BRIAN D. O. ANDERSON
SUMETH VONGPANITLERD

NetworK Analysis and Synthesis

A MODERN SYSTEMS THEORY APPROACH

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Editor

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NETWORK ANALYSIS AND SYNTHESIS

A Modern Systems Theory Approach

BRIAN D. O. ANDERSON

Professor of Electrical Engineering
University of Newcastle
New South Wales, Australia

SUMETH VONGPANITLERD

Director, Multidisciplinary Laboratories
Mahidol University
Bangkok, Thailand

PRENTICE-HALL, INC.
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To our respective parents

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Preface

For many years, network theory has been one of the more mathematically developed of the electrical engineering fields. In more recent times, it has shared this distinction with fields such as control theory and communication systems theory, and is now viewed, together with these fields, as a subdiscipline of modern system theory. However, one of the key concepts of modern system theory, the notion of state, has received very little attention in network synthesis, while in network analysis the emphasis has come only in recent times. The aim of this book is to counteract what is seen to be an unfortunate situation, by embedding network theory, both analysis and synthesis, squarely within the framework of modern system theory. This is done by emphasizing the state-variable approach to analysis and synthesis.

Aside from the fact that there is a gap in the literature, we see several important reasons justifying presentation of the material found in this book. First, in solving network problems with a computer, experience has shown that very frequently programs based on state-variable methods can be more easily written and used than programs based on, for example, Laplace transform methods. Second, the state concept is one that emphasizes the internal structure of a system. As such, it is the obvious tool for solving a problem such as finding all networks synthesizing a prescribed impedance; this and many other problems of internal structure are beyond the power of classical network theory. Third, the state-space description of passivity, dealt with at some length in this book, applies in such diverse areas as Popov stability, inverse optimal control problems, sensitivity reduction in control systems, and Kalman–Bucy or Wiener filtering (a topic of such rich application surely deserves treatment within a textbook framework). Fourth, the graduate major in systems science is better served by a common approach to different disciplines of systems science than by a multiplicity of different approaches with no common ground; a move injecting the state variable into network analysis and synthesis is therefore as welcome from the pedagogical viewpoint as recent moves which have injected state variables into communications systems.

The book has been written with a greater emphasis on passive than on active networks. This is in part a reflection of the authors' personal interests, and in part a reflection of their view that passive system theory is a topic about which no graduate systems major should be ignorant. Nevertheless, the inclusion of starred material which can be omitted with no loss of continuity offers the instructor a great deal of freedom in setting the emphasis in a course based on the book. A course could, if the instructor desires, de-emphasize the networks aspect to a great degree, and concentrate mostly on general systems theory, including the theory of passive systems. Again, a course could emphasize the active networks aspect by excluding much material on passive network synthesis.

The book is aimed at the first year graduate student, though it could certainly be used in a class containing advanced undergraduates, or later year graduate students. The background required is an introductory course on linear systems, the usual elementary undergraduate networks material, and ability to handle matrices. Proceeding at a fair pace, the entire book could be completed in a semester, while omission of starred material would allow a more leisurely coverage in a semester. In a course of two quarters length the book could be comfortably completed, while in one quarter, particularly with students of strong backgrounds, a judicious selection of material could build a unified course.

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Part I

INTRODUCTION

What is the modern system theory approach to network analysis and synthesis? In this part we begin answering this question.

1

Introduction

1.1 ANALYSIS AND SYNTHESIS

Two complementary functions of the engineer are analysis and synthesis. In analysis problems, one is usually given a description of a physical system, i.e., a statement of what its components are, how they are assembled, and what the laws of physics are that govern the behavior of each component. One is generally required to perform some computations to predict the behavior of the system, such as predicting its response to a prescribed input. The synthesis problem is the reverse of this. One is generally told of a behavioral characteristic that a system should have, and asked to devise a system with this prescribed behavioral characteristic.

In this book we shall be concerned with *network* analysis and synthesis. More precisely, the networks will be electrical networks, which *always* will be assumed to be (1) linear, (2) time invariant, and (3) comprised of a finite number of lumped elements. Usually, the networks will also be assumed to be passive. We presume the reader knows what it means for a network to be linear and time invariant. In Chapter 2 we shall define more precisely the allowed element classes and the notion of passivity.

In the context of this book the analysis problem becomes one of knowing the set of elements comprising a network, the way they are interconnected, the set of initial conditions associated with the energy storage elements, and the set of excitations provided by externally connected voltage and current generators. The problem is to find the response of the network, i.e., the resultant set of voltages and currents associated with the elements of the network.

In contrast, tackling the synthesis problem requires us to start with a statement of what responses should result from what excitations, and we are required to determine a network, or a set of network elements and a scheme for their interconnection, that will provide the desired excitation-response characteristic. Naturally, we shall state both the synthesis problem and the analysis problem in more explicit terms at the appropriate time; the reader should regard the above explanations as being rough, but temporarily satisfactory.

1.2 CLASSICAL AND MODERN APPROACHES

Control and network theory have historically been closely linked, because the methods used to study problems in both fields have often been very similar, or identical. Recent developments in control theory have, however, tended to outpace those in network theory. The magnitude and importance of the developments in control theory have even led to the attachment of the labels "classical" and "modern" to large bodies of knowledge within the discipline. It is the development of network theory paralleling modern control theory that has been lacking, and it is with such network theory that this book is concerned.

The distinction between modern and classical control theory is at points blurred. Yet while the dividing line cannot be accurately drawn, it would be reasonable to say that frequency-domain, including Laplace-transform, methods belong to classical control theory, while state-space methods belong to modern control theory. Given this division within the field of control, it seems reasonable to translate it to the field of network theory. Classical network theory therefore will be deemed to include frequency-domain methods, and modern network theory to include state-space methods.

In this book we aim to emphasize application of the notions of state, and system description via state-variable equations, to the study of networks. In so doing, we shall consider problems of both analysis and synthesis.

As already noted, modern theory looks at state-space descriptions while classical theory tends to look at Laplace-transform descriptions. What are the consequences of this difference? They are numerous; some are as follows.

First and foremost, the state-variable description of a network or system emphasizes the *internal structure* of that system, as well as its input–output performance. This is in contrast to the Laplace-transform description of a network or system involving transfer functions and the like, which emphasizes the input–output performance alone. The internal structure of a network must be considered in dealing with a number of important questions. For example, minimizing the number of reactive elements in a network synthesizing a prescribed excitation-response pair is a problem involving examination of the details of the internal structure of the network. Other pertinent examples include minimizing the total resistance of a network, examining the effect of nonzero initial conditions of energy storage or reactive elements on the externally observable behavior of the network, and examining the sensitivity of the externally observable behavior of the circuit to variations in the component values. It would be quite illogical to attempt to solve all these problems with the tools of classical network theory (though to be sure some progress could be made on some of them). It would, on the other hand, be natural to study these problems with the aid of modern network theory and the state-variable approach.

Some other important differences between the classical and modern approaches can be quickly summarized:

1. The classical approach to synthesis usually relies on the application of ingeniously contrived algorithms to achieve syntheses, with the number of variations on the basic synthesis structures often being severely circumscribed. The modern approach to synthesis, on the other hand, usually relies on solution, without the aid of too many tricks or clever technical artifices, of a well-motivated and easily formulated problem. At the same time, the modern approach frequently allows the straightforward generation of an infinity of solutions to the synthesis problem.
2. The modern approach to network analysis is ideally suited to implementation on a digital computer. Time-domain integration of state-space differential equations is generally more easily achieved than operations involving computation of Laplace transforms and inverse Laplace transforms.
3. The modern approach emphasizes the algebraic content of network descriptions and the solution of synthesis problems by matrix algebra. The classical approach is more concerned with using the tools of complex variable analysis.

The modern approach is not better than the classical approach in every way. For example, it can be argued that the intuitional pictures provided by Bode diagrams and pole-zero diagrams in the classical approach tend to be lost in the modern approach. Some, however, would challenge this argument on the grounds that the modern approach subsumes the classical approach. The modern approach too has yet to live up to all its promise. Above we listed problems to which the modern approach could logically be applied; some of these problems have yet to be solved. Accordingly, at this stage of development of the modern system theory approach to network analysis and synthesis, we believe the classical and modern approaches are best seen as being complementary. The fact that this book contains so little of the classical approach may then be queried; but the answer to this query is provided by the extensive array of books on network analysis and synthesis, e.g., [1–4], which, at least in the case of the synthesis books, are heavily committed to presenting the classical approach, sometimes to the exclusion of the modern approach.

The classical approach to network analysis and synthesis has been developed for many years. Virtually all the analysis problems have been solved, and a falling off in research on synthesis problems suggests that the majority of those synthesis problems that are solvable may have been solved. Much of practical benefit has been forthcoming, but there do remain practical problems that have yet to succumb. As we noted above, the modern approach has not yet solved all the problems that it might be expected to solve; particularly is it the case that it has solved few practical problems, although in isolated cases, as in the case of active synthesis discussed in Chapter 13, there have been spectacular results. We attribute the present lack of other tangible results to the fact that relatively little research has been devoted to the modern approach; compared with modern control theory, modern network theory is in its infancy, or, at latest, its early adolescence. We must wait to see the payoffs that maturity will bring.

1.3 OUTLINE OF THE BOOK

Besides the introductory Part I, the book falls into five parts. Part II is aimed at providing background in two areas, the first being m-port networks and means for describing them, and the second being state-space equations and their relation with transfer-function matrices. In Chapter 2, which discusses m-port networks, we deal with classes of circuit elements, such network properties as passivity, losslessness, and reciprocity, the immittance, hybrid and scattering-matrix descriptions of a network, as well as some important network interconnections. In Chapter 3 we discuss the description of lumped systems by state-space equations, solution of state-space equations, such properties as controllability, observability, and stability, and the relation of state descriptions to transfer-function-matrix descriptions.

Part III, consisting of one long chapter, discusses network analysis via state-space procedures. We discuss three procedures for analysis of passive networks, of increasing degree of complexity and generality, as well as analysis of active networks. This material is presented without significant use of network topology.

Part IV is concerned with translating into state-space terms the notions of passivity and reciprocity. Chapter 5 discusses a basic result of modern system theory, which we term the positive real lemma. It is of fundamental importance in areas such as nonlinear system stability and optimal control, as well as in network theory; Chapter 6 is concerned with developing procedures for solving equations that appear in the positive real lemma. Chapter 7 covers two matters; one is the bounded real lemma, a first cousin to the positive real lemma, and the other is the state-space description of the reciprocity property, first introduced in Chapter 2.

Part V is concerned with passive network synthesis and relies heavily on the positive real lemma material of Part IV. Chapter 8 introduces the general approaches to synthesis and disposes of some essential preliminaries. Chapters 9 and 10 cover impedance synthesis and reciprocal impedance synthesis, respectively. Chapter 11 deals with scattering-matrix synthesis, and Chapter 12 with transfer-function synthesis.

Part VI comprises one chapter and deals with active *RC* synthesis, i.e., synthesis using active elements, resistors, and capacitors. As with the earlier part of the book, state-space methods alone are considered.

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