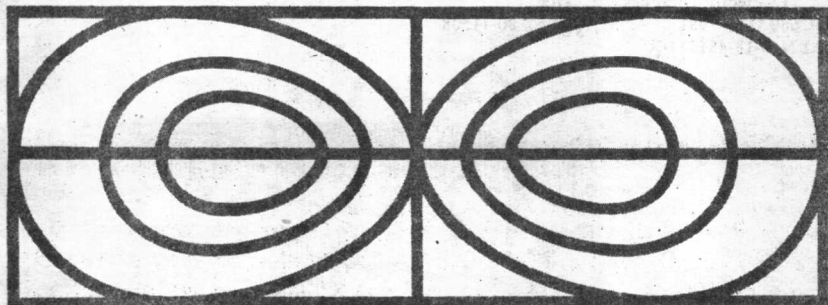


INTRODUCTION TO ELECTRODYNAMICS

GRIFFITHS

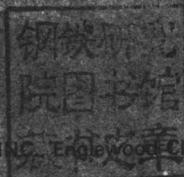
O 442
G 855



INTRODUCTION TO ELECTRODYNAMICS

DAVID J. GRIFFITHS

Physics Department
Read College



PRENTICE-HALL, INC. Englewood Cliffs 07632

304446

Library of Congress Cataloging in Publication Data

GRIFFITHS, DAVID, DATE

Introduction to electrodynamics.

Includes index.

1. Electrodynamics. I. Title.

QC680.G74 1981 537.6 80-11508

ISBN 0-13-481374-X

© 1981 by Prentice-Hall, Inc.
Englewood Cliffs, New Jersey 07632

All rights reserved. No part of this book
may be reproduced in any form or by any means
without permission in writing from the publisher.

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

Editorial Production/Supervision by Theodore Pastrick

Design by Sue Behnké

Page layout by Diane Doran Sturm

Manufacturing Buyer: John Hall

PRENTICE-HALL INTERNATIONAL, INC., *London*
PRENTICE-HALL OF AUSTRALIA PTY. LIMITED, *Sydney*
PRENTICE-HALL OF CANADA, LTD., *Toronto*
PRENTICE-HALL OF INDIA PRIVATE LIMITED, *New Delhi*
PRENTICE-HALL OF JAPAN, INC., *Tokyo*
PRENTICE-HALL OF SOUTHEAST ASIA PTE. LTD., *Singapore*
WHITEHALL BOOKS LIMITED, *Wellington, New Zealand*

VECTOR IDENTITIES

TRIPLE PRODUCTS

$$(1) \quad \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$$

$$(2) \quad \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

PRODUCT RULES

$$(3) \quad \nabla(fg) = f(\nabla g) + g(\nabla f)$$

$$(4) \quad \nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$$

$$(5) \quad \nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$$

$$(6) \quad \nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$$

$$(7) \quad \nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$$

$$(8) \quad \nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$$

SECOND DERIVATIVES

$$(9) \quad \nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$(10) \quad \nabla \times (\nabla f) = 0$$

$$(11) \quad \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

FUNDAMENTAL THEOREMS

$$\text{Gradient Theorem:} \quad \int_a^b (\nabla f) \cdot d\mathbf{l} = f(b) - f(a)$$

$$\text{Divergence Theorem:} \quad \int_{\text{volume}} (\nabla \cdot \mathbf{A}) \, d\tau = \int_{\text{surface}} \mathbf{A} \cdot d\mathbf{a}$$

$$\text{Curl Theorem:} \quad \int_{\text{surface}} (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint_{\text{line}} \mathbf{A} \cdot d\mathbf{l}$$

VECTOR DERIVATIVES

CARTESIAN. $d\mathbf{l} = dx \hat{i} + dy \hat{j} + dz \hat{k}$; $d\tau = dx dy dz$

Gradient. $\nabla t = \frac{\partial t}{\partial x} \hat{i} + \frac{\partial t}{\partial y} \hat{j} + \frac{\partial t}{\partial z} \hat{k}$

Divergence. $\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$

Curl. $\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \hat{i} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \hat{j} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \hat{k}$

Laplacian. $\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$

SPHERICAL. $d\mathbf{l} = dr \hat{r} + r d\theta \hat{\theta} + r \sin \theta d\phi \hat{\phi}$; $d\tau = r^2 \sin \theta dr d\theta d\phi$

Gradient. $\nabla t = \frac{\partial t}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial t}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial t}{\partial \phi} \hat{\phi}$

Divergence. $\nabla \cdot \mathbf{v} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta v_\theta) + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi}$

Curl. $\nabla \times \mathbf{v} = \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta v_\phi) - \frac{\partial v_\theta}{\partial \phi} \right] \hat{r}$
 $+ \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r} (r v_\phi) \right] \hat{\theta} + \frac{1}{r} \left[\frac{\partial}{\partial r} (r v_\theta) + \frac{\partial v_r}{\partial \theta} \right] \hat{\phi}$

Laplacian. $\nabla^2 t = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial t}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 t}{\partial \phi^2}$

CYLINDRICAL $d\mathbf{l} = dr \hat{r} + r d\phi \hat{\phi} + dz \hat{z}$; $d\tau = r dr d\phi dz$

Gradient. $\nabla t = \frac{\partial t}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial t}{\partial \phi} \hat{\phi} + \frac{\partial t}{\partial z} \hat{z}$

Divergence. $\nabla \cdot \mathbf{v} = \frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{1}{r} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$

Curl. $\nabla \times \mathbf{v} = \left[\frac{1}{r} \frac{\partial v_r}{\partial \phi} - \frac{\partial v_\phi}{\partial z} \right] \hat{r} + \left[\frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r} \right] \hat{\phi}$
 $+ \frac{1}{r} \left[\frac{\partial}{\partial r} (r v_\phi) - \frac{\partial v_r}{\partial \phi} \right] \hat{z}$

Laplacian. $\nabla^2 t = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial t}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$

CONTENTS

ADVERTISEMENT 1

ONE VECTOR ANALYSIS

1.1 VECTOR ALGEBRA 7

- 1.1.1 Vector Operations 7
- 1.1.2 Vector Algebra: Component Form 10
- 1.1.3 Triple Products 13
- 1.1.4 How Vectors Transform 14

1.2 DIFFERENTIAL CALCULUS 16

- 1.2.1 "Ordinary" Derivatives 16
- 1.2.2 Gradient 17
- 1.2.3 The Operator ∇ 20
- 1.2.4 Divergence 21
- 1.2.5 The Curl 22
- 1.2.6 Product Rules 24
- 1.2.7 Second Derivatives 26

1.3 INTEGRAL CALCULUS 28

- 1.3.1 "Ordinary" Integration 28
- 1.3.2 The Fundamental Theorem for Gradients 29
- 1.3.3 The Fundamental Theorem for Divergences 31
- 1.3.4 The Fundamental Theorem for Curls 35

1.3.5	<i>Relations Among the Fundamental Theorems</i>	38
1.3.6	<i>Divergence-Less and Curl-Less Fields</i>	40
1.4	CURVILINEAR COORDINATES	40
1.4.1	<i>Spherical Polar Coordinates</i>	40
1.4.2	<i>Cylindrical Coordinates</i>	45
1.5	THE ROLE OF VECTOR CALCULUS IN ELECTRODYNAMICS	46

TWO ELECTROSTATICS

2.1	THE ELECTROSTATIC FIELD	49
2.1.1	<i>Introduction</i>	49
2.1.2	<i>Coulomb's Law</i>	50
2.1.3	<i>The Electric Field</i>	51
2.1.4	<i>Continuous Charge Distributions</i>	52
2.2	DIVERGENCE AND CURL OF ELECTROSTATIC FIELDS	56
2.2.1	<i>Field Lines and Gauss's Law</i>	56
2.2.2	<i>The Divergence of \mathbf{E}</i>	60
2.2.3	<i>Applications of Gauss's Law</i>	61
2.2.4	<i>The Curl of \mathbf{E}</i>	66
2.3	ELECTRIC POTENTIAL	68
2.3.1	<i>Introduction to Potential</i>	68
2.3.2	<i>Comments on Potential</i>	69
2.3.3	<i>Poisson's Equation and Laplace's Equation</i>	73
2.3.4	<i>Potential of a Charge Distribution</i>	74
2.3.5	<i>Summary; Electrostatic Boundary Conditions</i>	77
2.4	WORK AND ENERGY IN ELECTROSTATICS	79
2.4.1	<i>The Work Done in Moving a Charge</i>	79
2.4.2	<i>The Energy of a Point Charge Distribution</i>	80
2.4.3	<i>The Energy of a Continuous Charge Distribution</i>	82
2.4.4	<i>Comments on Electrostatic Energy</i>	83
2.5	CONDUCTORS	85
2.5.1	<i>Basic Properties of Conductors</i>	85
2.5.2	<i>Induced Charges</i>	87

Contents

2.5.3	<i>The Surface Charge on a Conductor; the Force on a Surface Charge</i>	90
2.5.4	<i>Capacitors</i>	91

SPECIAL TECHNIQUES FOR CALCULATING POTENTIALS

3.1	LAPLACE'S EQUATION AND UNIQUENESS THEOREMS	96
3.1.1	<i>Introduction</i>	96
3.1.2	<i>Laplace's Equation in One Dimension</i>	97
3.1.3	<i>Laplace's Equation in Two Dimensions</i>	98
3.1.4	<i>Laplace's Equation in Three Dimensions</i>	100
3.1.5	<i>Boundary Conditions for Laplace's Equation</i>	101
3.1.6	<i>Conductors and the Second Uniqueness Theorem</i>	103
3.2	THE METHOD OF IMAGES	106
3.2.1	<i>The Classic Image Problem</i>	106
3.2.2	<i>The Induced Surface Charge</i>	108
3.2.3	<i>Force and Energy</i>	108
3.2.4	<i>Other Image Problems</i>	109
3.3	SEPARATION OF VARIABLES	112
3.3.1	<i>Cartesian Coordinates</i>	112
3.3.2	<i>Spherical Coordinates</i>	121
3.4	MULTIPOLE EXPANSION	129
3.4.1	<i>Approximate Potential at Large Distances</i>	129
3.4.2	<i>The Monopole and Dipole Terms</i>	131
3.4.3	<i>Origin of Coordinates in Multipole Expansions</i>	133
3.4.4	<i>The Electric Field of a Dipole</i>	134

FOUR ELECTROSTATIC FIELDS IN MATTER

4.1	POLARIZATION	138
4.1.1	<i>Dielectrics</i>	138
4.1.2	<i>Induced Dipoles</i>	139

4.1.3	<i>Alignment of Polar Molecules</i>	141
4.1.4	<i>Polarization</i>	144
4.2	THE FIELD OF A POLARIZED OBJECT	144
4.2.1	<i>Bound Charges</i>	144
4.2.2	<i>Physical Interpretation of Bound Charge</i>	147
4.2.3	<i>The Field Inside a Dielectric</i>	150
4.3	THE ELECTRIC DISPLACEMENT	152
4.3.1	<i>Gauss's Law in the Presence of Dielectrics</i>	152
4.3.2	<i>A Deceptive Parallel</i>	155
4.4	LINEAR DIELECTRICS	156
4.4.1	<i>Susceptibility, Permittivity, Dielectric Constant</i>	156
4.4.2	<i>Special Problems Involving Linear Dielectrics</i>	161
4.4.3	<i>Force and Energy in Dielectric Systems</i>	166
4.4.4	<i>Polarizability and Susceptibility</i>	170

FIVE

MAGNETOSTATICS

5.1	THE LORENTZ FORCE LAW	174
5.1.1	<i>Magnetic Fields</i>	174
5.1.2	<i>Magnetic Forces</i>	176
5.1.3	<i>Currents</i>	180
5.2	THE BIOT-SAVART LAW	184
5.2.1	<i>Steady Currents</i>	184
5.2.2	<i>The Magnetic Field of a Steady Current</i>	185
5.3	THE DIVERGENCE AND CURL OF \mathbf{B}	190
5.3.1	<i>Straight-Line Currents</i>	190
5.3.2	<i>The Divergence of \mathbf{B}</i>	192
5.3.3	<i>The Curl of \mathbf{B}</i>	193
5.3.4	<i>Ampère's Law</i>	194
5.3.5	<i>Comparison of Magnetostatics and Electrostatics</i>	201
5.4	MAGNETIC VECTOR POTENTIAL	203
5.4.1	<i>The Vector Potential</i>	203
5.4.2	<i>Summary; Magnetostatic Boundary Conditions</i>	208
5.4.3	<i>Multipole Expansion of the Vector Potential</i>	210

SIX MAGNETOSTATIC FIELDS IN MATTER

6.1	MAGNETIZATION	218
6.1.1	<i>Diamagnets, Paramagnets, Ferromagnets</i>	218
6.1.2	<i>Torques and Forces on Magnetic Dipoles</i>	219
6.1.3	<i>Effect of Magnetic Field on Atomic Orbits</i>	222
6.1.4	<i>Magnetization</i>	224
6.2	THE FIELD OF A MAGNETIZED OBJECT	225
6.2.1	<i>Bound Currents</i>	225
6.2.2	<i>Physical Interpretation of Bound Currents</i>	227
6.2.3	<i>The Magnetic Field Inside Matter</i>	229
6.3	THE AUXILIARY FIELD H	230
6.3.1	<i>Ampère's Law in Magnetized Materials</i>	230
6.3.2	<i>A Deceptive Parallel</i>	233
6.4	LINEAR AND NONLINEAR MEDIA	234
6.4.1	<i>Magnetic Susceptibility and Permeability</i>	234
6.4.2	<i>Ferromagnetism</i>	237

SEVEN ELECTRODYNAMICS

7.1	ELECTROMOTIVE FORCE	243
7.1.1	<i>Ohm's Law</i>	243
7.1.2	<i>Electromotive Force</i>	250
7.1.3	<i>Motional emf</i>	252
7.2	FARADAY'S LAW	257
7.2.1	<i>Electromagnetic Induction</i>	257
7.2.2	<i>Inductance</i>	263
7.2.3	<i>Energy in Magnetic Fields</i>	268

7.3	MAXWELL'S EQUATIONS	273
7.3.1	<i>Electrodynamics Before Maxwell</i>	273
7.3.2	<i>How Maxwell Fixed Up Ampère's Law</i>	274
7.3.3	<i>Maxwell's Equations and Magnetic Charge</i>	276
7.3.4	<i>Maxwell's Equations Inside Matter</i>	277
7.3.5	<i>Boundary Conditions</i>	280
7.4	POTENTIAL FORMULATION OF ELECTRODYNAMICS	282
7.4.1	<i>Scalar and Vector Potentials</i>	282
7.4.2	<i>Gauge Transformations</i>	283
7.4.3	<i>Coulomb Gauge and Lorentz Gauge</i>	284
7.4.4	<i>Lorentz Force Law in Potential Form</i>	286
7.5	ENERGY AND MOMENTUM IN ELECTRODYNAMICS	287
7.5.1	<i>Newton's Third Law in Electrodynamics</i>	287
7.5.2	<i>Poynting's Theorem</i>	288
7.5.3	<i>Maxwell's Stress Tensor</i>	291

EIGHT

ELECTROMAGNETIC WAVES

8.1	THE WAVE EQUATION	295
8.1.1	<i>Introduction</i>	295
8.1.2	<i>The Wave Equation in One Dimension</i>	297
8.1.3	<i>Sinusoidal Waves</i>	300
8.1.4	<i>Polarization</i>	304
8.1.5	<i>Boundary Conditions: Reflection and Transmission</i>	306
8.2	ELECTROMAGNETIC WAVES IN NONCONDUCTING MEDIA	309
8.2.1	<i>Monochromatic Plane Waves in Vacuum</i>	309
8.2.2	<i>Energy and Momentum of Electromagnetic Waves</i>	313
8.2.3	<i>Propagation Through Linear Media</i>	315
8.2.4	<i>Reflection and Transmission at Normal Incidence</i>	316
8.2.5	<i>Reflection and Transmission at Oblique Incidence</i>	318
	ELECTROMAGNETIC WAVES IN CONDUCTORS	324
8	<i>The Modified Wave Equation</i>	324
8.3.2	<i>Monochromatic Plane Waves in Conducting Media</i>	327
8.3.3	<i>Reflection and Transmission at a Conducting Surface</i>	330

8.4 DISPERSION 333

- 8.4.1 *The Frequency Dependence of ϵ , μ , and σ* 333
 8.4.2 *Dispersion in Nonconductors* 335
 8.4.3 *Free Electrons in Conductors and Plasmas* 340

NINE

ELECTROMAGNETIC RADIATION

9.1 DIPOLE RADIATION 345

- 9.1.1 *Retarded Potentials* 345
 9.1.2 *Electric Dipole Radiation* 350
 9.1.3 *Magnetic Dipole Radiation* 356
 9.1.4 *Radiation from an Arbitrary Distribution of Charges and Currents* 360

9.2 RADIATION FROM A POINT CHARGE 365

- 9.2.1 *Liénard-Wiechert Potentials* 365
 9.2.2 *The Fields of a Point Charge in Motion* 370
 9.2.3 *Power Radiated by a Point Charge* 375

9.3 RADIATION REACTION 380

- 9.3.1 *The Abraham-Lorentz Formula* 380
 9.3.2 *The Physical Origin of the Radiation Reaction* 384

TEN

ELECTRODYNAMICS AND RELATIVITY

10.1 THE SPECIAL THEORY OF RELATIVITY 388

- 10.1.1 *Einstein's Postulates* 388
 10.1.2 *The Geometry of Relativity* 396
 10.1.3 *The Lorentz Transformations* 406
 10.1.4 *The Structure of Spacetime* 412

10.2	RELATIVISTIC MECHANICS	420
10.2.1	<i>Proper Time and Proper Velocity</i>	420
10.2.2	<i>Relativistic Energy and Momentum</i>	422
10.2.3	<i>Relativistic Kinematics</i>	426
10.2.4	<i>Relativistic Dynamics</i>	430
10.3	RELATIVISTIC ELECTRODYNAMICS	435
10.3.1	<i>Magnetism as a Relativistic Phenomenon</i>	435
10.3.2	<i>How the Fields Transform</i>	437
10.3.3	<i>The Field Tensor</i>	445
10.3.4	<i>Electrodynamics in Tensor Notation</i>	448
10.3.5	<i>Potential Formulation of Relativistic Electrodynamics</i>	451

APPENDIX A

VECTOR CALCULUS IN CURVILINEAR COORDINATES

INTRODUCTION	454
NOTATION	454
GRADIENT	455
DIVERGENCE	456
CURL	458
LAPLACIAN	460

APPENDIX B

UNITS

INDEX	467
-------	-----



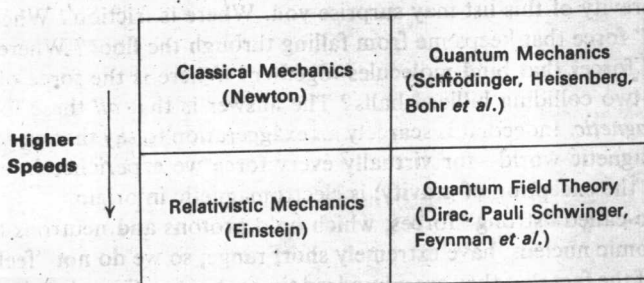
ADVERTISEMENT

What is electrodynamics, and where does it fit into the general scheme of physics?

What is electrodynamics and where does it fit into the general scheme of physics?

FOUR REALMS OF MECHANICS

In the diagram below I have sketched out the four great realms of mechanics:



Newtonian mechanics was found to be inadequate in the early years of this century—it's all right in "everyday life," but for objects moving at high

speeds (near the speed of light) it is incorrect, and must be replaced by special relativity (introduced by Einstein in 1905); for objects that are extremely small (near the size of atoms) it fails for different reasons, and is superseded by quantum mechanics (developed by Bohr, Schrödinger, Heisenberg, and many others in the twenties, mostly). For objects that are both very fast *and* very small (as is common in modern particle physics), a mechanics which combines relativity and quantum principles is in order: this relativistic quantum mechanics is known as quantum field theory—it was developed in the thirties and forties, primarily, but even today it cannot claim to be a completely satisfactory system. In this book, save for the last chapter, we shall work exclusively in the domain of classical mechanics, although the theory of electromagnetism extends with unique simplicity to the other three realms. (In fact, the theory in most respects *automatically* obeys special relativity, for which it was, historically, the main stimulus.)

FOUR KINDS OF FORCES

Mechanics tells us how a system will behave when subjected to a given *force*. There are just *four* basic forces known (presently) to physics: I list them in order of decreasing strength:

1. Strong
- 2. Electromagnetic
3. Weak
- 4. Gravitational

The brevity of this list may surprise you. Where is friction? Where is the “normal” force that keeps me from falling through the floor? Where are the chemical forces that bind molecules together? Where is the force of impact between two colliding billiard balls? The answer is that *all* these forces are *electromagnetic*. Indeed, it is scarcely an exaggeration to say that we live in an electromagnetic world—for virtually every force we experience in everyday life, with the exception of gravity, is electromagnetic in origin.

The so-called “strong” forces, which hold protons and neutrons together in the atomic nucleus, have extremely short range, so we do not “feel” them, in spite of the fact that they are a hundred times stronger than electrical forces. The “weak” forces, which account for certain kinds of radioactive decay, are not only of short range; they are far less powerful than electromagnetic ones to begin with. As for gravity, it is so pitifully feeble (compared to all the others) that it is only in virtue of huge mass concentrations (like the earth and the sun) that we ever notice it at all. The electrical repulsion between two electrons is 10^{42} times as large as their gravitational attraction.

Not only are electromagnetic forces overwhelmingly the dominant ones in everyday life, they are also, at present, the *only* ones that are completely understood. There is, of course, a classical theory of gravity (Newton's universal law of gravitation) and a relativistic one (Einstein's general relativity), but no quantum mechanical theory of gravity has ever been very successful (though many people are still working on it). At the present time there is a rather cumbersome candidate theory for the weak interactions, and a strikingly beautiful one (called "chromodynamics") for the strong interactions. Both theories draw their inspiration from electrodynamics; neither can claim conclusive experimental confirmation at this stage. So electrodynamics, a beautifully complete and successful theory, has become a kind of reference point for physicists: an ideal model that all other theories strive to emulate.

Classical electrodynamics was worked out in bits and pieces by Franklin, Coulomb, Ampère, Faraday, and others, but the man who put it all together and built it into the compact and consistent theory it is today, was James Clerk Maxwell. The theory is now a little over a hundred years old.

THE UNIFICATION OF PHYSICAL THEORIES

In the beginning, **electricity** and **magnetism** were entirely separate subjects. The one dealt with glass rods and cat's fur, pith balls, batteries, currents, electrolysis, and lightning; the other with bar magnets, iron filings, compass needles, and the North Pole. But in 1820 Oersted noticed that an *electric* current could deflect a *magnetic* compass needle. Soon afterward, Ampère correctly postulated that *all* magnetic phenomena are due to electric charges in motion. Then, in 1831, Faraday discovered that a moving *magnet* generates an *electric* current. By the time Maxwell and Lorentz put the finishing touches on the theory, electricity and magnetism were inextricably intertwined. They could no longer be regarded as separate subjects, but rather as two *aspects* of a *single* subject: **electromagnetism**.

Faraday had speculated that light, too, is electrical in nature. Maxwell's theory provided spectacular justification for this hypothesis, and soon **optics**—the study of lenses, mirrors, and prisms, and interference and diffraction—was incorporated into electromagnetism. Hertz, who presented the decisive experimental confirmation for Maxwell's theory in 1888, put it this way: "The connection between light and electricity is now established In every flame, in every luminous particle, we see an electrical process Thus, the domain of electricity extends over the whole of nature. It even affects ourselves intimately: we perceive that we possess . . . an electrical organ—the eye." By 1900, then, three great branches of physics, electricity, magnetism, and optics, had merged into a single unified theory. (And it was soon apparent that visible light represents only a tiny "window" in the vast

spectrum of electromagnetic radiation, from radio through microwaves, infrared and ultraviolet, to X-rays and gamma rays.)

Einstein dreamed of an even grander unification, which would combine **gravity and electrodynamics**, in much the same way as electricity and magnetism had been combined a century earlier. His “unified field theory” was not particularly successful, but in recent years the same impulse has led to a very promising scheme which joins electromagnetic and *weak* forces. If this theory (for which Weinberg, Salam, and Glashow won the Nobel Prize in 1979) survives the test of time, the catalog of forces will shrink to three: strong, electromagnetic-weak, and gravitational. And who knows, perhaps one day we shall recognize that these are all really manifestations of a *single* force.

THE FIELD FORMULATION OF ELECTRODYNAMICS

The fundamental problem a theory of electromagnetism hopes to solve is this: I hold up a bunch of electric charges *here* (and maybe shake it)—what happens to some *other* charge, *over there*? The classical solution takes the form of a *field theory*: we say that the space around an electric charge is permeated by electric and magnetic “fields” (the electromagnetic “odor,” as it were, of the charge). A second charge, in the presence of these fields, experiences a force; the fields, then, transmit the influence from one charge to the other.

When a charge undergoes *acceleration*, a portion of the field “detaches” itself, in a sense, and travels off at the speed of light, carrying with it energy, momentum, and angular momentum. We call this **electromagnetic radiation**. Its existence invites (if not *compels*) us to regard the fields as independent dynamical entities in their own right, every bit as “real” as atoms or baseballs. Our interest accordingly shifts from the study of forces between charges to the theory of the fields themselves. But it takes a charge to *produce* an electromagnetic field, and it takes another charge to *detect* one, so we had best begin by considering the nature of electric charge.

PROPERTIES OF ELECTRIC CHARGE

1. *Charge comes in two varieties*, which we call plus and minus, and they are such that a minus tends to *cancel* a plus (if we have $+q$ and $-q$ at the same point, electrically it is the same as having no charge there at all). This may seem too “obvious” to warrant comment, but I encourage you to think sometime of the other possibilities: what if there were eight or ten different species of charge? Or what if the two kinds did not tend to cancel? The astonishing fact is that plus and minus charges occur in *exactly* equal