

Some attempt has been made to meet the needs of the beginning student of fishery biology by working out certain examples in detail, even where this consists largely of standard mathematical procedures. To be used as an introductory text book, however, this Bulletin should be "cut down" by omitting less frequently used methods and by choosing one among several alternative procedures where these exist. The choice would depend partly on local problems and interests.

This Bulletin is of course not intended as a complete text book for fishery biologists. Methods of measuring fish, determining their age, marking, tagging, collecting, and tabulating catch statistics — all these are mentioned only incidentally, although they provide the data from which the vital statistics of a stock must be estimated. Books that treat these matters include Chugunova (1959), Gulland (1966), Lagler (1956), Rounsefell and Everhart (1953), Royce (1972), and the International Biological Program handbook edited by Ricker (1971a). Nor are we concerned here with other animals and plants of the environment, with the flow of nutrient energy which maintains an aquatic population, or with the overall productivity of bodies of water. These subjects are discussed in most textbooks of ecology, and various aspects pertinent to fish production are treated in works by Dickie and Paloheimo (1974), Gulland (1971), Moiseev (1969), Nikolsky (1965), Regier (1974), Ricker (1946, 1969b), Walford (1958), Weatherley (1972), and Winberg (1956).

The 1958 Bulletin included a list of individuals who had assisted in various ways. Some of the same people have made further contributions, and I wish to acknowledge also discussions or written comments from K. R. Allen, D. H. Cushing, J. A. Gulland, G. J. Paulik, H. A. Regier, and B. J. Rothschild.

Computer calculations in this Bulletin have been done by K. R. Allen or J. A. C. Thomson. A. A. Denbigh assisted in preparing the new figures. Mrs Barbara Korsvoll prepared the typescript and has assisted with many of the computations.



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