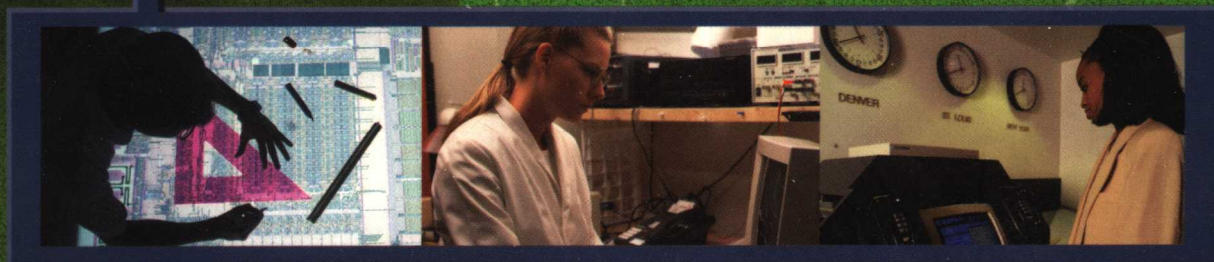


INTERMEDIATE

Algebra

Third Edition



**Streeter
Hutchison
Hoelzle**

INTERMEDIATE

Algebra

Third Edition

James Streeter

Late Professor of Mathematics
Clackamas Community College

Donald Hutchison

Clackamas Community College

Louis Hoelzle

Bucks County Community College

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Instructor's Edition for Intermediate Algebra

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ABOUT THE AUTHORS

About the Authors

While a graduate student at the University of Washington, **James Streeter** paid for his education as a math tutor. It was here that he began to formulate the ideas that would eventually become this package. Upon graduation, he taught for 2 years at Centralia Community College. In 1968 he moved on to Clackamas Community College to become the school's first mathematics chair.

At the college, Jim recognized that he faced a very different population than the one he had tutored at the University of Washington. Jim was convinced that to reach the maximum number of these students, he would have to utilize every medium available to him. Jim opened a math lab that included CAI, original slides and tapes (which were eventually published by Harper & Row), and original worksheets and text materials. With the assistance of the people at McGraw-Hill, that package has been refined to include media and supplements that did not even exist when this project began.

Donald Hutchison spent his first 10 years of teaching working with disadvantaged students. He taught in an intercity elementary school and an intercity high school. He also worked for 2 years at Wassaic State School in New York and 2 years at the Portland Habilitation Center. He worked with both physically and mentally disadvantaged students in these two settings.

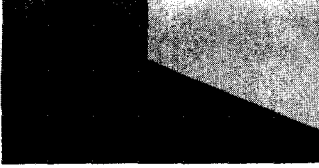
In 1982, Don was hired by Jim Streeter to teach at Clackamas Community College. In 1989, Don became Chair of the Mathematics Department at the college. It was here at Clackamas that Don discovered two things that, along with his family, form the focus for his life. Jim introduced Don to the joy of writing (with the first edition of *Beginning Algebra*), and Jack Scrivener converted him to a born-again environmentalist.

Don is also active in several professional organizations. He was a member of the ACM committee that undertook the writing of computer curriculum for the 2-year college. From 1989 to 1994 he was the Chair of the Technology in Mathematics Education Committee for AMATYC. He was President of ORMATYC from 1996 to 1998.

Louis Hoelzle has been teaching at Bucks County Community College for 27 years. In 1989, Lou became Chair of the Mathematics Department at Bucks County Community College. He has taught the entire range of courses from Arithmetic to Calculus, giving him an excellent view of the current and future needs of developmental students.

Over the past 34 years, Lou has also taught Physics courses at 4-year colleges, which has enabled him to have the perspective of the practical applications of mathematics. In addition, Lou has extensively reviewed manuscripts and written several solutions manuals for major textbooks. In these materials he has focused on writing for the student.

Lou is also active in professional organizations. He has served on the Placement and Assessment Committee for AMATYC since 1989.



PREFACE

Statement of Philosophy

We believe that the key to learning mathematics, at any level, is active participation. When students are active participants in the learning process, they are directly involved in the development of mathematical ideas and can make connections to previously studied material. Such participation leads to understanding, success, and confidence. We developed this text with this philosophy in mind and integrated many features throughout the book to reflect this philosophy. The *Check Yourself* exercises are designed to keep the student involved and active with every page of exposition. The graphing calculator references involve the student actively in the development of mathematical ideas. We have placed great emphasis on a visual approach since it helps students see a problem and its solution in a less abstract setting than symbolic manipulation alone permits. Furthermore, the combination of graphing calculators and the function approach allows us to put a fresh face on some topics that might otherwise be all too familiar to students who are taking their third or fourth algebra class.

Early Introduction of Functions

In the study of college algebra or precalculus, the concept of function plays a central role. Since functions of various types are frequently applied in many fields of study, we believe it essential that our students understand what a function is and be able to distinguish several types of functions before they enter that next level of mathematics. These concepts, coupled with the appropriate use of graphing technology, empower students to solve many new types of equations, inequalities, and application problems. The function theme should therefore permeate an intermediate algebra course. It is a thread that can successfully tie together the traditional topics taught here.

For these reasons, we introduce the concept of function very early in the course. In Chapter 1, relations, linear functions, and their graphs are developed. In Chapter 2, students are introduced to graphical solutions for equations and inequalities. This is a new approach to old problems for almost every student. We find that this new approach has two benefits. For many students, looking at a graphical solution provides a key for understanding the concept of “solution.” For other students it provides a fresh look at material that they have seen many times before. Chapters 1 and 2 are the foundation for the rest of the course.

Why We Changed Our Approach

We were certainly impressed by the AMATYC Standards. Much of what we have included—group exercises, writing exercises, data analysis, functions, elementary statistics concepts, and use of technology—comes directly from the Standards. But seeing these items in the Standards was more confirmation than inspiration. Most of these changes in our text have evolved from the way we teach the course at Clackamas Community College. We find students to be more receptive to new ideas than ever in our Intermediate Algebra courses primarily because of the degree to which this approach involves the students in their education. They are forced to become active learners.

Graphing Calculators

In this text we assume that each student holds, or has access to, a graphing calculator. We consider such machines invaluable aids in the study (and teaching!) of col-

lege mathematics. Since visualization is such an important theme in this course, the instructor will find frequent opportunities to illustrate, or encourage discovery of, important concepts via the graphing calculator. Margin notes accompanying the text also urge the student to view a particular graph on a calculator. Furthermore, with these powerful tools students are able to solve problems that involve messy, real-world data.

We do, however, view the graphing calculator as a tool (albeit a wonderful one), one of many tools to be used in the study of algebra. Our goal is the understanding of concepts, aided by a visual approach. Whether such visualization is provided by a calculator, or by a sketch based on graphing principles, is not critical. Instructors and students must continually decide whether to turn on the calculator or to make a sketch of the problem by hand. This in itself is a valuable process.

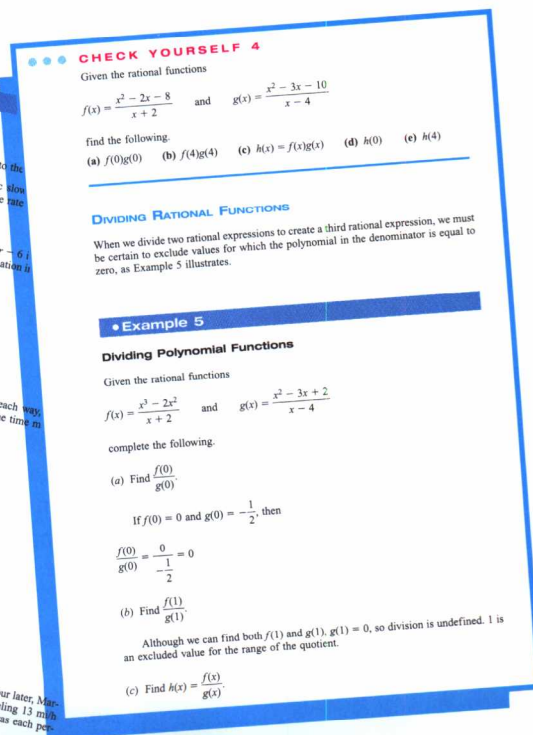
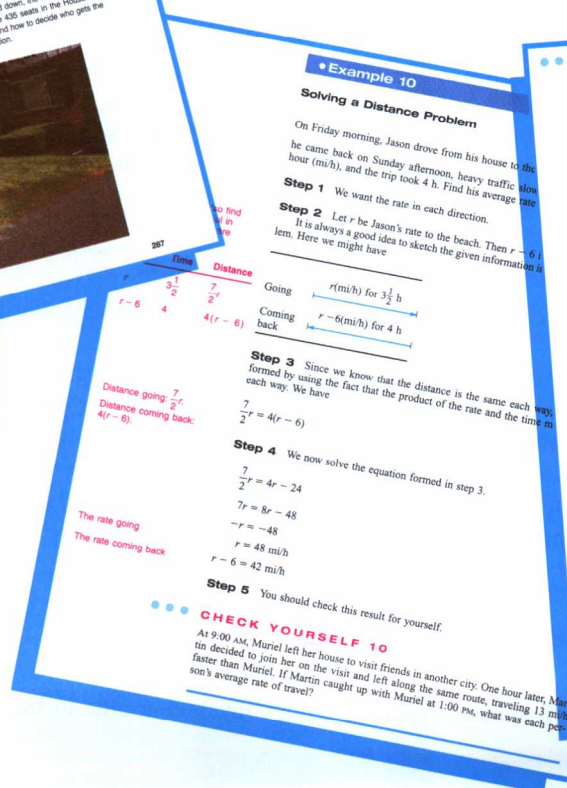
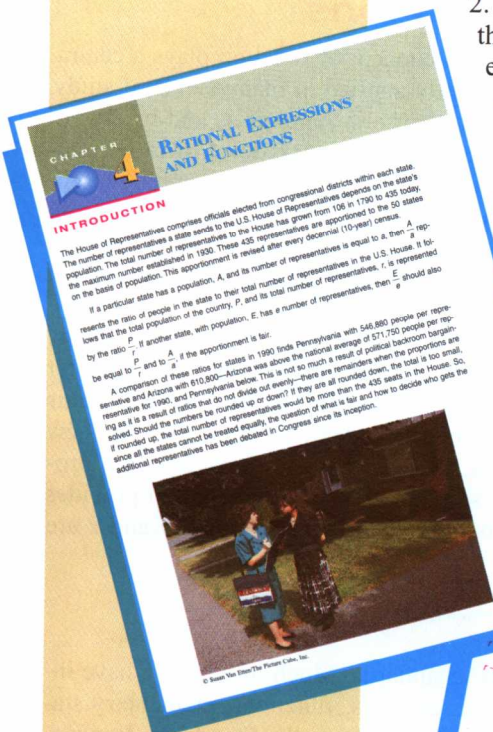
Important Features

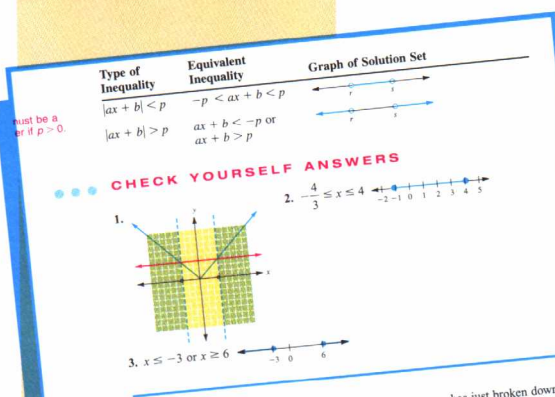
Functions-Based Approach

Students are introduced to functions in Chapter 1 and graphical solutions in Chapter 2. This emphasis on functions and graphing is carried throughout the rest of the text in every subsequent chapter. The exposition and the check yourself exercises have been extensively revised to incorporate this approach.

Vignettes, Applications, Check Yourself Exercises

Those familiar with the Streeter series will recognize most of the pedagogical elements of this text. The chapter-opening **vignettes** provide a sketch of a real event or scenario that requires the use of the concepts of the current chapter as part of the solution. They also provide fodder for interesting **applications** in the exercise sections. The **check yourself exercises** in each chapter engage the students in practice after every example. Answers are provided at the end of the section for immediate feedback.



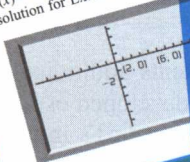


Determining Heating Costs. Ed and Sharon's heating system has just broken down. They can replace their current electrical heat pump system with another electrical heat pump system or replace it with an oil heating system or a gas system. Ed knows that in order to install an oil heating system, he will need a new air conditioner and extensive duct remodeling in his house. He also knows that a gas heating system will require that gas lines be laid on his street and then connected to the house. Both Ed and Sharon are concerned about installing another heat pump system because it does not seem to work well in terms that will be economical in terms of installation costs and yearly

70. Break-Even Analysis A company that produces $C = 40x + 150$ and its revenue in dollars is even?

Graphing Calculator Use your calculator to graph $f(x)$.

71. $f(x) = 3x^2 - 24x + 36$. Note the solution for Exercise 47.



A circle can be graphed on a calculator by plotting the example, to graph $x^2 + y^2 = 16$, we solve for y :

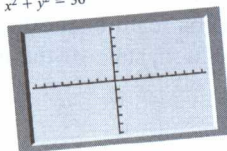
$$y = \pm \sqrt{16 - x^2}$$

This is then graphed as two separate functions,

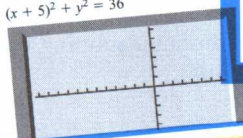
$$y = \sqrt{16 - x^2} \quad \text{and} \quad y = -\sqrt{16 - x^2}$$

In Exercises 39 to 42, use that technique to graph

39. $x^2 + y^2 = 36$



41. $(x + 5)^2 + y^2 = 36$



Self-Test for Chapter 4

The purpose of this self-test is to help you check your progress and to review for a chapter test in yourself about 1 hour to take the test. When you are done, check your answers in the back of the book. If any answers, be sure to go back and review the appropriate sections in the chapter and the exercises that

Simplify each of the following rational expressions.

1. $\frac{-21x^2y^3}{28xy^5}$

2. $\frac{3w^2 + w - 2}{3w^2 - 8w + 4}$

3. $\frac{x^3 + 2x^2 - 3x}{x^3 - 3x^2 + 2x}$

4. Graph the following.

$f(x) = \frac{x^2 - 5x + 4}{x - 4}$

Comprehensive Exercise Sets

Extended **group projects** are also included for more comprehensive application of the section material. The **exercise sets** are complete and varied, providing a reinforcement of basic skills and an opportunity for development of critical thinking and communication abilities. They include multiple views of every concept in the section, including **graphing calculator exercises**, **challenging exercises**, and **writing exercises**. Many of the exercise sets also contain application exercises that are extensions of the chapter-opening vignettes. The **group exercises** provide opportunities to work on teamwork, exploration, and conjecture.

Study Aids

Every chapter ends with a **summary**, **summary exercises**, and a **chapter self-test**. **Cumulative tests** are placed after every chapter starting at Chapter 2 and help students retain mastery of earlier topics so that they are better prepared for the final exam at the end of the course.



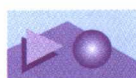
Summary

Simplifying Rational Expressions [4.1]

$\frac{x^2 - 5x}{x - 3}$ is a rational expression. The variable x cannot have the value 3.

Rational expressions have the form $\frac{P}{Q}$ where P and Q are polynomials and Q cannot be 0.

We can multiply a rational expression by the same nonzero polynomial to simplify it.



Summary Exercises

This summary exercise set is provided to give you practice with each of the objectives in the chapter. It is keyed to the appropriate chapter section. The answers are provided in the *Instructor's Manual*.

[4.1] For what value of the variable will each of the following rational expressions be undefined?

1. $\frac{x}{2}$

2. $\frac{3}{y}$

3. $\frac{2}{x - 5}$

4. $\frac{3x}{2x - 5}$

[4.1] Simplify each of the following.

5. $\frac{18x^5}{24x^3}$



Cumulative Test for Chapters 1-2

This test is provided to help you in the process of reviewing the previous chapters. Answers are provided in the back of the book. If you missed any answers, be sure to go back and review the appropriate chapter sections.

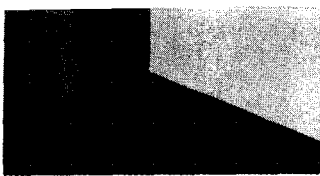
For each of the following sets of numbers, identify the domain and range.

1. $\{(2, 7), (3, 5), (-1, 1), (-2, 0), (4, 5)\}$

2. $\{(x, y) | 6x - 5y = 60\}$

Solve each of the following equations.

3. $2(3x + 9) = 8(2 + x)$



SUPPLEMENTS

Supplements

A comprehensive set of ancillary materials for both the student and the instructor is available with this text.

Instructor's Edition

This ancillary includes answers to all exercises and tests. These answers are printed in a second color for ease of use by the instructor and are located on the appropriate pages throughout the text.

Instructor's Solutions Manual

The manual provides worked-out solutions to the even-numbered exercises in the text.

Instructor's Resource Manual

The resource manual contains multiple-choice placement tests for three levels of testing: (1) a diagnostic pretest for each chapter and three forms of multiple-choice and open-ended chapter tests; (2) two forms of multiple-choice and open-ended cumulative tests; and (3) two forms of multiple-choice and open-ended final tests. Also included is an answer section and appendixes that cover collaborative learning and the implementation of the new standards.

Print and Computerized Testing

The testing materials provide an array of formats that allow the instructor to create tests using both algorithmically generated test questions and those from a standard testbank. This testing system enables the instructor to choose questions either manually or randomly by section, question type, difficulty level, and other criteria. Testing is available for IBM, IBM-compatible, and Macintosh computers. A softcover print version of the testbank provides most questions found in the computerized version.

Streeter Video Series

The video series is completely new to this edition. It gives students additional reinforcement of the topics presented in the book. The videos were developed especially for the Streeter pedagogy, and features are tied directly to the main text's individual chapters and section objectives. The videos feature an effective combination of learning techniques, including personal instruction, state-of-the-art graphics, and real-world applications.

Multimedia Tutorial

This interactive CD-ROM is a self-paced tutorial specifically linked to the text and reinforces topics through unlimited opportunities to review concepts and practice problem solving. It requires virtually no computer training on the part of the students and supports IBM and Macintosh computers.

MathWorks

This DOS-based interactive tutorial software is available and specifically designed to accompany the Streeter pedagogy. The program supports IBM, IBM-compatible, and Macintosh computers as well as a variety of networks. MathWorks can also be used with its companion program, the Instructor's Management System, to track and record the progress of students in the class.

In addition, a number of other technology and Web-based ancillaries are under development; they will support the ever-changing technology needs in developmental mathematics. For further information about these or any supplements, please contact your local WCB/McGraw-Hill sales representative.

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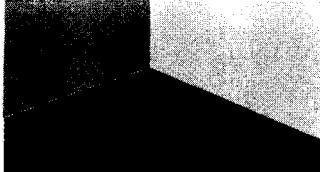
Through three editions and a revision, the list of contributors to this text has grown steadily. Foremost are the users and the reviewers. We do not have space to individually thank each user, but suffice it to say that every comment that has reached us has helped in the evolution of this text. We do have space for thanks to each reviewer. Every person on the list has helped by questioning, correcting, or complimenting. Our thanks to each of them.

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Kenneth Hufham, Cape Fear Community College (NC)
Robert Farinelli, Community College of Allegheny County (PA)
James Warren Fightmaster, Virginia Western Community College
Roseann Foglio, Gloucester Community College (NJ)
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Debra Sample, Arkansas State University
Hugh Sanders, Georgia College
Richard Semmler, Northern Virginia Community College
Randy Taylor, Las Positas College (CA)
Carol Walker, Hinds Community College (MS)
Raymond Whaley, Waubesa College (IL)

The staff at WCB/McGraw-Hill has had an incredible knack for providing what was needed. Sometimes it was a new direction, sometimes it was a friend, always it was time. Thanks to Tom, Maggie, Eva, Jack, and Paul. As always, we encourage prospective authors to talk with the people at WCB/McGraw-Hill. It will be a valuable use of your time.

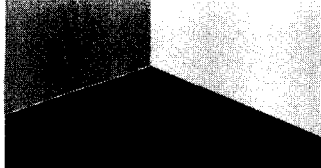
In this age of the high-tech mathematics lab, the supplements and the supplement authors have become as important as the text itself. We are proud that our names appear on the supplements together with the following authors: John Garlow, Tarrant County Junior College; Gloria Langer; John Robert Martin, Tarrant County Junior College; and Mark Stevenson, Oakland Community College.

We must also thank all the students who have been taught, talked to, questioned, and tested. This text was created for them, this text was created by them. We also want



to recognize the contributions of Zanae Rodrigo, whose attention to detail made this a much better text. Finally, we want to thank our wives, Claudia and Rose, and our children, Trinh, Micol, Jacob, and Christine Hutchison and Beth, Amy, Meg, Jo, and Patrick Hoelzle. This group has been patient beyond reason and wise beyond their years.

Donald Hutchison
Louis Hoelzle



TO THE STUDENT

To The Student

Intermediate Algebra is the final step in your preparation for college transfer mathematics. In this course you will cover many new concepts and see the expansion of several familiar ones. All the topics in this text are designed to prepare you for subsequent courses in Pre-Calculus, College Algebra, or Statistics. We have provided situations and applications that will help you understand the relevance of what you are learning. As your mathematical expertise expands in succeeding courses, the applications of your skills will become even more diverse and interesting. Your ability to succeed in those later courses will very much depend on how well you understand the material covered in this text. The following suggestions are designed to help you succeed in this, and future, mathematics classes.

1. If you are in a lecture class, take the time to read the appropriate section before the lecture. Take careful notes of every example your instructor presents in class.
2. When you sit down to study, have your calculator, pencil, and paper ready. Work through the examples in the text. Do every *Check Yourself* exercise, checking your answer against the one at the end of the section. If you have difficulty, go back and reread the previous example. Make certain you understand what you are doing and why. The best test of whether you do understand a concept lies in your ability to explain that concept to a classmate. Try working together.
3. At the end of each section is a set of exercises. Work these carefully in order to check your progress on the section just completed. The answers to odd-numbered exercises are at the end of the section. If you have difficulty with any of the exercises, review the appropriate parts of the section. If your confusion is not completely cleared up, by all means do not become discouraged. Ask your instructor or an available tutor for assistance. A word of caution: Work the exercises on a regular (preferably daily) basis. Learning algebra requires active participation on your part. As is the case with the learning of any skill, the main ingredient is practice.
4. When you have completed a chapter, review by using the *Summary*. Following the summary are *Summary Exercises* for further practice. The exercises are keyed to chapter sections so you will know where to turn if you are still struggling.
5. When finished with the Summary Exercises, try the *Self-Test* that appears at the end of each chapter. Answers, with section references, are in the back of the book.
6. Finally, an important element of success in studying algebra is the process of regular review. We provide a series of *Cumulative Reviews* throughout this book. Use these tests to prepare for any midterms or finals. If it appears that you have forgotten some concepts from earlier chapters, do not worry. Go back and review the section where the idea was first explained, or the appropriate chapter summary; that is the purpose of the *Cumulative Tests*.

We hope you will find these suggestions helpful as you work through the material in this text. Best of luck in this course!

Donald Hutchison
Louis Hoelzle

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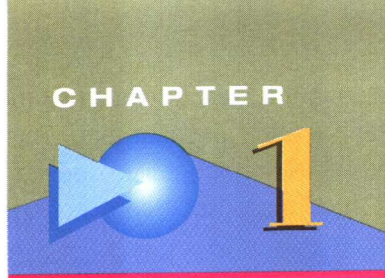
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CHAPTER

ORDERED PAIRS AND LINEAR FUNCTIONS

INTRODUCTION

Data are usually presented in one of three forms: a table, a formula, or a graph. Graphs often show trends that may not be easy to see when data are read from a table, or when a rule is given in a formula.

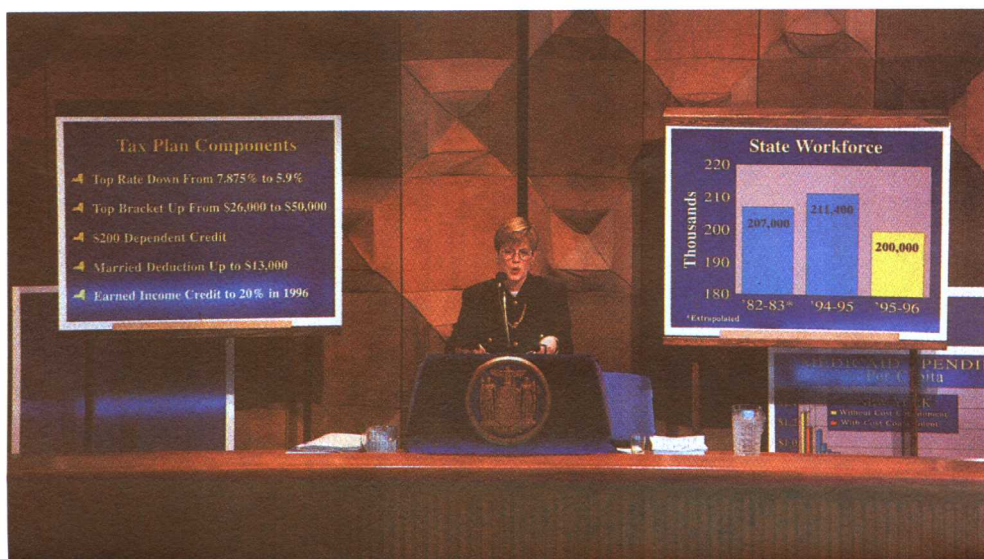
Economists are among the many professionals who use graphs to show connections between two sets of data. For example, an economist may use a graph to look for a connection between two different measures for the standard of living in various countries.

One way of measuring the standard of living in a country is the per capita gross domestic product (GDP). The GDP is the total value of all goods and services produced by all businesses and individuals over the course of 1 year. To find the per capita GDP, we divide that total value by the population of the country.

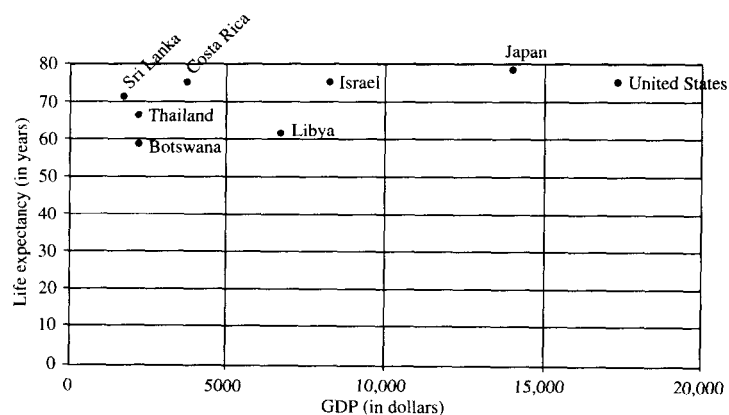
Other economists, including some who wrote an article in *Scientific American* in May 1993, question this method. Rather than comparing GDP among countries, they use survival rate (life expectancy) to measure the quality of life. The following table compares life expectancy and GDP.

Country	Life Expectancy at Birth (in years)	Per Capita GDP (in dollars)
Botswana	59	\$ 2,496
Libya	62	7,250
Sri Lanka	71	2,053
Costa Rica	75	3,760
Thailand	66	2,576
Israel	76	9,182
United States	76	17,615
Japan	78	13,135

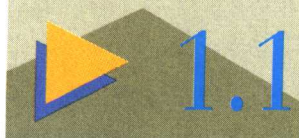
Adapted from *The Human Development Report*, United Nations, Oxford Press: Cambridge, England, 1990.



Not many people could quickly read and interpret the information in the table. Although all the information is available, it is not as easy to see as in the following graph (called a scatter plot). This graph contains the same information, but in a form that is easier to interpret. Note that, if life expectancy is a good measure of the standard of living, productivity may not be a good measure. It does not appear to be the case that higher productivity leads to a longer life.



A graph such as this one helps us to visualize complex information. This kind of graph is created using ordered pairs. We will introduce ordered pairs in this first chapter and then use them throughout the book.



Ordered Pairs and Relations

1.1 OBJECTIVES

1. Identify ordered pairs
2. Identify relations
3. Identify the domain and range of a relation

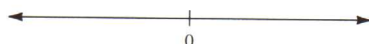


The number line extends infinitely in both directions.

ALGEBRA

Algebra is a system consisting of a set, together with operations, that follow certain properties. The word **algebra** comes from the title of a book written in the ninth century by Al-Khowarazmi. It described the techniques used in solving equations. Even after more than a 1000 years, solving equations and simplifying expressions continue to be the primary topics of an algebra course taught at this level.

In intermediate algebra, most of our work will be with the set of real numbers. These are all the numbers that can be found on a number line.



In set notation, we refer to the set of all real numbers as $\{x \mid x \in \mathbb{R}\}$, which is read, "The set of all numbers x such that x is a real number."

ORDERED PAIRS

We will usually be interested in the relationship between two numbers. Whenever this is the case, the order in which we present the two numbers will be important, which is why we will call them **ordered pairs**.

Given two related numbers, x and y , we will write the pair of numbers as (x, y) . The set of all possible ordered pairs of real numbers will then be written as follows: $\{(x, y) \mid x \in \mathbb{R}, y \in \mathbb{R}\}$.

The ordered pair $(2, -3)$ is different from the ordered pair $(-3, 2)$. Whereas, the set of ordered pairs $\{(1, 3), (4, -1)\}$ is the same as the set of ordered pairs $\{(4, -1), (1, 3)\}$. Order is important in an ordered pair. Order is not important in a set.

Note that sets always use **braces** to enclose their contents.

• Example 1

Identifying Ordered Pairs

Which of the following are ordered pairs?

- (a) $(2, -\pi)$ (b) $\{2, -4\}$ (c) $(1, 3, -1)$
(d) $\{(1, -5), (9, 0)\}$ (e) $2, 5$

Only (a) is an ordered pair. (b) is a set (it uses braces instead of parentheses), (c) has three numbers instead of two, (d) is a set of ordered pairs, and (e) is simply a list of two numbers.

• • • CHECK YOURSELF 1

Which of the following are ordered pairs?

- (a) $\left\{\frac{1}{2}, -3\right\}$ (b) $\left(-3, \frac{1}{3}\right)$ (c) $\{(5, 0)\}$ (d) $(1, -5)$ (e) $-3, 6$

RELATIONS

A set of ordered pairs is called a **relation**.

The ordered pairs can either be listed explicitly, or implied. Examples of relations include

$$A = \{(-2, 8), (1, 2), (-3, -5)\}$$

$$R = \{(1, 3), (1, 4), (1, 5)\}$$

$$B = \{(x, y) | y = x\}$$

Relation B is comprised of every ordered pair in which x and y are equal. This relation includes $(0, 0)$, $(\frac{1}{2}, \frac{1}{2})$, $(-3, -3)$, and (π, π) .

DOMAIN AND RANGE

In most cases, the domain is the set of all x values, and the range is the set of all y values.

The **domain** of a relation is the set of all the first elements of the ordered pairs. The **range** of a relation is the set of all the second elements of the ordered pairs.

• Example 2

Finding the Domain and Range of a Relation

Find the domain and range for each relation.

(a) $A = \{(-2, 8), (1, 2), (-3, -5)\}$

The domain is the set $\{-2, 1, -3\}$. The range is the set $\{8, 2, -5\}$.

(b) $R = \{(1, 3), (1, 4), (1, 5)\}$

The domain is the set $\{1\}$. The range is the set $\{3, 4, 5\}$.

• • • CHECK YOURSELF 2

Find the domain and range for each relation.

(a) $A = \{(-5, 4), (-4, 7), (-4, 9)\}$ (b) $R = \{(1, 0), (3, 0), (5, 0)\}$

It is also possible to find the domain when the relation is described by rule rather than by list. When looking for the domain from a rule, the question to ask is, "What values can I substitute for x ?" When looking for the range, we ask, "What values can be substituted for y ?"