Robert M. Caddell

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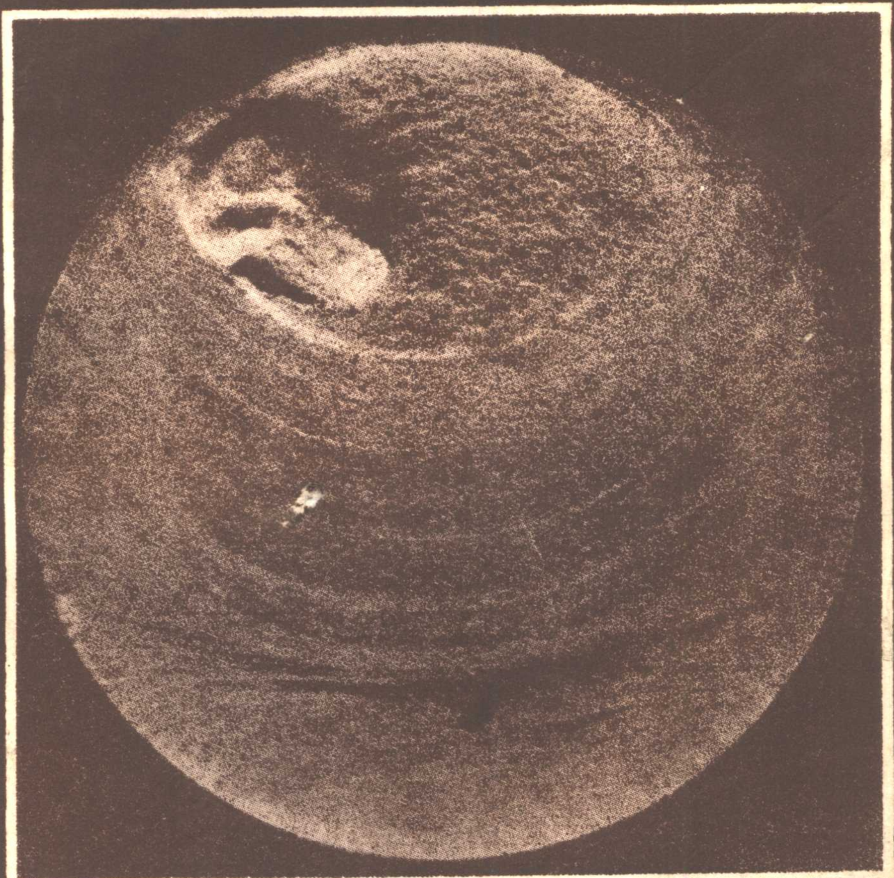

DEFORMATION AND FRACTURE OF SOLIDS

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PREFACE

The mechanical behavior of solids, as expressed by deformation and fracture, is a field of fundamental importance in engineering. Yet, most undergraduate curricula provide only a cursory exposure to this subject. This conclusion stems from my personal experience that includes over twenty years of teaching both undergraduate and graduate students from a variety of engineering disciplines and whose prior background was obtained at numerous universities and colleges. My major teaching involvement with this subject has been with an elective course that is populated with seniors and first-year graduate students. They have studied in such fields of engineering as Mechanical, Naval Architecture, Materials and Metallurgy, Aerospace and Applied Mechanics. Perhaps the major challenge that arises in such a situation is to accommodate the variety of backgrounds of such a diverse group of individuals. Most have had a course in the Science of Engineering Materials and a traditional Strength of Materials approach to design. Some have pursued further work in metallurgy, mechanical design, and stress analysis; however, practically none has been exposed to the important subject of fracture. Yet even in regard to the most basic concepts that have been covered in earlier studies, I have found an unawareness and confusion that is perplexing but which must be overcome before proceeding with other topics. The format and content of this book constitute an approach that has been successful in practice.

Chapters 1 through 3 provide a concise review of stress, strain and elasticity that should clarify any misconceptions that students usually exhibit. The use of Mohr's circle, is more detailed than one would find in most similar texts, the reason arising from the observation that most students neither appreciate the limitations of this technique nor fully understand the basis behind it. Because traditional courses in design and materials have stressed elastic behavior of solids, the importance of plasticity is usually ignored. For that reason, Chaps. 4 and 5 contain more extensive coverage than is usually found in other texts of this nature. Time-dependent behavior is discussed in Chap. 6 where the main emphasis is on the importance of rate equations in the study of all the parameters that are influenced by time effects. Chapter 7 is not intended to satisfy experts in dislocation theory but the approach taken does provide a useful and direct way to explain many important macroscopic observations of importance to engineers. The major concepts of fracture and fracture mechanics are covered in Chap. 8. This has become such an important subject in certain areas of modern design that it should be a part of undergraduate study, yet it receives little if any attention at most schools. Besides covering the concepts and use of the stress intensity factor and strain energy release rate, an approach using an energy balance is presented. The latter is not found in other books yet it provides a better introduction to the physical meaning of fracture toughness as determined by experiment. Chapter 9 introduces some elementary ideas in regard to composite materials. There is no doubt that their use in engineering applications will increase in the years ahead and the brief coverage in this chapter will point out their desirable features as well as limitations. A broad overview of fatigue constitutes Chap. 10. Both the traditional design approach and the more recent concepts involving fracture mechanics are covered to a meaningful extent. A brief summary of failure analysis completes this chapter.

Having used several other texts in the past, the opinions and complaints of many students have guided me in writing this book. Often, because of their length, similar texts cannot be covered reasonably in a single semester. Yet students must pay for the full book, usually with some chagrin. To overcome this complaint, I have attempted to pare topics to a minimal but essential level and have omitted some completely. This is more of a personal choice than an implication that such topics are of lesser importance. Any teacher using this book can easily fill this void with a minimal number of handout notes. In response to other student complaints, a number of example problems are provided in each chapter and, where necessary, derivations are included in full measure. Numerous end-of-chapter problems include some that are directed towards basic definitions and others that pose more of a challenge.

Both the English and SI unit notations are used and important conversion factors are summarized after this preface. If one is objective, there is no question that the SI system is more convenient. To a large extent a bias for English units, based upon nothing more than tradition, must be overcome. Interspersing the use of both systems should assist in reducing reluctance towards what appears to be inevitable.

Most problems are self-contained with regard to property values needed for a solution, but in a number of instances deliberate omissions have been made. This forces students to check other sources and anyone who has practiced engineering will agree that this is often a critical part of real problem solving. Students must be made aware of this and the sooner the better.

References are limited to papers on basic developments or to other texts that provide further detail on topics of concern. Except in research problems, students seem disinterested in checking references. By limiting the number, it is hoped that these younger readers will develop this important habit.

My indebtedness to certain colleagues and former students must now be paid. In particular Drs. A. G. Atkins, D. K. Felbeck, W. F. Hosford Jr., K. C. Ludema, and Y. W. Mai have provided many incisive comments and discussions during our years of acquaintance. To segregate their inputs in a specific way is an impossible task; it must suffice to pay my thanks to each of them. Of the many students who have given constructive aid I must mention Drs. R. S. Raghava and A. R. Woodliff. They have provided the type of student input that all teachers desire. Finally I must thank Ms. Karen Almas, Ms. Mary Anne Brocious, Ms. Gloria Hartman, and Mrs. Karen Chapin for the many hours of excellent assistance they provided in the typing of this text. Their patience and care have been invaluable.

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Some Basic Units and Their Abbreviations as Specified for The International System of Units (SI)

Unit	*Standard*	*Abbreviation*
length	meter	m
mass	kilogram	kg
time	second	s
*force	Newton	N = $kg \cdot m/s^2$
*stress	$Newton/meter^2$	N/m^2
*stress or pressure	Pascal = 1 N/m^2	Pa
*energy	Joule	$N \cdot m$

**These are derived from basic units.*

Multiplication Factors Used in the SI System

Factor	*Prefix*	*Symbol*	*Factor*	*Prefix*	*Symbol*
10^{12}	tera	T	10^{-6}	micro	μ
10^{9}	giga	G	10^{-9}	nano	n
10^{6}	mega	M	10^{-12}	pico	p
10^{3}	kilo	k	10^{-15}	femto	f
10^{-3}	milli	m	10^{-18}	atto	a

Useful Conversion Factors—English to SI Units

To convert from	*to*	*multiply by*
inch	meter	2.54×10^{-2}
feet	meter	3.048×10^{-1}
$inch^2$	$meter^2$	6.452×10^{-4}
$feet^2$	$meter^2$	9.29×10^{-2}
$inch^3$	$meter^3$	1.639×10^{-5}
$feet^3$	$meter^3$	2.832×10^{-2}
pound-force	Newton	4.448
$pounds/inch^2$	$Newton/meter^2$	6.895×10^{3}

Other Useful Relations

1 micron = 10^{-4} cm = 10^{-6} m
1 Angstrom = 10^{-8} cm = 10^{-10} m
1 $dyne/cm^2$ = 1.44×10^{-5} $pounds/inch^2$
10 Angstroms = 1 nm
For body-centered cubic unit cell, $a_0 = 4r/\sqrt{3}$
For face-centered cubic unit cell, $a_0 = 4r/\sqrt{2}$
where r = atomic radius and a_0 = lattice parameter

CONTENTS

1

STRESS

1-1 INTRODUCTION

Stress is defined as the intensity of force per unit area and many problems have been solved with the use of this simple concept. Experiments involving uniaxial tension, compression and torsion are most often used to provide a physical illustration of the above definition. Yet this introductory concept does not provide a full appreciation of the nature of stress. Biaxial or triaxial states of stress are often encountered and excessive emphasis on one-dimensional situations can lead to decided misconceptions.

Initially we used the words *force* and *area* above; both are vector quantities requiring magnitude and direction for their full description. Stress requires two sets of direction cosines for its complete description and if it is desired to transform the stress components from one set of coordinate axes to a different set, then certain mathematical *rules* must be followed. This is called tensor transformation and a knowledge of these rules and accompanying notation provides great convenience, since simple shorthand expressions are used to describe the state of stress. A background in tensor analysis is not essential for the purposes of this text but it is important to realize that:

1. *Stress* is a second-order tensor (usually just called a tensor) involving two sets of direction cosines for transformation purposes.
2. *Force* is a first-order tensor (usually called a vector) involving one set of direction cosines for transformation purposes.
3. *Temperature* is a zero-order tensor (usually called a scalar) and is independent of any direction notation.

Other physical examples could be used, but the major purpose of these remarks is to indicate that stress is *not* a vector.

Situations can be visualized where the state of stress varies throughout a body subjected to loading. Consider the simple tension test of a ductile metal after necking has occurred. Although the *force* is common throughout the length, the change in area (plus the geometry) causes the stress to vary from point to point. To accommodate this variation the concept of the *state of stress at a point* is used.

Consider an elemental force, dF, acting at a point P included in an elemental area, dA, as shown in Fig. 1-1(a). In the most general sense, dF is neither

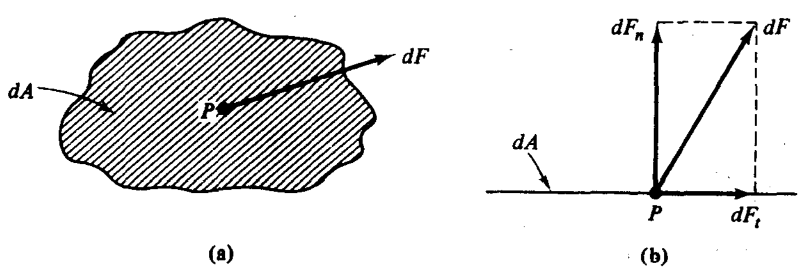

Figure 1-1 Forces acting on an elemental area showing total force (a) and resolved forces (b).

normal to dA nor parallel to it, as in Fig. 1-1(b), and dF may be reduced to the normal and tangential components dF_n and dF_t. As $dA \longrightarrow 0$ in the limit, stress is *defined* as follows:

$$\sigma = \frac{dF}{dA}, \qquad \sigma_n = \frac{dF_n}{dA}, \qquad \sigma_t = \frac{dF_t}{dA}$$

where σ is the *total state of stress* at P and σ_n and σ_t are the normal and tangential components. Note that the same *area* is involved in each of the above definitions.

Now if P is envisioned to lie at the centroid of a very small parallelepiped (this is a mathematical concept only) where any changes across this model are very small, the state of stress at point P is considered to be homogeneous and is described by the model. If numerous forces act on a body, their effect at P (using an arbitrary coordinate system) is described by Fig. 1-2. For

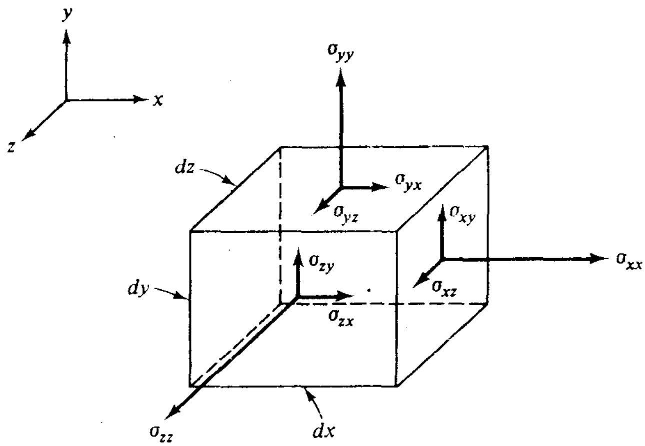

Figure 1-2 Stress element for a homogeneous state of stress at a point. As shown, all stress components are positive by convention.

stationary equilibrium, each face opposite to the three shown has identical stresses acting such that a force balance would give $\sum F_x = 0$ while the absence of rotation means $\sum M_{P_z} = 0$. Similar relations apply with regard to the y and z directions.

The nine components of stress which comprise the *stress tensor* are often described as σ_{ij} where:

$$\sigma_{ij} = \begin{vmatrix} \sigma_{xx} & \sigma_{yx} & \sigma_{zx} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{zy} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{vmatrix}$$

with i and j being iterated over x, y, and z. Since tensor analysis will not be used in this text, the usual tensor notation need not be strictly adhered to. In most instances, normal stresses (tensile or compressive) will be denoted as σ_x, σ_y, σ_z and shear stresses as τ_{xy}, τ_{yz}, τ_{zx}. Note that if $\sum M_p = 0$, $\tau_{xy} = \tau_{yx}$, and so forth, thus the nine components of the stress tensor reduce to six. Conventionally, tensile normal stresses are considered positive while compressive stresses are negative, so all normal stresses in Fig. 1-2 are positive. *It*

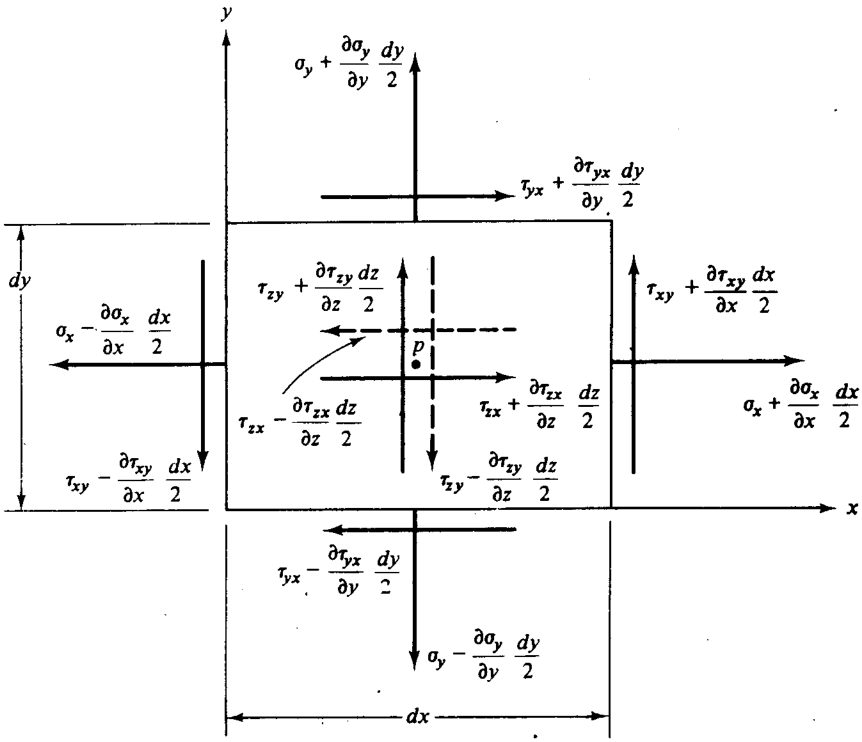

Figure 1-3 Equilibrium of stresses at a point as seen in the *x-y* plane.

is also essential to follow a consistent convention in regard to shear stresses. In this text, positive stresses are *defined* by Fig. 1-2.

If the state of stress varies from point to point this type of change must be accommodated as shown in Fig. 1-3, using the *x-y* plane for reference. Note the following:

1. In moving away from P, any rate of change is assumed to be linear.
2. The sign before any term showing a rate of change does not mean that the change itself is dictated by that sign. For example, the term $\sigma_x + (\partial\sigma_x/\partial x)(dx/2)$ does not imply that the change in σ_x is necessarily increasing as one moves in the positive x direction.
3. Stress components involving σ_z, τ_{yz}, and τ_{xz} are all perpendicular to the *x-y* plane and have no force components parallel to this plane.

Considering a force summation in the x direction, only the terms including σ_x, τ_{yx}, and τ_{zx} are of concern. If the element is in equilibrium, then

$$\sum F_x = 0 - \sum \text{(stress) (area)}$$

This produces the following, where the derivation is left as an exercise:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0 \qquad (1\text{-}1a)^*$$

An identical procedure, using the y and z directions, would produce:

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} = 0 \qquad (1\text{-}1b)$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0 \qquad (1\text{-}1c)$$

These are the *equilibrium equations* and although the terms all involve stresses, it should be remembered that they were derived using a force balance.

Three approaches to stress analysis are presented in the rest of this chapter. The first involves an elementary approach to the generalized three-dimensional problem where a known stress state, as related to a particular coordinate system, is transformed to another coordinate system. The second development provides the systematic *transformation* equations which are no more than a series of mathematical expressions that fully encompass the findings of the elementary approach. Finally, because it is such a useful and powerful tool, we devote more attention than usual to the use of Mohr's circle. Although there may appear to be a degree of redundancy in the developments that follow, it is intended that the coverage of stress using different approaches will provide a sound understanding of this important subject.

1-2 A GENERALIZED THREE-DIMENSIONAL ANALYSIS OF A HOMOGENEOUS STRESS STATE

Often, it is important to determine the state of stress on a plane at some angle to those upon which the applied stresses act. Although the state of stress at a point P is not altered, the components of stress acting upon planes other than those which comprise the elemental model will differ. This can be shown most readily by passing an arbitrary plane through the model as shown in Fig. 1-4(a) where the stresses σ_x, τ_{xy}, etc., are known, and the stresses acting upon ABC are to be found in terms of the known stresses.

Considering OP to be normal to ABC, its line of orientation with respect to the x-y-z coordinate system is defined by the three direction cosines shown in Fig. 1-4(b). Let the *total* stress acting on ABC (for the most general analysis) be neither perpendicular nor parallel to this plane; it is indicated as S. The total *force* acting on ABC would be the area of this plane multiplied by

*Effects of gravity, acceleration, etc., are of no concern in this book.

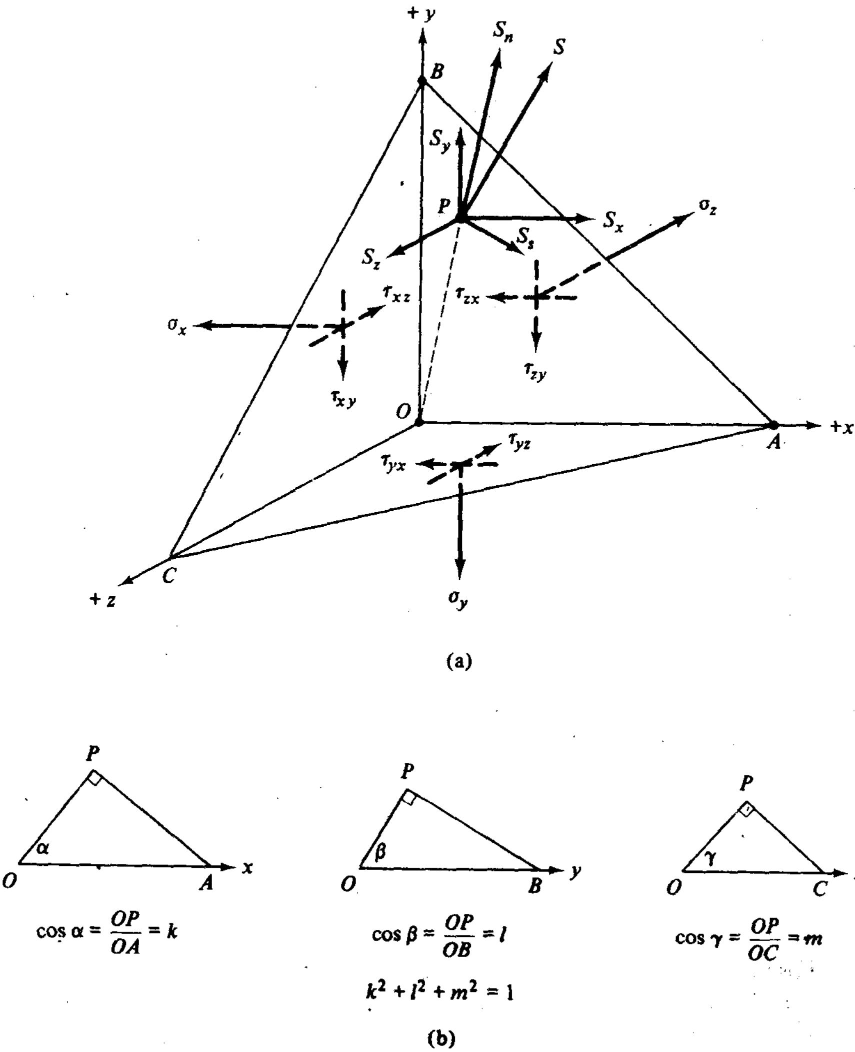

Figure 1-4 (a) Stresses acting on plane *ABC* arising from applied stresses shown acting upon three sides of the original stress element; (b) Definition of direction cosines of line *OP*.

S; this force could then be described by components parallel to the original coordinate system. If each of these force components were then divided by the area of *ABC*, this would produce stress components S_x, S_y, and S_z as indicated. Because a common *area* is used throughout, we get the following result:

$$S^2 = S_x^2 + S_y^2 + S_z^2 \tag{1-2}$$