Fuzzy Set Theory—and Its Applications

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Kluwer-Nijhoff Publishing
a member of the Kluwer Academic Publishers Group

Boston-Dordrecht-Lancaster

Distributors for North America:
Kluwer Academic Publishers
190 Old Derby Street
Hingham, MA 02043, U.S.A.

Distributors outside North America:
Kluwer Academic Publishers Group
Distribution Centre
P.O. Box 322
3300 AH Dordrecht
THE NETHERLANDS

Library of Congress Cataloging in Publication Data

Zimmermann, H.-J. (Hans-Jürgen), 1934-
Fuzzy set theory and its applications.

(International series in management science/operations research)
Bibliography: p.
Includes index.
1. Fuzzy sets. 2. Operations research. I. Title.
II. Series.
QA248.Z55 1984 511.3'2 84-29711
ISBN 0-89838—150-9

Printed in the United States of America

Foreword

As its name implies, the theory of fuzzy sets is, basically, a theory of graded concepts—a theory in which everything is a matter of degree or, to put it figuratively, everything has elasticity.

In the two decades since its inception, the theory has matured into a wide-ranging collection of concepts and techniques for dealing with complex phenomena which do not lend themselves to analysis by classical methods based on probability theory and bivalent logic. Nevertheless, a question which is frequently raised by the skeptics is: Are there, in fact, any significant problem-areas in which the use of the theory of fuzzy sets leads to results which could not be obtained by classical methods?

Professor Zimmermann's treatise provides an affirmative answer to this question. His comprehensive exposition of both the theory and its applications explains in clear terms the basic concepts which underlie the theory and how they relate to their classical counterparts. He shows through a wealth of examples the ways in which the theory can be applied to the solution of realistic problems, particularly in the realm of decision analysis, and motivates the theory by applications in which fuzzy sets play an essential role.

An important issue in the theory of fuzzy sets which does not have a counterpart in the theory of crisp sets relates to the combination of fuzzy sets through disjunction and conjunction or, equivalently, union and intersection. Professor Zimmermann and his associates at the Technical University of Aachen have made many important contributions to this problem and were the first to introduce the concept of a parametric family of connectives which can be chosen to fit a particular application. In recent years, this issue has given rise to an extensive literature dealing with t-norms and related concepts which link some apsects of the theory of fuzzy sets to the theory of probabilistic metric spaces developed by Karl Menger.

Another important issue which is addressed in Professor Zimmermann's treatise relates to the distinction between the concepts of probability and possibility, with the latter concept having a close connection with that of membership in a fuzzy set. The concept of possibility plays a particularly important role in the representation of meaning, in the management of uncertainty in expert systems, and in applications of the theory of fuzzy sets to decision analysis.

As one of the leading contributors to and practitioners of the use of fuzzy sets in decision analysis, Professor Zimmermann is uniquely qualified to address the complex issues arising in fuzzy optimization problems and, especially, fuzzy mathematical programming and multicriterion decision-making in a fuzzy environment. His treatment of these topics is comprehensive, up-to-date and illuminating.

In sum, Professor Zimmermann's treatise is a major contribution to the literature of fuzzy sets and decision analysis. It presents many original results and incisive analyses. And, most importantly, it succeeds in providing an excellent introduction to the theory of fuzzy sets—an introduction which makes it possible for an uninitiated reader to obtain a clear view of the theory and learn about its applications in a wide variety of fields.

The writing of this book was a difficult undertaking. Professor Zimmermann deserves to be congratulated on his outstanding accomplishment and thanked for contributing so much over the past decade to the advancement of the theory of fuzzy sets as a scientist, educator, administrator and organizer.

L.A. Zadeh
Berkeley, March 1985

Preface

Since its inception 20 years ago the theory of fuzzy sets has advanced in a variety of ways and in many disciplines. Applications of this theory can be found in artificial intelligence, computer science, control engineering, decision theory, expert systems, logic, management science, operations research, pattern recognition, robotics and others. Theoretical advances have been made in many directions. In fact it seems extremely difficult for a newcomer to the field or for somebody who wants to apply fuzzy set theory to his problems to recognize properly the present "state of the art." Therefore, many applications use fuzzy set theory on a much more elementary level than appropriate and necessary. On the other hand, theoretical publications are already so specialized and assume such a background in fuzzy set theory that they are hard to understand. The more than 4,000 publications that exist in the field are widely scattered over many areas and in many journals. Existing books are edited volumes containing specialized contributions or monographs that only focus on specific areas of fuzzy sets, such as pattern recognition [Bezdek 1981], switching functions [Kandel, Lee 1979], or decision making [Kickert 1978]. Even the excellent survey book by Dubois and Prade [1980] is primarily intended as a research compendium for insiders rather than an introduction to fuzzy set theory or a textbook. This lack of a comprehensive and modern text is particularly recognized by newcomers to the field and by those who want to teach fuzzy set theory and its applications.

The primary goal of this book is to help to close this gap—to provide a textbook for courses in fuzzy set theory and a book that can be used as an introduction.

One of the areas in which fuzzy sets have been applied most extensively is in modeling for managerial decision making. Therefore, this area has been selected for more detailed consideration. The information has been divided into two volumes. The first volume contains the basic theory of fuzzy sets and some areas of application. It is intended to provide extensive coverage of the theoretical and applicational approaches to fuzzy sets. Sophisticated formalisms have not been included. I have tried to present the basic theory and its extensions as detailed as necessary to be comprehended by those who have not been exposed to fuzzy set theory. Examples and exercises serve to illustrate the concepts even more clearly. For the interested or more advanced reader, numerous references to recent literature are included that should facilitate studies of specific areas in more detail and on a more advanced level.

The second volume is dedicated to the application of fuzzy set theory to the area of human decision making. It is self-contained in the sense that all concepts used are properly introduced and defined. Obviously this cannot be done in the same breadth as in the first volume. Also the coverage of fuzzy concepts in the second volume is restricted to those that are directly used in the models of decision making.

It is advantageous but not absolutely necessary to go through the first volume before studying the second. The material in both volumes has served as texts in teaching classes in fuzzy set theory and decision making in the United States and in Germany. Each time the material was used, refinements were made, but the author welcomes suggestions for further improvements.

The target groups were students in business administration, management science, operations research, engineering, and computer science. Even though no specific mathematical background is necessary to understand the books, it is assumed that the students have some background in calculus, set theory, operations research, and decision theory.

I would like to acknowledge the help and encouragement of all the students, particularly those at the Naval Postgraduate School in Monterey and at the Institute of Technology in Aachen (F.R.G.), who improved the manuscripts before they became textbooks. I also thank Mr. Hintz who helped to modify the different versions of the book, worked out the examples, and helped to make the text as understandable as possible. Ms. Grefen typed the manuscript several times without losing her patience. I am also indebted to Kluwer Nijhoff Publishing Company for making the publication of this book possible.

H.-J. Zimmermann
Aachen, March 1985

Contents

1 INTRODUCTION TO FUZZY SETS

1.1 Crispness, Vagueness, Fuzziness, Uncertainty

Most of our traditional tools for formal modelling, reasoning, and computing are crisp, deterministic and precise in character. By crisp we mean dichotomous, that is, yes-or-no-type rather than more-or-less type. In conventional dual logic, for instance, a statement can be true or false—and nothing in between. In set theory an element can either belong to a set or not, and in optimization a solution is either feasible or not. Precision assumes that the parameters of a model represent exactly either our perception of the phenomenon modelled or the features of the real system that has been modelled. Generally precision also implies that the model is unequivocal, that is, that it contains no ambiguities.

Certainty eventually indicates that we assume the structures and parameters of the model to be definitely known, and that there are no doubts about their values or their occurrence. If the model under consideration is a formal model [Zimmermann 1980, p. 127], that is, if it does not pretend to model reality adequately, then the model assumptions are in a sense arbitrary, that is, the model builder can freely decide which model characteristics he chooses. If, however, the model or theory asserts to be factual [Popper 1959, Zimmermann 1980], that is, conclusions drawn

Teaching aids and errata are available free of charge from the author directly: Prof. H.-J. Zimmermann, Templergraben 55, 5100 Aachen, Germany (F.R.G.).

from these models have a bearing on reality and they are supposed to model reality adequately, then the modelling language has to be suited to model the characteristics of the situation under study appropriately.

The utter importance of the modelling language is recognized by Apostel, when he says:

> The relationship between formal languages and domains in which they have models must in the empirical sciences necessarily be guided by two considerations that are by no means as important in the formal sciences:
> (a) The relationship between the language and the domain must be closer because they are in a sense produced through and for each other;
> (b) extensions of formalisms and models must necessarily be considered because everything introduced is introduced to make progress in the description of the objects studied. Therefore we should say that the formalization of the concept of approximate constructive necessary satisfaction is the main task of semantic study of models in the empirical sciences [Apostel 1961, p. 26].

Because we request that a modelling language is unequivocal and nonredundant on one hand and, at the same time, catches semantically in its terms all that is important and relevant for the model we seem to have the following problem. Human thinking and feeling, in which ideas, pictures, images, and value systems are formed, first of all has certainly more concepts or comprehensions than our daily language has words. If one considers, in addition, that for a number of notions we use several words (synonyms) then it becomes quite obvious that the power (in a set theoretic sense) of our thinking and feeling is much higher than the power of a living language. If in turn we compare the power of a living language with the logical language, then we will find that logic is even poorer. Therefore it seems to be impossible to guarantee a one to one mapping of problems and systems in our imagination and a model using a mathematical or logical language.

One might object that logical symbols can arbitrarily be filled with semantic contents and that by doing so the logical language becomes much richer. It will be shown that it is very often extremely difficult to appropriately assign semantic contents to logical symbols.

The usefulness of the mathematical language for modelling purposes is undisputed. However, there are limits of the usefulness and of the possibility of using the classical mathematical language, based on the dichotomous character of set theory, to models in particular systems and phenomena in the social sciences:" There is no idea or proposition in the field, which can not be put into mathematical language, although the utility of doing so can very well be doubted" [Brand 1961]. Schwarz [Schwarz 1962] brings up another argument against the unreflected use of mathematics, if he states: "An argument, which is only convincing if it is precise loses all its force if the assumptions on which it is based are slightly changed, while an argument, which is convincing but imprecise may well be stable under small perturbations of its underlying axioms." For factual models or modelling languages two major complications arise:

1. Real situations are very often not crisp and deterministic and they cannot be described precisely.
2. The complete description of a real system often would require by far more detailed data than a human being could ever recognize simultaneously, process and understand.

This situation has already been recognized by thinkers in the past. In 1923 the philosopher B. Russell [Russell 1923] referred to the first point when he wrote:

> All traditional logic habitually assumes that precise symbols are being employed. It is therefore not applicable to this terrestrial life but only to an imagined celestial existence.

L. Zadeh referred to the second point when he wrote: "As the complexity of a system increases, our ability to make precise and yet significant statements about its behaviour diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics" [Zadeh 1973].

Let us consider characteristic features of real world systems again: Real situations are very often uncertain or vague in a number of ways. Due to lack of information the future state of the system might not be known completely. This type of uncertainty (stochastic character) has long been handled appropriately by probability theory and statistics. This Kolmogoroff type probability is essentially frequentistic and bases on set-theoretic considerations. Koopman's probability refers to the truth of statements and therefore bases on logic. On both types of probabilistic approaches it is assumed, however, that the events (elements of sets) or the statements, respectively, are well defined. We shall call this type of uncertainty or vagueness *stochastic uncertainty* by contrast to the vagueness concerning the description of the semantic meaning of the events, phenomena or statements themselves, which we shall call *fuzziness*.

Fuzziness can be found in many areas of daily life, such as in engineering [see, for instance, Blockley 1980], in medicine [see Vila, Delgado 1983], in meteorology [Cao, Chen 1983], in manufacturing [Mamdani 1981] and others. It is particularly frequent, however, in all areas in which human judgment, evaluation, and decisions are important. There are the areas of decision making, reasoning, learning, and so on. Some reasons for this have already been mentioned. Others are that most of our daily communication uses "natural languages" and a good part of our thinking is done in it. In these natural languages the meaning of words is very often vague. The meaning of a word might even be well defined, but when using the word as a label for a set, the boundaries within which objects belong to the set or do not become fuzzy or vague. Examples are words such as "birds" (how about penguins, bats, etc.?), "red roses," but also terms such as "tall men," "beautiful women," "creditworthy customers." In this context we can probably distinguish two kinds of fuzziness with respect to their origins: intrinsic fuzziness and informational fuzziness. The former is the fuzziness to which Russell's remark referred and it is illustrated by "tall men." This term is fuzzy because the meaning of tall is fuzzy and dependent on the context (height of observer, culture, etc.). An example of the latter is the term "creditworthy customers": A creditworthy customer can possibly be described completely and crisply if we use a large number of descriptors. These are more, however, than a human being could handle simultaneously. Therefore the term, which in psychology is called a "subjective category" becomes fuzzy. One could imagine that the subjective category creditworthiness is decomposed into two smaller subjective categories, each of which needs fewer descriptors to be completely described. This process of decomposition could be continued until the descriptions of the subjective categories generated are reasonably defined. On the other hand, the notion "creditworthiness" could be constructed by starting with the smallest subjective subcategories and aggregating them hierarchically.

For creditworthiness this concept structure, which has a symmetrical structure, was developed together with 50 credit clerks of banks.

Credit experts distinguish between the financial basis and the personality of an applicant. The financial basis comprises all realities, movables, assets, liquid funds, and others. The evaluation of the economic situation depends on the actual securities, that is, the difference between property and debts, and on the liquidity, that is, the continuous difference between income and expenses.

On the other hand, personality denotes the collection of traits by which a potent and serious person distinguishes itself. The achievement potential bases on the mental and physical capacity as well as on the individual's motivation. The business conduct includes economical standards. While the former means setting of realistic goals, reasonable planning, and economic criteria success, the latter is directed toward the applicant's disposition to obey business laws and mutual agreements. Hence a creditworthy person lives in secure circumstances and guarantees a successful, profit-oriented cooperation. See Figure 1-1.

In chapter 14 we will return to this figure and elaborate on the type of aggregation.

1.2 Fuzzy Set Theory

The first publications in fuzzy set theory by Zadeh [Zadeh 1965] and Goguen [Goguen 1967, 1968] show the intention of the authors to generalize the classical notion of a set and a proposition (statement) to accommodate fuzziness in the sense described in paragraph 1.1.

Zadeh [Zadeh 1965, p. 339] writes: "The notion of a fuzzy set provides a convenient point of departure for the construction of a conceptual framework which parallels in many respects the framework used in the case of ordinary sets, but is more general than the latter and, potentially, may prove to have a much wider scope of applicability, particularly in the fields of pattern classification and information processing. Essentially, such a framework provides a natural way of dealing with problems in which the source of imprecision is the absence of sharply defined criteria of class membership rather than the presence of random variables."

"Imprecision" here is meant in the sense of vagueness rather than the lack of knowledge about the value of a parameter as in tolerance analysis. Fuzzy set theory provides a strict mathematical framework (there is nothing fuzzy about fuzzy set theory!) in which vague conceptual phenomena can be precisely and rigorously studied. It can also be considered as a modelling language well suited for situations in which fuzzy relations, criteria, and phenomena exist.

Fuzziness has so far not been defined uniquely semantically, and probably never will. It will mean different things, depending on the application area and the way it is measured. In the meantime, numerous authors have contributed to this theory. In 1984 as many as 4,000 publications may already exist. The specialization of those publications conceivably increases, making it more and more difficult for newcomers to this area to find a good entry and to understand and appreciate the philosophy, formalism, and applicational potential of this theory. Roughly speaking, fuzzy set theory in the last two decades has developed along two lines:

1. As a formal theory which, when maturing, became more sophisticated and specified and which was enlarged by original ideas and concepts as

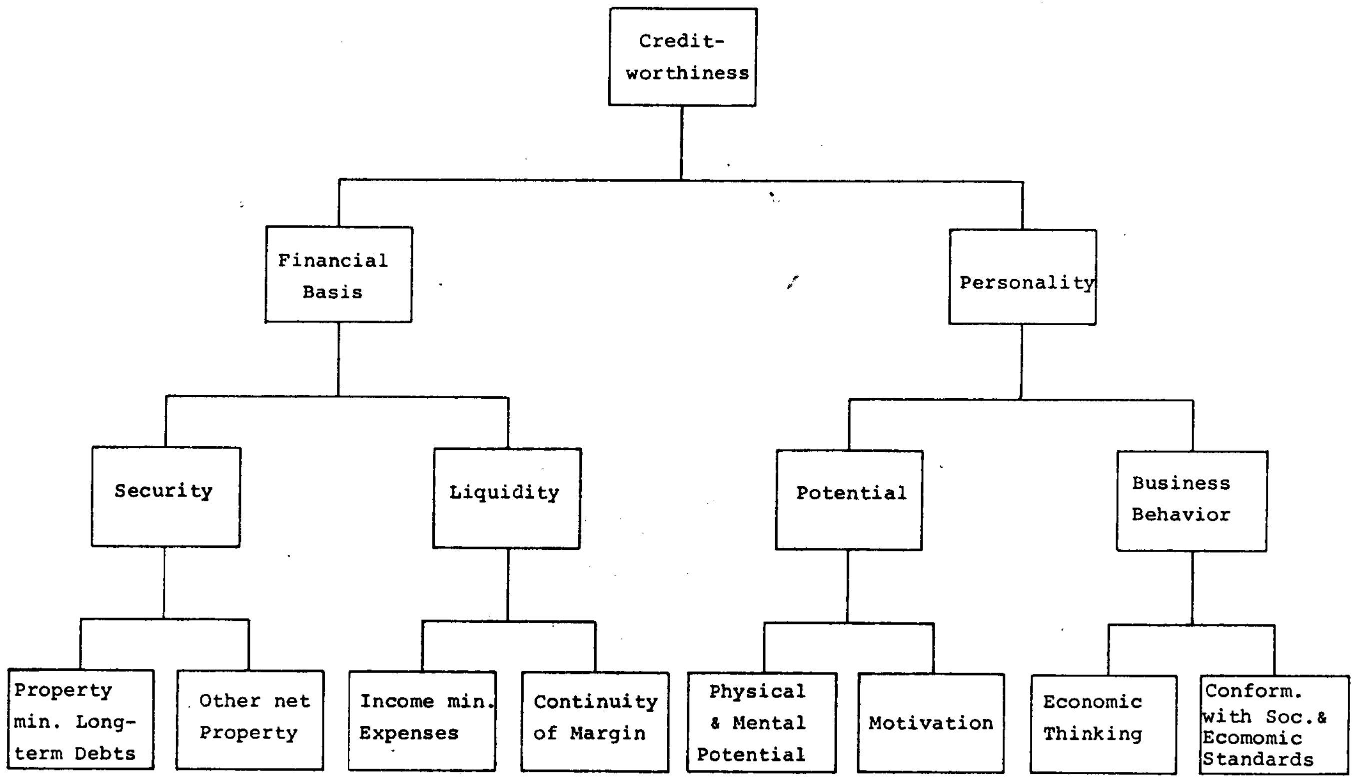

Figure 1-1. Concept Hierarchy of Creditworthiness

well as by "embracing" classical mathematical areas such as algebra, graph theory, topology, and so on by generating (fuzzifying) them.

2. As a very powerful modelling language, which can cope with a large fraction of uncertainties of real-life situations. Because of its generality it can be well adapted to different circumstances and contexts. In many cases this will mean, however, the context-dependent modification and specification of the original concepts of the formal fuzzy set theory. Regrettably this adaption has not yet progressed to a satisfactory level, leaving an abundance of challenges for the ambitious researcher and practitioner.

It seems desirable that an introductionary textbook be available to help students to get started and find their way around. Obviously such a textbook cannot cover the entire body of the theory as available now in appropriate detail. This book will therefore proceed as follows:

Part I of this book, containing chapters 2 and 8, will develop the formal framework of fuzzy mathematics. Due to space limitations and for didactical reasons two restrictions will be observed:

1. Topics which are of high mathematical interest but which require a very solid mathematical background and those which are not of obvious applicational relevance will not be discussed.
2. Most of the discussion will proceed along the lines of the early concepts of fuzzy set theory. At appropriate times, however, the additional potential of fuzzy set theory by using other axiomatic frameworks resulting in other operators will be indicated or described. The character of these chapters will obviously have to be formal.

The second part, chapters 9 to 14, of the book will then survey the most interesting applications of fuzzy set theory. At that stage the student should be in a position to recognize possible extensions and improvements of the applications presented. Chapter 12 on decision making in fuzzy environments might be considered as unduly brief, compared with the available literature. This area, however, will be taken up in a second volume and discussed in much more detail. This seems justified since on one hand it might not be of interest to a good number of persons being interested in fuzzy set theory from another angle and on the other hand it can be considered as the most advanced of the application areas of fuzzy set theory.