Linear Control Systems

A Computer-aided Approach

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PERGAMON PRESS

OXFORD · NEW YORK · BEIJING · FRANKFURT
SÃO PAULO · SYDNEY · TOKYO · TORONTO

U.K.	Pergamon Press, Headington Hill Hall, Oxford OX3 0BW, England
U.S.A.	Pergamon Press, Maxwell House, Fairview Park, Elmsford, New York 10523, U.S.A.
PEOPLE'S REPUBLIC OF CHINA	Pergamon Press, Qianmen Hotel, Beijing, People's Republic of China
FEDERAL REPUBLIC OF GERMANY	Pergamon Press, Hammerweg 6, D-6242 Kronberg, Federal Republic of Germany
BRAZIL	Pergamon Editora, Rua Eça de Queiros, 346, CEP 04011, São Paulo, Brazil
AUSTRALIA	Pergamon Press Australia, P.O. Box 544, Potts Point, N.S.W. 2011, Australia
JAPAN	Pergamon Press, 8th Floor, Matsuoka Central Building, 1-7-1 Nishishinjuku, Shinjuku-ku, Tokyo 160, Japan
CANADA	Pergamon Press Canada, Suite 104, 150 Consumers Road, Willowdale, Ontario M2J 1P9, Canada

First edition 1986

Library of Congress Cataloging in Publication Data

Jamshidi, Mohammad.
Linear control systems.
(International series on systems and control; v. 7)
Includes bibliographical references.
I. Automatic control — Data processing. 2. Control theory — Data processing. I. Malek-Zavarei, Manu. II. Title. III. Series.
TJ213.J32 1984 629.8'312 84–6164

British Library Cataloguing in Publication Data

Jamshidi, Mohammad
Linear control systems. — (International series on systems and control ; v. 7).
1. Automatic control — Data processing. I. Title
II. Malek-Zavarei, Manu. III. Series
629.8'32'02854 TJ213

ISBN 0–08–028701–8 (Hardcover)
ISBN 0–08–028702–6 (Flexicover)

Printed in Great Britain by A. Wheaton & Co. Ltd., Exeter

International Series on

SYSTEMS AND CONTROL, Volume 7

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PREFACE

This text is intended to give an introduction to classical and modern control systems modeling, analysis and design. The bulk of the manuscript is based on two courses on classical/modern control and linear systems at the *University of New Mexico's Department of Electrical and Computer Engineering*. The book's three main themes are: control, systems and computer-aided design and analysis.

The text has evolved over several years of the author's combined effort at various locations including Shiraz University (Shiraz, Iran), IBM Thomas J. Watson Research Center (Yorktown Heights, NY) and the University of New Mexico (Albuquerque, NM). The book is intended to bridge the gap between a first course in classical control and a theoretical-oriented graduate course such as optimal control. Continuous-time and discrete-time as well as time-domain and frequency-domain presentations of linear systems are considered on a balanced basis. The intended audience are thus the advanced undergraduate and the first-year graduate electrical engineering students. There has been no attempt to teach computer programming in the text although there are over 100 BASIC subroutines and driver programs listed in the manuscript. All the computer programs have been originally written for an HP-9845A or B desk-top computers. These programs are now being put together in the form of two integrated and interconnected packages called FREDOM© and TIMDOM© to treat frequency-domain and time-domain techniques of analysis and design. The packages are also being tailored for a wide range of personal and other computers such as IBM/PC, HP-2816, Apple IIe and SUN workstation. For more information on the availability and acquisition of these packages, the interested reader may write directly to the Publisher.

The detailed plan of the book is shown on page 4. A typical chapter of the book is structured as follows. The chapter begins with an Introduction Section which is followed by a theoretical development or a mathematical description, which are further illustrated by numerical examples. Occasionally certain important techniques are summarized by an algorithm. The various system and control concepts are coded in extended BASIC and the respective main or driver programs are listed within the text. In the cases where a given subroutine is called once the subroutines is also listed along with the main program. All the subroutines called more than once within the programs and all the utility routines are listed in Appendix A. Indices to all programs and subroutines are given at the end of the book. Each chapter ends with a Problems Section and a list of cited references.

Chapter 1 serves as an introduction to the text. In this chapter a system is defined and important classes of systems are presented. Chapter 2 and 3 present a review of linear algebra and the transforms (Laplace and z) theories. A well familiar reader can skip these two chapters without any loss of continuity. System modeling utilizing both transfer functions (frequency domain) and statespace representation (time-domain) have been presented in Chapter 4. State transformations and linearization of nonlinear systems are among other topics discussed here.

In Chapter 5 model reduction of large-scale linear time-invariant systems is discussed. Both perturbation (singular and regular) and aggregation (time-domain and frequency-domain) are presented here. The solution of the state equations for both continuous-time and discrete-time systems are discussed in Chapter 6. Both time-invariant and time-varying cases are considered.

Chapter 7 is devoted to system stability. This chapter includes notions of stability in the classical control such as Routh-Hurwitz and Jury-Blanchard criteria as well as the root locus method, Bode and Nyquist diagrams and the Nyquist criterion. It also includes modern system theoretical stability concepts such as zero-input and zero-state stability, the Lyapunov method and the circle criterion are. The important notions of controllability and observability are discussed in Chapter 8. The canonical decomposition of an uncontrollable (unobservable) system into controllable (observable) and uncontrollable (unobservable) subsystem are discussed here. Minimal realization and duality are among other topics discussed in Chapter 8.

The next three chapters of the book are generally concerned with the design of linear single-input single-output (SISO) or multi-input multi-output (MIMO) systems. In Chapter 9, design of a SISO linear control system via series and feedback compensations is presented. The design using root locus and frequency responses (Bode and Nyquist diagrams) as design tools with the aid of available computer programs are illustrated by a number of detail design problems. The computer-aided design of a SISO system via feedback compensation is illustrated as well.

Design of linear MIMO systems via modern system concepts such as pole placement along with state estimation are taken up in Chapter 10. The design of a state estimator (observer) using full or reduced order formulations are presented here.

Chapter 11 is concerned with design of linear SISO systems through optimization of system criteria. Here functional minimization techniques are utilized to design a controller for a SISO control system subject to plant parameter variations. Parseval's theorems for both continuous-time and discrete-time systems have been utilized for this purpose. In short, Chapter 11 is concerned with design of a linear SISO system via parameter optimization.

There are two main schemes to utilize the text: classical control and modern system engineering. The suggested sequence of chapters and sections to be used for the above two schemes are given below.

Classical Control (a senior-level course): Chapters 1, 3, Sections 4.2, 4.3, 4.5, 7.4, 7.5, 7.6 and 7.7, Chapters 9 and 11.

Linear System (a first-year graduate course): Chapters 1, 2, Sections 3.6 through 3.9, 4.2 through 4.4, 4.6 and 4.7, Chapters 6, Sections 7.2, 7.3, and 7.9, Chapter 8, Chapter 10, Sections 5.3 and 5.4.

The manuscript has been used in two such courses at the Department of Electrical and Computer Engineering of the University of New Mexico. The text can be even more useful to the reader when it is used in conjunction with the packages FREDOM© and TIMDOM©.

The authors are indebted to many people for their various contributions. We would like to thank Dean Jerry May of the University of New Mexico's College of Engineering and Peter Dorato, Chairperson of the Electrical and Computer Engineering Department for their leadership and continuous support. The authors would like to thank Professor Madan Singh of the University of Manchester Institute of Technology for reviewing the manuscript and many helpful suggestions. We would like to thank Professor Singh in another capacity — the series editor of Pergamon's International Series on Systems and Control for suggesting to write the book. We would like to thank many of our former students who have contributed to the text in various forms. In particular we like to thank D. Behroozi, R.-W. Chang, S.-K. Chiang, G. Eisler, V. Gieri, M. Masukawa, R. Morel, S. Otis, R. Owen, P.-E. Tang and J. Wilcoxen for writing the original versions of some of the programs.

The manuscript was prepared using a DEC 11/780 VAX digital computer. We would like to thank Mrs. Gladys Ericksen for many hours of tireless and persistent text processing at the computer's terminals.

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3 REVIEW OF THE LAPLACE AND z-TRANSFORMS

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4 SYSTEM MODELING

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CHAPTER 1

PRELIMINARIES

1.1 INTRODUCTION

This chapter serves as a general introduction to the book. It is intended to provide a framework within which the analysis and design of linear systems can be explained. As such it may refer to some concepts which are not defined yet but will be defined in later chapters.

In the following sections, first the concept of a system will be introduced. Then the notation to be used in the book will be explained. A section on systems classification and a final section on the scope of the book will conclude this chapter.

1.2 WHAT IS A SYSTEM?

Webster's dictionary defines a *system* as "a regularly interacting or independent group of items forming a unified whole." From our point of view a system is an entity that can be characterized by a finite number of attributes. An example of a system is a lumped electric circuit. Some attributes of this system are the values of the elements used in the circuit. Another example is a weight resting on a spring. Some attributes of this system are the mass of the weight and the spring constant. Sufficient attributes should be given for each system to wholly characterize it.

A *control system* is a system capable of monitoring or regulating the operation of a process or a plant. An early example of a control system is the centrifugal steam engine governor invented by James Watt in the eighteenth century [1.1,1.2]. This control system controlled the flow of steam by the application of centrifugal force to a lever, to maintain the speed of a steam engine at a relatively constant level. A modern example of a control system is the autopilot system of an airplane.

1.3 NOTATIONS

In this section first the method of numbering and cross-referencing in the book will be explained. Then the notations and abbreviations used throughout the book will be compiled.

1.3.1 Numbering and Cross-Referencing

Chapters of the book are consecutively numbered. The sections in each chapter are numbered consecutively by two digits, the first of which is the chapter number. Subsections in each section are similarly numbered consecutively by three digits. The equations in each section are numbered on the right-hand side consecutively. Also, the figures in each section are numbered consecutively. For reference to an equation in the *same* section, only the number of the equation will be referred to. But to refer to an equation or a figure in *another* section, the two-digit section number will also precede the number of the equation or the figure.

Also in each section consecutive numbers are used to identify definitions, theorems and examples. In the *same* section, these are referred to by their numbers. In *other* sections, these are referred to by a three-digit number consisting of their number preceded by the two-digit number of the section in which they appear.

1.3.2 Conventions

Capital letters denote sets or vector spaces, e.g., S, V.

Lower case and Greek letters indicate scalars and scalar-valued functions, e.g., m, α, $f(t)$.

Bold lower case letters indicate vectors, e.g., $\mathbf{x}$, $\mathbf{y}$.

Bold capital letters indicate matrices, e.g., $\mathbf{A}$, $\mathbf{B}$.

The Laplace transform of a function is denoted by the corresponding capital letter, e.g., $G(s) = L[g(t)]$.

The z-transform of a function is indicated by the corresponding capital letter, e.g., $H(z) = Z[h(t)]$.

Superscript "'" denotes the transpose of a vector or a matrix, e.g., $\mathbf{x}'$, $\mathbf{A}'$.

Superscript "*" denotes the conjugate transpose of a vector or a matrix, e.g., $\mathbf{x}^*$, $\mathbf{A}^*$.

Superscript "−1" denotes the inverse of a matrix or a transformation, e.g., $\mathbf{A}^{-1}$, L^{-1}.

Superscript "⊥" denotes the orthogonal complement of a subspace, e.g., $V^{\perp}$.

A dot over a time function denotes its derivative with respect to time, e.g., $\dot{x}$, $\dot{\mathbf{x}}$.

Two or three dots over a time function denote its second or third derivative with respect to time, respectively, e.g., $\ddot{\mathbf{x}}$, $\dddot{\mathbf{x}}$. For higher-order derivatives a superscript with the corresponding derivative order in parenthesis may also be used, e.g., $x^{(3)}$, $\mathbf{y}^{(4)}$. A bar over a scalar or a vector denotes its complex conjugate, e.g., $\bar{\alpha}$, $\bar{\mathbf{x}}$.

Braces indicate sets, e.g., $\{x\}$.

1.3.3 Abbreviations and Symbols

(.,.), (.,.] and [.,.] denote, respectively, open, semiclosed, and closed interval, respectively, e.g., t in the interval $(a,b]$ means $a < t \leq b$.

$||.||$: norm of a vector or a matrix, e.g., $||\mathbf{x}||$, $||\mathbf{A}||$.
$\underline{\Delta}$: equal by definition
$\Rightarrow$: implies
$\Leftarrow$: is implied by
$\Leftrightarrow$: implies and is implied by
$\exists$: there exists
$\forall$: for all
$\ni$: such that
ϵ: belongs to, e.g., $x \epsilon S$
$\notin$: does not belong to
$\supset$: contains, e.g., $S_1 \supset S_2$
$\subset$: is contained in
$\cup$: union
$\cap$: intersection
$\oplus$: direct sum
Δ : end of an example or discussion
$*$: convolution
$(.,.)$: inner product
j: the imaginary number $\sqrt{-1}$
adj: adjoint of a matrix, e.g., *adj* **A**
det (.): determinant of a matrix, e.g., det (**A**)
$\rho(.)$: rank of a matrix, e.g., $\rho(\mathbf{A})$
$\gamma(.)$: nullity of a matrix, e.g. $\gamma(\mathbf{A})$
tr (.): trace of a matrix, e.g., *tr* (**A**)
$D^{-n}(.)$: nth-order integration of a function, e.g., $D^{-2}(f)$
diag: diagonal matrix
q.e.d. : quod erat demonstrandum (which was to be proved)
l.t.i.: linear time-invariant
w.r.t.: with respect to
KCL: Kirchoff's current law
KVL: Kirchoff's voltage law
r.h.p.: right-half plane
l.h.p.: left-half plane
exp(.): exponential of a scalar or a matrix, e.g., exp $(\mathbf{A}t) \underline{\Delta} e^{\mathbf{A}t}$
SISO: single-input single-output
MIMO: multi-input multi-output
AMRE: algebraic matrix Riccati equation
DMRE: discrete matrix Riccati equation
TPBVP: two-point boundary value problem

1.4 CLASSIFICATION OF SYSTEMS

From this point on the word "system" means a system model, i.e., a mathematical representation of a physical system. (See Chapter 4.) Systems can be classified according to the type of equations describing them. To this end there are five different ways of classifying systems.

1.4.1 Lumped-Parameter and Distributed-Parameter Systems

Lumped-parameter systems are those which can be described by ordinary differential (or difference) equations. Such systems are also referred to as finite-dimensional systems for reasons which will become clear later. In contrast, *distributed-parameter* systems, referred to as infinite-dimensional systems, are those which require partial differential equations for their characterization.

A lumped electric circuit, i.e., one in which the element sizes are negligible compared to the wavelength of the highest frequency of operation, is an example of a lumped-parameter system. A transmission line is an example of a distributed-parameter system. We will be concerned only with lumped-parameter systems in this book.

1.4.2 Deterministic and Stochastic Systems

In *deterministic* systems all the parameters can be described exactly. In *stochastic* systems, however, some (or all) parameters can be described only probabilistically, i.e., as random variables. We will deal only with deterministic systems in this book.

1.4.3 Continuous-Time and Discrete-Time Systems

In *continuous-time* systems, described by differential equations, all the variables are defined for all values of time in typically a semi-infinite interval $[t_o, \infty)$. In contrast, the variables in *discrete-time* systems are defined only at discrete instants of time. Discrete-time systems are described by difference equations.

1.4.4 Linear and Nonlinear Systems

A *linear* system is one to which the superposition principle applies. That is, a linear system exhibits proportionality of input and output. This, however, is an over-simplification; for a rigorous definition of system linearity will be given in Chapter 4. A linear system can be described by a linear differential or difference equation. A system which is not linear is called a *nonlinear* system.

1.4.5 Time-Invariant and Time-Varying System

A system is called *time-invariant* or *stationary* if all its parameters are constant. Such systems can be described by constant-coefficient differential or difference equations. If one or more of the system parameters vary with time, the system will be called *time-varying* or *nonstationary*. Such systems are described by differential or difference equations with time-varying coefficients.

It should be noted that a given system falls into one of the two categories in each of the above classifications. For example a system may be lumped, stochastic, continuous-time, linear and time-varying.

1.5 SCOPE OF THE BOOK

The main theme of the book is the application of computers in the analysis and design of linear systems. The book is divided into three parts:

Part I. Mathematical Background
Part II. Analysis
Part III. Design

Part I provides the mathematical background on which Parts II and III are based. This part includes a review of linear algebra, Laplace and z transforms. Each topic is supported by several computer programs and accompanied by many numerical examples.

Part II deals with the analysis of linear systems. Topics such as system modeling, model reduction in large-scale linear systems as well as the solution of system equations and stability are included in this part. Also, controlability and observability of linear systems are addressed in Part II. Relevant ready-to-use computer programs and numerical examples support each topic.

Design of linear control systems from the classical and modern points of view is considered in Part III. This part treats the design of linear systems by compensation, state estimation in linear systems, parameter optimization and optimal control. As in Parts I and II, every topic in Part III is accompanied by computer programs as well as numerical examples.

The book is mainly concerned with lumped, deterministic, linear time-invariant systems. Both continuous-time and discrete-time systems are treated in each topic. At some points, digressions may be made to nonlinear and/or time-varying systems.

The computer programs are written in BASIC language. An appendix is included at the end of the book which contains the source code for various subroutines.

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