Elements of Pulse Circuits

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ERRATUM

On pp. vii, 33–35, 90, 106 and Index for 'cascade' read 'cascode'.

[Publisher's note : This error was inadvertently introduced after the author had corrected his proofs for press.]

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PREFACE

THIS book is addressed primarily to physicists and research workers who wish to obtain an introduction to pulse circuits. It is assumed that the reader is already familiar with radio valves and elementary receiving technique, and accordingly the fundamentals of radio practice are either taken for granted or reviewed briefly: the application to pulse waveforms is then tackled immediately.

Although mathematical statement is used occasionally in the interests of brevity and precision, the approach is mainly non-mathematical, the emphasis being on a direct understanding of the physical principles involved. It is hoped that the book will be of service to radio workers generally and, therefore, while connections are made with advanced physics they are never essential to the argument. In particular certain topics are omitted because the general reader will not have the equipment to deal with them (notably the use of transmission lines for pulse shaping).

In an introductory volume such as this it is impossible to give a detailed acknowledgement to all sources of information. No attempt is made to trace circuits to their source and references are given only for the benefit of the reader who wants to pursue the subject further. I would, however, like to express here my thanks to all those who have contributed either in conversation or in print to my studies and therefore to this book. In particular I gratefully acknowledge the assistance received in composing the manuscript from my colleagues Mr. J. B. Earnshaw and Dr. H. A. Whale.

F. J. M. FARLEY.

Harwell,
Nov., 1955

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CHAPTER I

BASIC CONCEPTS

1.1. Pulse waveforms. In radio communication the usual practice has been to consider only sinusoidal waveforms. We are well aware, of course, that a typical speech waveform is far from sinusoidal; the simultaneous presence of several

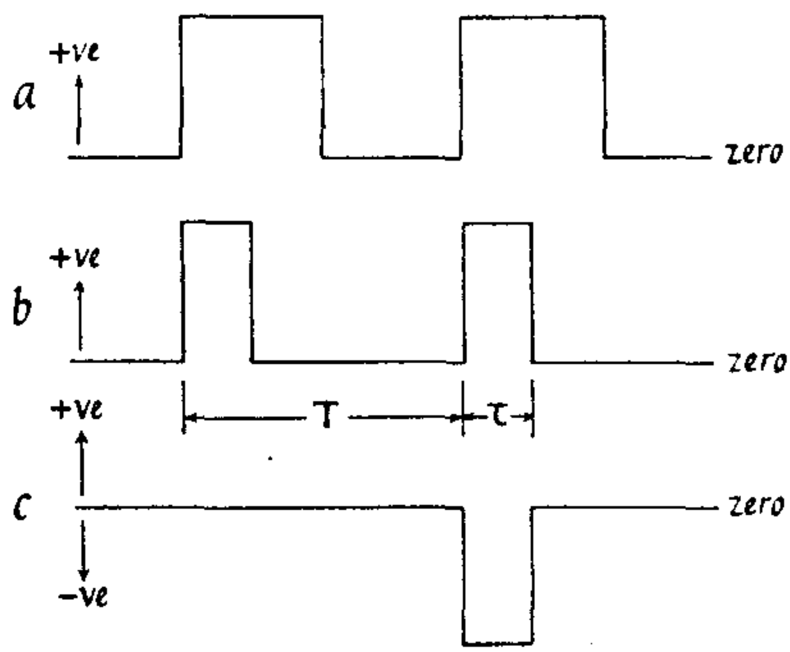

FIG. 1.—Pulse waveforms.

different frequencies makes the wave deviate from the pure sine-wave shape. Nevertheless, in analysing circuit performance it is often sufficient to confine the discussion to sine-waves. The sine-wave has been chosen as the basic waveform for circuit analysis because it has the property of passing through linear electrical networks without changing its shape. We shall see below, when we consider other waveforms, that in general their shape is changed by a linear network.

In pulse circuits we are not interested so much in the fundamental frequency of a wave and its harmonic content, if any; our attention is concentrated mainly on the exact shape of the wave and its variation from point to point in the circuit. The basic waveform is now the *square-wave* (Fig. 1). In the ideal square-wave the voltage changes in an infinitely short time from one steady level to another: that is, the wave has a perfectly flat top and bottom and infinitely steep sides. Fig. 1*a* shows a symmetrical square-wave, while in Fig. 1*b* we see an asymmetrical form in which the positive voltage regime is of shorter duration. This latter is often regarded as a square (or rectangular) *pulse* of amplitude V

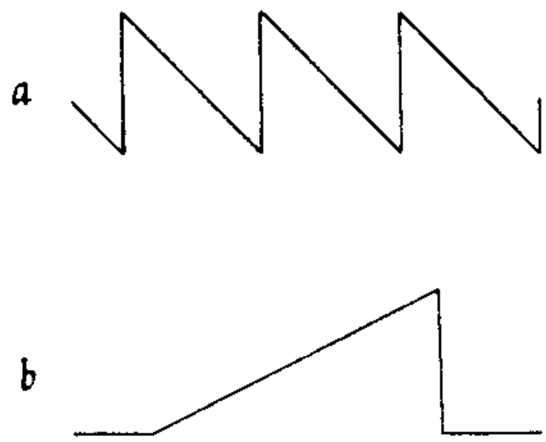

FIG. 2.—Triangular or saw-tooth waveforms.

and duration τ, rising from a steady *base-line*. In this example, the pulse recurs regularly after a time interval T, called the recurrence period. The quantity τ/T is called the *mark-space* ratio: thus a symmetrical square wave would have a mark space ratio of $\frac{1}{2}$. In many cases, however, the pulses are not regular, but occur singly, or at irregular intervals. It is often convenient, therefore, to discuss the response of a circuit to a single square pulse, which may, of course, be positive as in Fig. 1*b*, or negative as in Fig. 1*c*.

Another common pulse waveform is the triangular wave, Fig. 2*a*, often called the saw-tooth, or time base, waveform. If this wave is applied to the X-deflection plates of a cathode ray tube, it causes the spot to sweep across the screen at a uniform rate, and then to fly back infinitely quickly (in the ideal case) to repeat the process. Thus, we may use the cathode ray tube to plot the shape of another waveform applied to the Y-deflection plates. In such applications it is important that the time base waveform be exactly linear: methods of generating and handling linear saw-toothwaves will be discussed in Chap. V. Fig. 2*b* shows another common type of triangular wave.

Both the square and the triangular waves have a large harmonic content. In the ideal case of infinitely sharp

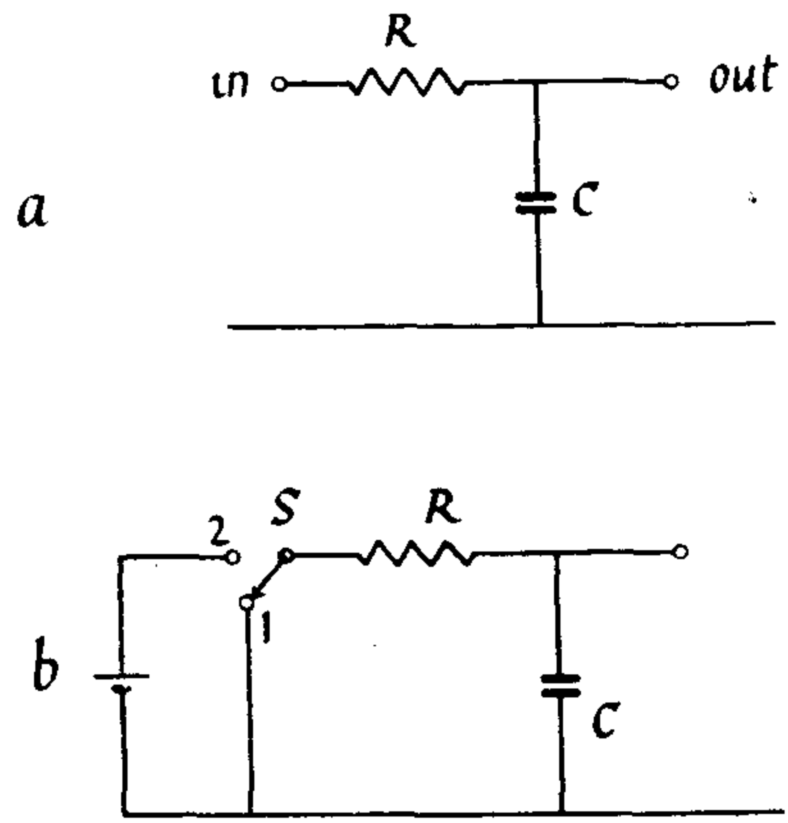

FIG. 3.—Integration circuit.

corners, harmonics of infinite frequency must be present. In practice there is some rounding of the corners, but it remains true that our circuits must handle a wide range of frequency if the waveform is to be transmitted at all faithfully. Attenuation of the higher frequencies produces a rounding of the corners; whereas attenuation of the lower frequencies results in distortion of the base-line or nonlinearity of a saw-tooth.

Postponing the general discussion of these effects to Chap. VI, we now consider the effect on the pulse shape of two simple resistance-condenser combinations.

1.2. Integration. Suppose a positive square pulse of amplitude V is applied to the circuit of Fig. 3*a*. To determine the output waveform we may suppose that the pulse has been produced from a battery of voltage V, by the operation of the switch S, as shown in Fig. 3*b*. Initially, with the switch in position 1, the output voltage will be zero. When the switch is moved to position 2, current i flows through resistance R, through condenser C, and back to the battery. The result

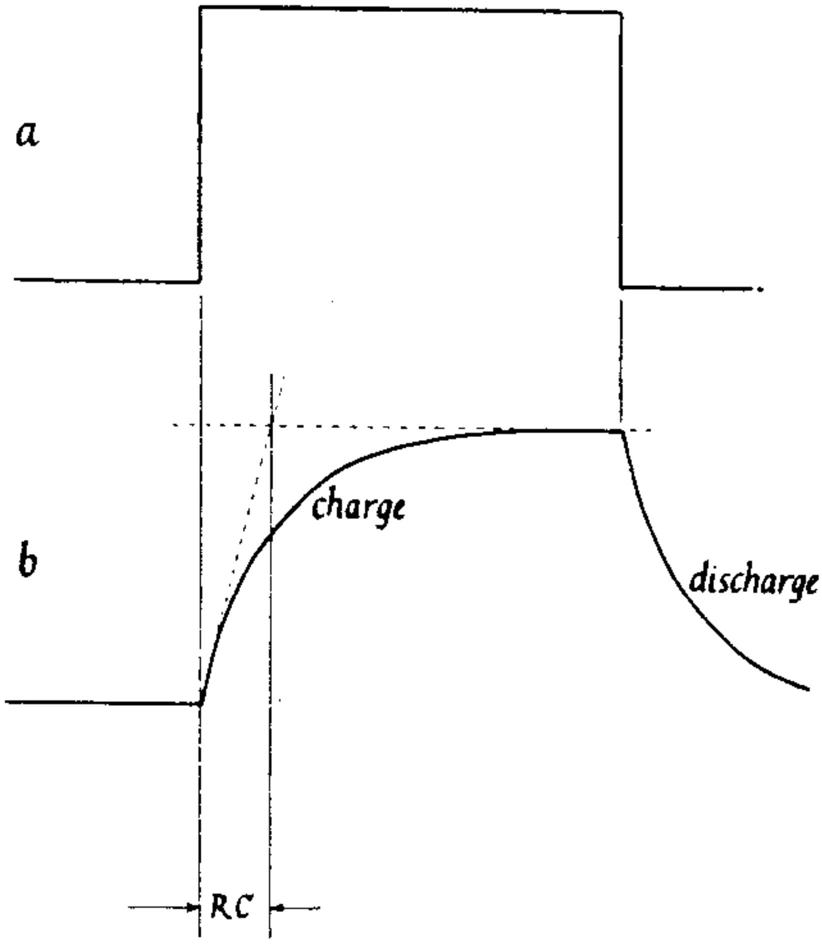

FIG. 4.—Integration of a square pulse.

is that C is slowly charged and the output voltage, v, slowly rises. The differential equation for the output voltage is:

$$\frac{dv}{dt} = \frac{i}{C} = \frac{V - v}{RC} \quad . \quad . \quad . \qquad (1)$$

Hence, $\log (V - v) = - t/RC + \text{const.}$

The condition, $v = 0$ when $t = 0$, shows that the constant of integration is $\log V$, and we obtain finally

$$v = V(1 - e^{-t/RC}) * \quad . \quad . \qquad (2)$$

* Using the expansion series for the exponential this gives $v \simeq Vt/RC$ at the beginning of the wave where $t \ll RC$.

The output voltage therefore rises linearly at first, but then more slowly, eventually reaching the steady value V, equal to the input voltage. This is the well-known *exponential* charging of a condenser, and is illustrated in Fig. 4*b*.

The time scale of this output waveform is entirely determined by RC, called the *time constant* of the circuit. The product of resistance and capacity is a quantity of dimensions *time*; if the resistance is in ohms and the capacity in farads the product gives the time in seconds. It is useful to remember that the output voltage rises initially at the rate V/RC; that is as if to reach the final voltage after time RC. (See equation (1).) The percentage of the final voltage reached, and remaining, after various times is given in Table 1.

TABLE 1

Time in Units RC	*Voltage change per cent.*	*Subsequent voltage change per cent.*
0	0	100
0·2	18·1	81·9
0·5	39·4	60·6
1	63·2	36·8
2	86·5	13·5
3	95·0	5·0
4	98·2	1·8
5	99·3	0·7
7	99·9	0·1

When switch S is returned to position 1 (corresponding to the end of the input pulse), the condenser C discharges through resistance R until the output voltage, v, is again zero. If we solve the differential equation as before we find

$$v = Ve^{-t/RC} \quad . \quad . \quad . \quad . \qquad (3)$$

This waveform is shown in Fig. 4*b*. We see that it is similar in shape to the positive going waveform at the beginning of the output pulse, and Table 1 applies in this case also.*

The net result is that the ideally square input pulse of Fig. 4*a* is transformed at the output to the shape of Fig. 4*b*. We have so far supposed that the input pulse duration is much

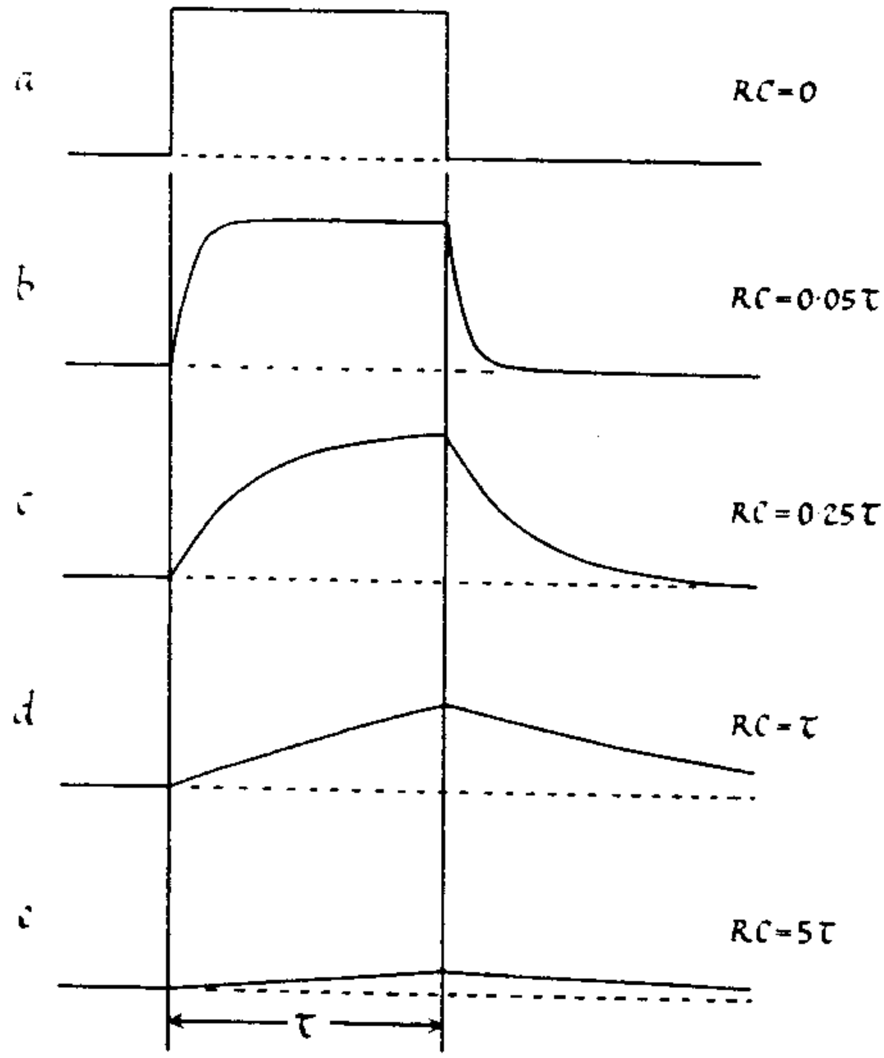

FIG. 5.—Integration of a square pulse by various time constants.

larger than the time constant *RC*. If these times are comparable, the discharge phase sets in before the final positive voltage is reached. Fig. 5 shows the output waveforms for a fixed input pulse and various values of the time constant *RC*. We see that the distortion is negligible when *RC* is very short, and increases progressively as *RC* is increased.

* This is an example of the general rule that for linear networks positive and negative pulse fronts produce effects which are the same, but inverted; and the end of a positive pulse can be regarded as an isolated negative going pulse front. The symmetry between positive and negative disappears, however, when valves and other non-linear elements are included in the circuit.

When the time constant is very long, the peak output voltage is proportional to the area under the input pulse, that is, to V. This follows from equation (1) which shows that

$$\frac{v}{t} = \frac{V}{RC} \text{ for } v \ll V. \qquad (4)$$

$\therefore v = V\tau/RC$ at the end of the pulse if $\tau \ll RC$.

Physically, the result follows because the condenser C is charged at a rate proportional to V and for a time τ. More generally, for an input pulse of arbitrary shape, the peak output voltage is proportional to the area under the pulse, provided always that $RC \gg$ pulse length.

Because of this property the circuit of Fig. 3*a* is called an *integrating* circuit. The whole process, resulting in the whole series of output waveforms in Fig. 5, is known for convenience as *integration*. It is important to realize that exact mathematical integration is approached only in the extreme case of Fig. 5*e*. In the more typical case of Fig. 5*b* there is no connection with mathematical integration, but we still use this term for convenience.

We now consider the effect of the integrating circuit on a triangular wave. Suppose that the input to the circuit of Fig. 3*a* has the form $V = kt$. The differential equation for the output voltage v then becomes

$$\frac{dv}{dt} = \frac{kt - v}{RC} \qquad (5)$$

or, in standard form

$$\frac{dv}{dt} + \frac{1}{RC}v = \frac{k}{RC}t.$$

Multiplying both sides by $e^{t/RC}$,

$$\frac{d}{dt}(v \,.\, e^{t/RC}) = \frac{k}{RC}te^{t/RC}.$$

Therefore,

$$(ve^{t/RC}) = k\int_0^t td(e^{t/RC}).$$

which on integration by parts yields

$$v = kt - k \,.\, RC(1 - e^{-t/RC}) \quad . \quad . \quad (6)$$

This behaviour is illustrated in Fig. 6. Initially dv/dt is zero, but after a transition period of order a few times RC, the

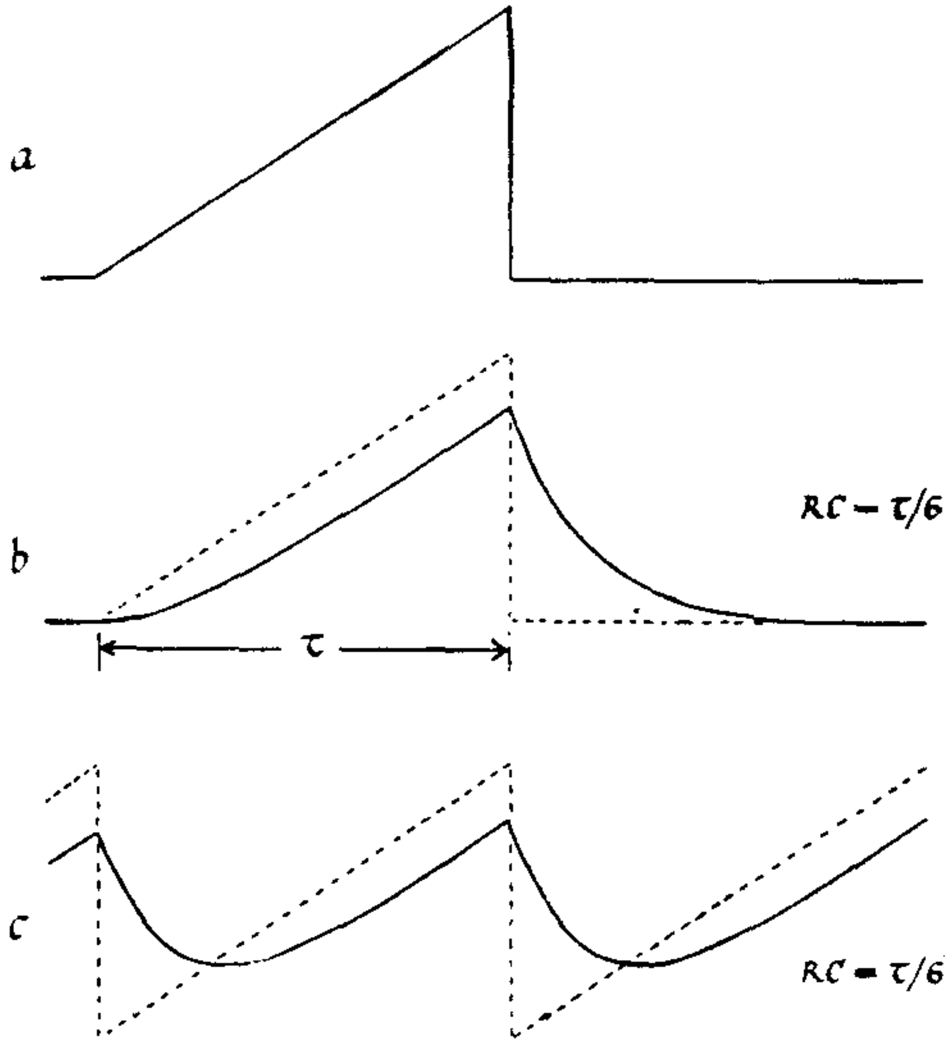

FIG. 6.—Integration of a saw-tooth.

output voltage (Fig. 6*b*) rises at the same rate as the input voltage (Fig. 6*a*), but lags by the constant amount $k \,.\, RC$; that is by the amount the input saw-tooth rises in time RC. In effect, therefore, the output voltage lags in time by the interval RC.

$$v = k(t - RC) \quad . \quad . \quad . \quad . \quad (7)$$

Physically, this happens because a steady current $C\dfrac{dv}{dt}$ is needed to charge condenser C at the constant rate, and this current is provided by the voltage drop $RC\frac{dv}{dt}$ across resistance R.

In Fig. 6*a* we show a typical input triangular waveform, and Fig. 6*b* gives the corresponding output. At the end of the sweep the input voltage returns to zero and the output decays exponentially as in the case of the square pulse. In the case of a recurrent saw-tooth the exponential decay at the end of one saw-tooth has to join smoothly to the lagging rise of the following saw-tooth with the result shown in Fig. 6*c*. Note that dv/dt is zero at the output when the input and output voltages are equal because then there is no current through R. In all cases, the integrating effect of the circuit produces departures from linearity at the beginning of the time base for the duration of several time constants.

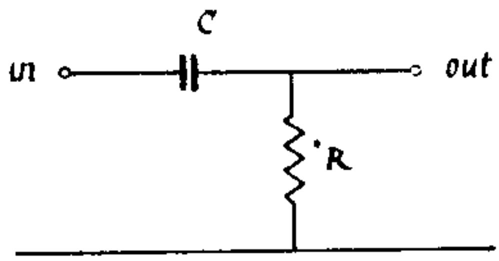

FIG. 7.—Differentiation circuit.

1.3. Differentiation. We now consider the circuit of Fig. 7 which differs from Fig. 3*a* in that condenser and resistance have been interchanged. Instead of observing the voltage across the condenser, we are now interested in the voltage across the resistance.

We can proceed as before by solving the differential equation for the output voltage but it is simpler to observe that (voltage across R) + (voltage across C) = input voltage. The voltage across C is already known from our work on the integrating circuit (see equations (2) and (3)) and we obtain for a square wave input the result

$$\left.\begin{aligned} v &= Ve^{-t/RC} \text{ at the front of the pulse} \\ v &= -\ Ve^{\ t/RC} \text{ at the tail} \end{aligned}\right\} \quad . \qquad (8)$$

The input and output waveforms are plotted in Figs. 8*a* and *b*.

Physically we can explain this behaviour by noting that the voltage across a condenser cannot in general change instantaneously. It can only change when the charge changes, and this usually happens gradually as current flows through the condenser. This means that rapid changes in voltage are transmitted by a condenser without attenuation: as far as rapid voltage changes are concerned, we can regard the condenser as a direct connection. This is an important principle which we shall use again and again in analysing pulse circuits.

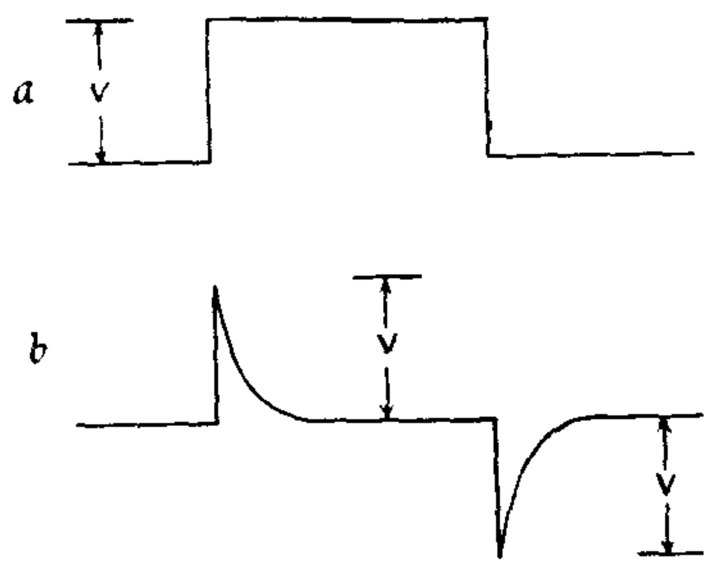

FIG. 8.—Differentiation of a square pulse.

In the present case, the steep front of the input pulse is transmitted completely by the condenser and appears undiminished at the output. The output voltage causes current to flow through R: this current must flow also through C, so that the condenser is charged and the output voltage gradually returns to zero. On the tail of the output pulse the action is similar. As before the time scale is determined by the time constant RC, and Table 1 again applies.

The output pulse obtained with a fixed input pulse and various values of the time constant RC is shown in Fig. 9. Here the pulse is modified only slightly if $RC \gg \tau$; it is the short time constants that give distortion. In the extreme case $RC \ll \tau$, the output approximates to the mathematical differential coefficient of the input waveform. For this reason the process is known as *differentiation*. Here again, we must regard the word as a technical term applying to the whole family of distortions; the pure mathematical meaning should be kept well in the background.

Let us now consider the *differentiation* of a triangular wave, $V = kt$. We can obtain the output voltage by subtraction as before, using equation (6). This yields

$$v = k \, . \, RC(1 - e^{-t/RC}) \quad . \quad . \quad . \quad (9)$$

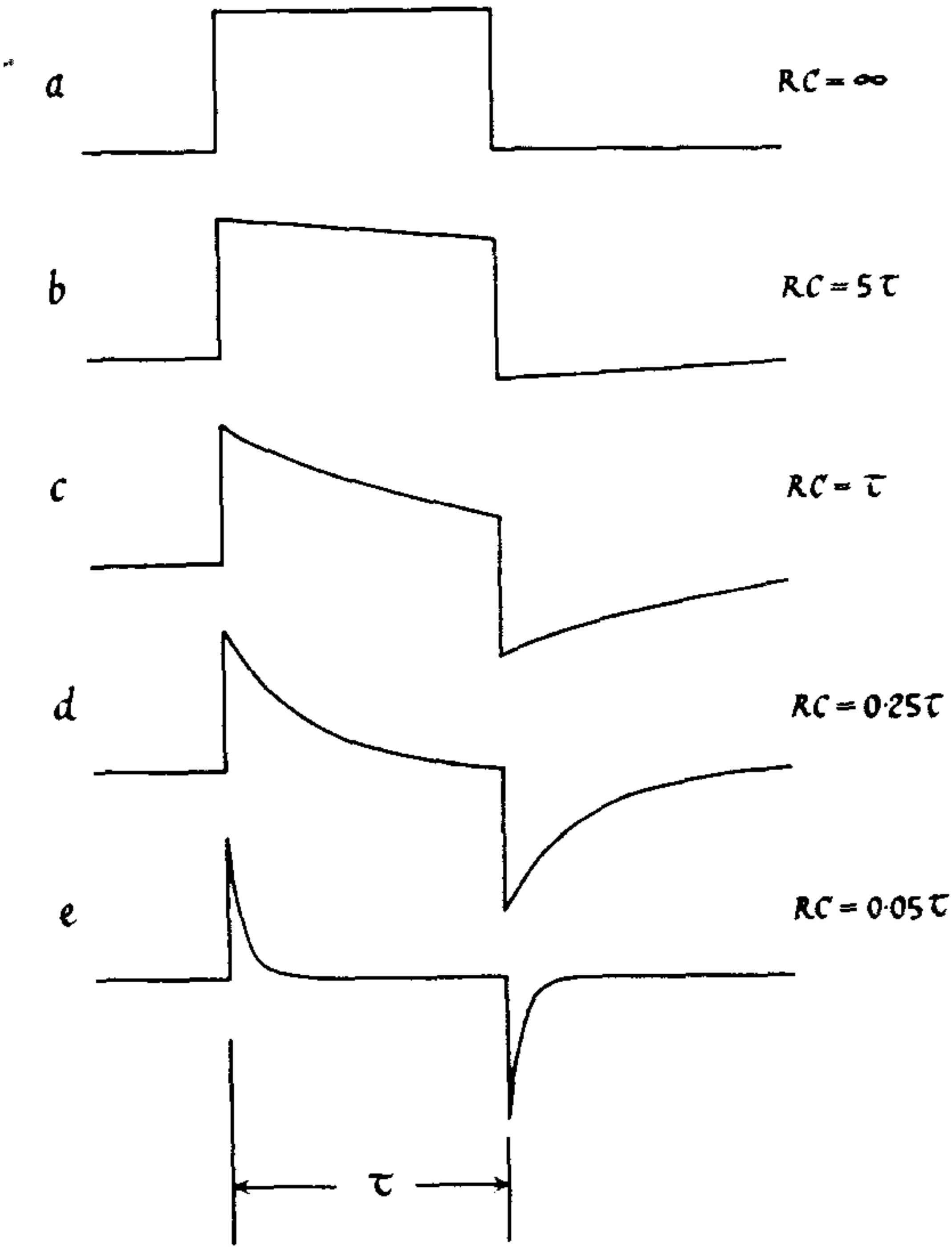

FIG. 9.—Differentiation of a square pulse by various time constants.

Figs. 10*a* and *b* show the input and output waveforms. The output voltage rises initially at the same rate as the input voltage, but after a time of order RC settles down to the steady value $k \, . \, RC$. Physically, the charging of the condenser at constant rate requires a current $C \dfrac{dv}{dt}$ flowing through