INTRODUCTION TO ELECTRODYNAMICS

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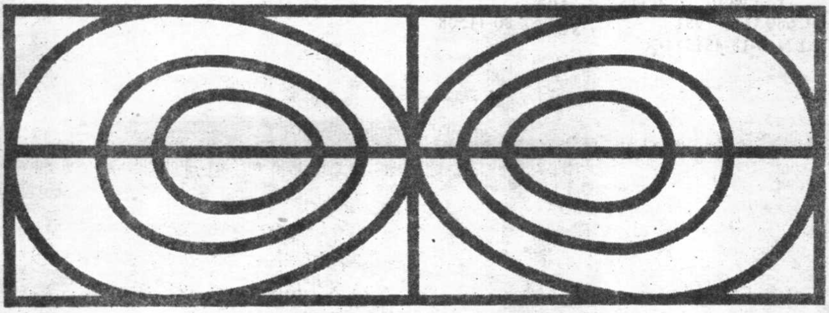

INTRODUCTION TO ELECTRODYNAMICS

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VECTOR IDENTITIES

TRIPLE PRODUCTS

(1) $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})$

(2) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$

PRODUCT RULES

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$

(5) $\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$

(6) $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$

(7) $\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f)$

(8) $\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A})$

SECOND DERIVATIVES

(9) $\nabla\cdot(\nabla\times\mathbf{A}) = 0$

(10) $\nabla\times(\nabla f) = 0$

(11) $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem: $\int_a^b (\nabla f)\cdot d\mathbf{l} = f(b) - f(a)$

Divergence Theorem: $\int_{\text{volume}} (\nabla\cdot\mathbf{A})\,d\tau = \int_{\text{surface}} \mathbf{A}\cdot d\mathbf{a}$

Curl Theorem: $\int_{\text{surface}} (\nabla\times\mathbf{A})\cdot d\mathbf{a} = \oint_{\text{line}} \mathbf{A}\cdot d\mathbf{l}$

VECTOR DERIVATIVES

CARTESIAN. $d\mathbf{l} = dx\,\hat{\imath} + dy\,\hat{\jmath} + dz\,\hat{k}$; $d\tau = dx\,dy\,dz$

Gradient. $$\nabla t = \frac{\partial t}{\partial x}\hat{\imath} + \frac{\partial t}{\partial y}\hat{\jmath} + \frac{\partial t}{\partial z}\hat{k}$$

Divergence. $$\nabla\cdot\mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl. $$\nabla\times\mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\imath} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{k}$$

Laplacian. $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

SPHERICAL. $d\mathbf{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$; $d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient. $$\nabla t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

Divergence. $$\nabla\cdot\mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

Curl. $$\nabla\times\mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r}$$
$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) + \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

Laplacian. $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

CYLINDRICAL $d\mathbf{l} = dr\,\hat{r} + r\,d\phi\,\hat{\phi} + dz\,\hat{z}$; $d\tau = r\,dr\,d\phi\,dz$

Gradient. $$\nabla t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial\phi}\hat{\phi} + \frac{\partial t}{\partial z}\hat{z}$$

Divergence. $$\nabla\cdot\mathbf{v} = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$$

Curl. $$\nabla\times\mathbf{v} = \left[\frac{1}{r}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{r} + \left[\frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}\right]\hat{\phi}$$
$$+ \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\phi) - \frac{\partial v_r}{\partial\phi}\right]\hat{z}$$

Laplacian. $$\nabla^2 t = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}$$

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ADVERTISEMENT

What is electrodynamics, and where does it fit into the general scheme of physics?

What is **electrodynamics** and where does it fit into the general scheme of physics?

FOUR REALMS OF MECHANICS

In the diagram below I have sketched out the four great realms of mechanics:

	Classical Mechanics (Newton)	Quantum Mechanics (Schrödinger, Heisenberg, Bohr *et al.*)
Higher Speeds ↓	Relativistic Mechanics (Einstein)	Quantum Field Theory (Dirac, Pauli Schwinger, Feynman *et al.*)

→ Smaller Distances

Newtonian mechanics was found to be inadequate in the early years of this century—it's all right in "everyday life," but for objects moving at high speeds (near the speed of light) it is incorrect, and must be replaced by special relativity (introduced by Einstein in 1905); for objects that are extremely small (near the size of atoms) it fails for different reasons, and is superseded by quantum mechanics (developed by Bohr, Schrödinger, Heisenberg, and many others in the twenties, mostly). For objects that are both very fast *and* very small (as is common in modern particle physics), a mechanics which combines relativity and quantum principles is in order: this relativistic quantum mechanics is known as quantum field theory—it was developed in the thirties and forties, primarily, but even today it cannot claim to be a completely satisfactory system. In this book, save for the last chapter, we shall work exclusively in the domain of classical mechanics, although the theory of electromagnetism extends with unique simplicity to the other three realms. (In fact, the theory in most respects *automatically* obeys special relativity, for which it was, historically, the main stimulus.)

FOUR KINDS OF FORCES

Mechanics tells us how a system will behave when subjected to a given *force*. There are just *four* basic forces known (presently) to physics: I list them in order of decreasing strength:

1. Strong
2. Electromagnetic
3. Weak
4. Gravitational

The brevity of this list may surprise you. Where is friction? Where is the "normal" force that keeps me from falling through the floor? Where are the chemical forces that bind molecules together? Where is the force of impact between two colliding billiard balls? The answer is that *all* these forces are *electromagnetic*. Indeed, it is scarcely an exaggeration to say that we live in an electromagnetic world—for virtually every force we experience in everyday life, with the exception of gravity, is electromagnetic in origin.

The so-called "strong" forces, which hold protons and neutrons together in the atomic nucleus, have extremely short range, so we do not "feel" them, in spite of the fact that they are a hundred times stronger than electrical forces. The "weak" forces, which account for certain kinds of radioactive decay, are not only of short range; they are far less powerful than electromagnetic ones to begin with. As for gravity, it is so pitifully feeble (compared to all the others) that it is only in virtue of huge mass concentrations (like the earth and the sun) that we ever notice it at all. The electrical repulsion between two electrons is 10^{42} times as large as their gravitational attraction.

Not only are electromagnetic forces overwhelmingly the dominant ones in everyday life, they are also, at present, the *only* ones that are completely understood. There is, of course, a classical theory of gravity (Newton's universal law of gravitation) and a relativistic one (Einstein's general relativity), but no quantum mechanical theory of gravity has ever been very successful (though many people are still working on it). At the present time there is a rather cumbersome candidate theory for the weak interactions, and a strikingly beautiful one (called "chromodynamics") for the strong interactions. Both theories draw their inspiration from electrodynamics; neither can claim conclusive experimental confirmation at this stage. So electrodynamics, a beautifully complete and successful theory, has become a kind of reference point for physicists: an ideal model that all other theories strive to emulate.

Classical electrodynamics was worked out in bits and pieces by Franklin, Coulomb, Ampère, Faraday, and others, but the man who put it all together and built it into the compact and consistent theory it is today, was James Clerk Maxwell. The theory is now a little over a hundred years old.

THE UNIFICATION OF PHYSICAL THEORIES

In the beginning, **electricity** and **magnetism** were entirely separate subjects. The one dealt with glass rods and cat's fur, pith balls, batteries, currents, electrolysis, and lightning; the other with bar magnets, iron filings, compass needles, and the North Pole. But in 1820 Oersted noticed that an *electric* current could deflect a *magnetic* compass needle. Soon afterward, Ampère correctly postulated that *all* magnetic phenomena are due to electric charges in motion. Then, in 1831, Faraday discovered that a moving *magnet* generates an *electric* current. By the time Maxwell and Lorentz put the finishing touches on the theory, electricity and magnetism were inextricably intertwined. They could no longer be regarded as separate subjects, but rather as two *aspects* of a *single* subject: **electromagnetism**.

Faraday had speculated that light, too, is electrical in nature. Maxwell's theory provided spectacular justification for this hypothesis, and soon **optics**—the study of lenses, mirrors, and prisms, and interference and diffraction—was incorporated into electromagnetism. Hertz, who presented the decisive experimental confirmation for Maxwell's theory in 1888, put it this way: "The connection between light and electricity is now established .... In every flame, in every luminous particle, we see an electrical process .... Thus, the domain of electricity extends over the whole of nature. It even affects ourselves intimately: we perceive that we possess . . . an electrical organ—the eye." By 1900, then, three great branches of physics, electricity, magnetism, and optics, had merged into a single unified theory. (And it was soon apparent that visible light represents only a tiny "window" in the vast spectrum of electromagnetic radiation, from radio through microwaves, infrared and ultraviolet, to X-rays and gamma rays.)

Einstein dreamed of an even grander unification, which would combine **gravity** and **electrodynamics**, in much the same way as electricity and magnetism had been combined a century earlier. His "unified field theory" was not particularly successful, but in recent years the same impulse has led to a very promising scheme which joins electromagnetic and *weak* forces. If this theory (for which Weinberg, Salam, and Glashow won the Nobel Prize in 1979) survives the test of time, the catalog of forces will shrink to three: strong, electromagnetic-weak, and gravitational. And who knows, perhaps one day we shall recognize that these are all really manifestations of a *single* force.

THE FIELD FORMULATION OF ELECTRODYNAMICS

The fundamental problem a theory of electromagnetism hopes to solve is this: I hold up a bunch of electric charges *here* (and maybe shake it)—what happens to some *other* charge, over *there*? The classical solution takes the form of a *field theory*: we say that the space around an electric charge is permeated by electric and magnetic "fields" (the electromagnetic "odor," as it were, of the charge). A second charge, in the presence of these fields, experiences a force; the fields, then, transmit the influence from one charge to the other.

When a charge undergoes *acceleration*, a portion of the field "detaches" itself, in a sense, and travels off at the speed of light, carrying with it energy, momentum, and angular momentum. We call this **electromagnetic radiation**. Its existence invites (if not *compels*) us to regard the fields as independent dynamical entities in their own right, every bit as "real" as atoms or baseballs. Our interest accordingly shifts from the study of forces between charges to the theory of the fields themselves. But it takes a charge to *produce* an electromagnetic field, and it takes another charge to *detect* one, so we had best begin by considering the nature of electric charge.

PROPERTIES OF ELECTRIC CHARGE

1. *Charge comes in two varieties,* which we call plus and minus, and they are such that a minus tends to *cancel* a plus (if we have $+q$ and $-q$ at the same point, electrically it is the same as having no charge there at all). This may seem too "obvious" to warrant comment, but I encourage you to think sometime of the other possibilities: what if there were eight or ten different species of charge? Or what if the two kinds did not tend to cancel? The astonishing fact is that plus and minus charges occur in *exactly* equal