APPLIED CALCULUS

THIRD EDITION

FOR BUSINESS AND ECONOMICS,
LIFE SCIENCES, AND SOCIAL SCIENCES

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DELLEN PUBLISHING COMPANY
San Francisco, California

COLLIER MACMILLAN PUBLISHERS
London

divisions of Macmillan, Inc.

Preface

Many colleges and universities now offer mathematics courses that emphasize useful topics for students in business and economics, life sciences, and social sciences. Because of this trend, we have surveyed instructors, course outlines, and college catalogs from a large number of colleges and universities and, on the basis of these surveys, selected the topics, applications, and emphasis in this text.

The material in this book is suitable for a two-quarter or two-semester course in calculus for students in business and economics, computer science, life sciences, and/or social sciences. The contents consist of a careful but well-motivated presentation of the calculus for functions of one variable, including the exponential and logarithmic functions (trigonometric functions are included in an appendix), and selected topics in multivariable calculus, differential equations, infinite series, numerical techniques, and probability. The choice and organization of topics make the book readily adaptable to a variety of courses. (See the diagram on page xvii for chapter dependencies.)

The book is designed for students who have had $1\frac{1}{2}-2$ years of high school algebra or the equivalent. However, because much of this material is forgotten due to lack of use, Chapter 0 contains a review of the basic topics in intermediate algebra that are most pertinent to the course, and portions of Appendix A review more fundamental algebraic concepts. Any of this material can be studied systematically at the beginning of a course or referred to throughout a course as needed. In addition, certain key topics are reviewed immediately before their use (see Section 3-3).

■ Major Changes from the Second Edition

The third edition of *Applied Calculus for Business and Economics, Life Sciences and Social Sciences* reflects the experiences and recommendations of a large number of the users of the first two editions. Much of the material has been reorganized to provide a more efficient presentation of the topics. Many examples and exercises have been changed to provide clear illustrations of mathematical concepts without undue algebraic complexity. Special attention has been paid to increasing the quantity and quality of applications throughout the book. It is impossible to use actual real-world models in many applied problems, but it is possible to provide simplified versions of such models that reflect the important features of the applica-

tion and involve mathematical operations appropriate for the level of this book. Such simplified, yet realistic, applications have been included in every chapter of the book.

The intermediate algebra review material has been reorganized. Chapter 0 now contains the most frequently used algebra topics, including quadratic equations and logarithmic and exponential functions, and accurately reflects the level of algebraic ability necessary for the material in the remainder of the book. Portions of Appendix A review more fundamental algebra concepts for students whose algebraic skills have become rusty due to lack of use.

The material on differentiation (Chapters 1–3) has been reorganized; the treatment of limits and asymptotes has been simplified; and the chain rule is now covered twice, first in Chapter 1 as the general power rule and again in Chapter 3, after the derivatives of the exponential and logarithmic functions have been introduced.

In Chapters 4 and 5, integration by substitution is now covered earlier (Section 4-2) and in a more fundamental manner and the Riemann sum approach to integration has been moved to Chapter 5. The discussion of consumers' and producers' surplus has been expanded and placed in a new section (Section 5-2), along with a discussion of continuous income streams. The integral table has been expanded to provide more flexibility and to give the students a better idea of the nature of a full-sized integral table.

In Chapters 6–10, the sections on total differentials and joint probability density functions have been deleted, the treatment of L'Hôpital's rule has been simplified, and new sections on binomial and normal probability distributions have been added.

■ General Comments

Chapters 1–3 present the differential calculus for functions of one variable, including the exponential and logarithmic functions. Limits and continuity are presented in an intuitive fashion, utilizing numerical approximations and graphical concepts. The basic rules of differentiation for algebraic functions are covered in Chapter 1. The relationship between derivatives and graphs is discussed in Sections 2-1 and 2-2 and then applied to optimization problems in Section 2-3 and to curve sketching in Section 2-4. A short section on differentials is included to facilitate their use in integration by substitution and by parts. The chain rule is first introduced as the general power rule in Chapter 1 and then generalized in Chapter 3 to reinforce this important rule as students learn to apply it to derivatives involving exponential and logarithmic functions, implicit differentiation, and related rates.

Chapters 4 and 5 deal with integral calculus. Differential equations and exponential growth and decay are included as an application of antidiffer-

entiation. In Chapter 4, the definite integral is intuitively introduced in terms of an area function and then, in Chapter 5, it is formally defined as the limit of a Riemann sum. Integration by parts, some additional applications of integration, improper integrals, and integral tables are also covered in Chapter 5.

Chapter 6 introduces multivariable calculus, including partial derivatives, optimization, Lagrange multipliers, least squares, and double integrals. If desired, this chapter can be covered immediately after Chapter 4. (See the diagram on page xvii.) No previous experience with three-dimensional coordinate systems is assumed. Three-dimensional figures are included to illustrate the concepts, but graphing techniques are not discussed.

Differential equations are first introduced in Section 4-3, but Chapter 7 contains a much more thorough presentation of this important topic, including second-order linear differential equations and systems of first-order linear equations. All the growth laws discussed in Section 4-3 are reviewed and many other applications are considered.

Chapter 8 begins by discussing the approximation of functions by Taylor polynomials. Infinite series follow naturally from this presentation, eliminating the need for a lengthy treatment of series of constants and all the associated convergence tests. The operations that can be performed on Taylor series are carefully discussed since this is the most practical procedure for finding the Taylor series for many functions.

A variety of numerical techniques are covered in Chapter 9. This material can be presented effectively using only a hand-held calculator, but sample computer output is included in each section as an illustration of the way these techniques are put to use in actual practice. If students have access to computing facilities, it is a simple matter to incorporate the use of a computer in the presentation of this material. An interactive computer applications package containing the programs used in this chapter, additional calculus related programs, and additional examples and exercises is available from the publisher (see the lists of student and instructor aids later in this Preface).

Chapter 10 covers basic probability concepts, including both discrete and continuous probability distributions. The presentation, though comprehensive, assumes no previous experience with probability.

Finally, Appendix B reviews the basic properties of the trigonometric functions and then develops differentiation and integration formulas for these functions. This material can be discussed any time after Chapter 4 has been covered (see the diagram on page xvii).

■ Important Features

Emphasis

Emphasis is on computational skills, ideas, and problem solving rather than mathematical theory. Most derivations and proofs are omitted except

where their inclusion adds significant insight into a particular concept. General concepts and results are usually presented only after particular cases have been discussed.

Examples and Matched Problems

More than 350 completely worked examples are included. Each example is followed by a similar problem for the student to work while reading the material. This actively involves the student in the learning process. The answers to these matched problems are included at the end of each section for easy reference.

Exercise Sets

This book contains more than 3,600 exercises. Each exercise set is designed so that an average or below-average student will experience success and a very capable student will be challenged. The sets are usually divided into A (routine, easy mechanics), B (more difficult mechanics), and C (difficult mechanics and some theory) levels.

Applications

Enough applications are included to convince even the most skeptical student that mathematics is really useful. The majority of the applications are included at the end of exercise sets and are generally divided into business and economics, life science, and social science groupings. An instructor with students from all three disciplines can let them choose applications from their own fields of interest; if most students are from one of the three areas, then special emphasis can be placed there. Most of the applications are simplified versions of real-world problems taken from professional journals and books. No specialized experience is required to solve any of the applications in this book.

■ Student and Instructor Aids

Student Aids

Dashed **"think boxes"** are used to enclose steps that are usually performed mentally (see Section 0-2).

Examples and developments are often **annotated** to help students through critical stages (see Section 0-2).

A **second color** is used to indicate key steps (see Section 0-2).

Boldface type is used to introduce new terms and highlight important comments.

Answers to odd-numbered problems are included in the back of the book.

Chapter review sections include a review of all important terms and symbols and a comprehensive review exercise. Answers to all review exercises are included in the back of the book.

A **student's solutions manual** by G. J. Etgen is available at a nominal cost through a book store. The manual includes detailed solutions to all odd-numbered problems and all review exercises.

A manual for an **Interactive Computer Applications Package** by Carolyn L. Meitler is available at a nominal cost through a book store. The manual contains instructions, examples, and exercises that demonstrate the use of the programs on the *ICAP for Applied Calculus* disks. The disks containing the programs are distributed free of charge to institutions using this book. No previous computing experience is necessary to use this package.

Instructor Aids

A unique **computer-generated random test system** is available to instructors without cost. The system, utilizing an IBM PC™ computer and a number of commonly used dot matrix printers, will generate an almost unlimited number of chapter tests and final examinations, each different, quickly and easily. At the same time, the system produces an answer key and a student worksheet with an answer column that exactly matches the answer column on the answer key. Graphing grids are included on the answer key and on the student worksheet for problems requiring graphs.

A **printed and bound test battery** is also available to instructors without cost. The battery contains several chapter tests for each chapter, answer keys, and student worksheets with answer columns that exactly match the answer columns on the answer keys. Graphing grids are included on the answer key and on the student worksheet for problems requiring graphs.

An **instructor's answer manual** containing answers to the even-number problems not included in the text is available to instructors without charge.

A **student's solutions manual** by G. J. Etgen (see Student Aids) is available to instructors without charge from the publisher.

An **Interactive Computer Applications Package** by Carolyn L. Meitler (see Student Aids) is available to instructors without charge from the publisher. The programs in this package are available on diskettes for APPLE II™ and IBM PC™ computers. The publisher will supply one of these diskettes without charge to institutions using this book.

Acknowledgments

In addition to the authors, many people are involved in the successful publication of a book. We wish to thank personally all those who reviewed the manuscript: Carl Bedell, Philadelphia College of Textiles and Science; Alberto Beron, Moorpark College; Frank Dangello, Shippensburg University; Duane Deal, Ball State University; Mel Mitchell, Clarion University of Pennsylvania; Thomas Plavchek, Wilkes College; Thomas Spradley, American River College; and Delores Williams, Pepperdine University.

We also wich to thank:

John Williams for a strong and effective cover design.

John Drooyan for the many sensitive and beautiful photographs throughout the book.

Philip Bender, Stephen Merrill, and Robert Mullins for carefully checking all examples and problems (a tedious but extremely important job).

Carolyn Meitler for assisting in the preparation of the computer-generated testing system and for developing the *ICAP for Applied Calculus*.

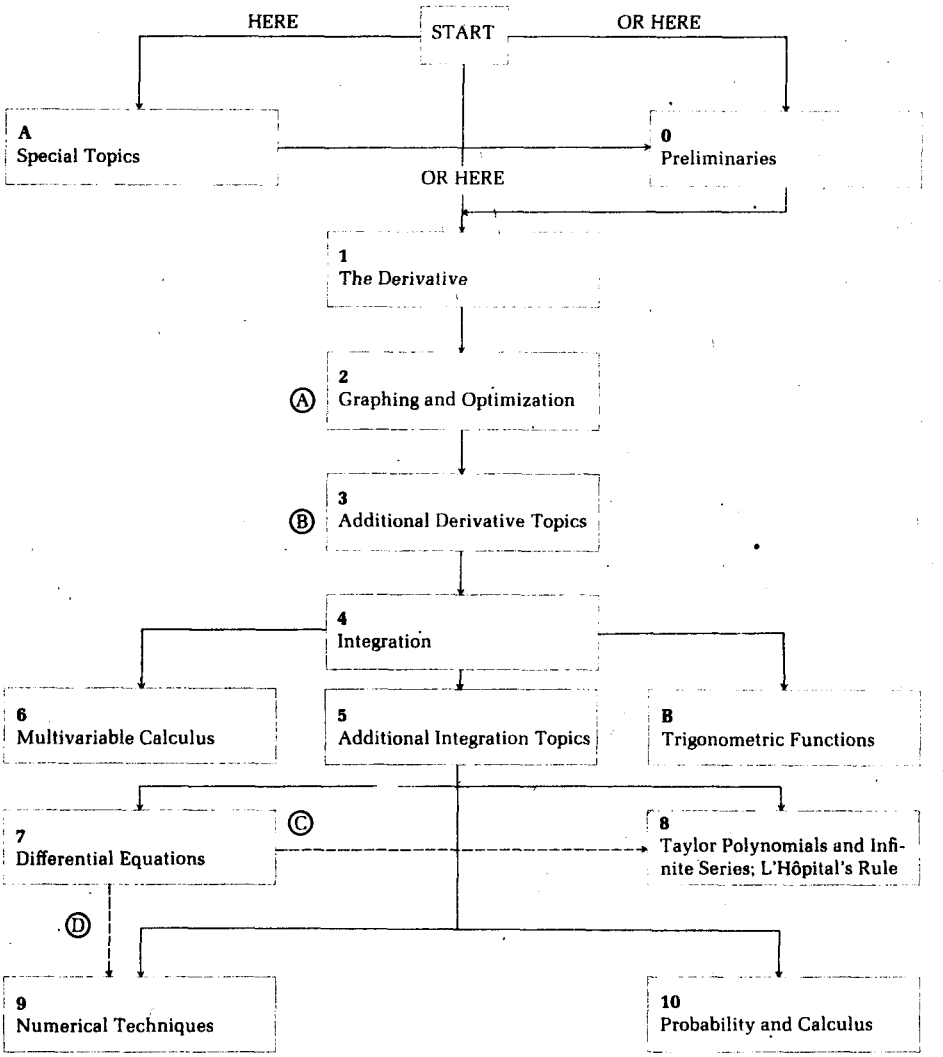
Phyllis Niklas and Janet Bollow for another outstanding text design and for guiding the book smoothly through all production details.

Don Dellen, the publisher, who continues to provide all the support services and encouragement an author could hope for.

Producing this new edition with the help of all these extremely competent people has been a most satisfying experience.

R. A. Barnett
M. R. Ziegler

Chapter Dependencies



Sections 3-6, 5-6, 6-4, and 6-7 are optional. They can be omitted without loss of continuity, and they are not covered in the chapter reviews, in the computer-generated tests, or in the printed test bank. Certain other sections are also easily omitted:

Ⓐ Section 2-4 can be omitted without loss of continuity.

Ⓑ Sections 3-4 and 3-5 can be omitted without loss of continuity.

Chapters 6–10 are relatively independent. The following comments pertain to topic selection from these chapters:

Ⓒ Section 8-4 contains a brief treatment of Taylor series solutions of differential equations. This is easily omitted if Chapter 7 has not been covered.

Ⓓ Section 9-4 deals with Euler's method for differential equations. This can be omitted if Chapter 7 has not been covered.

Contents

Preliminaries

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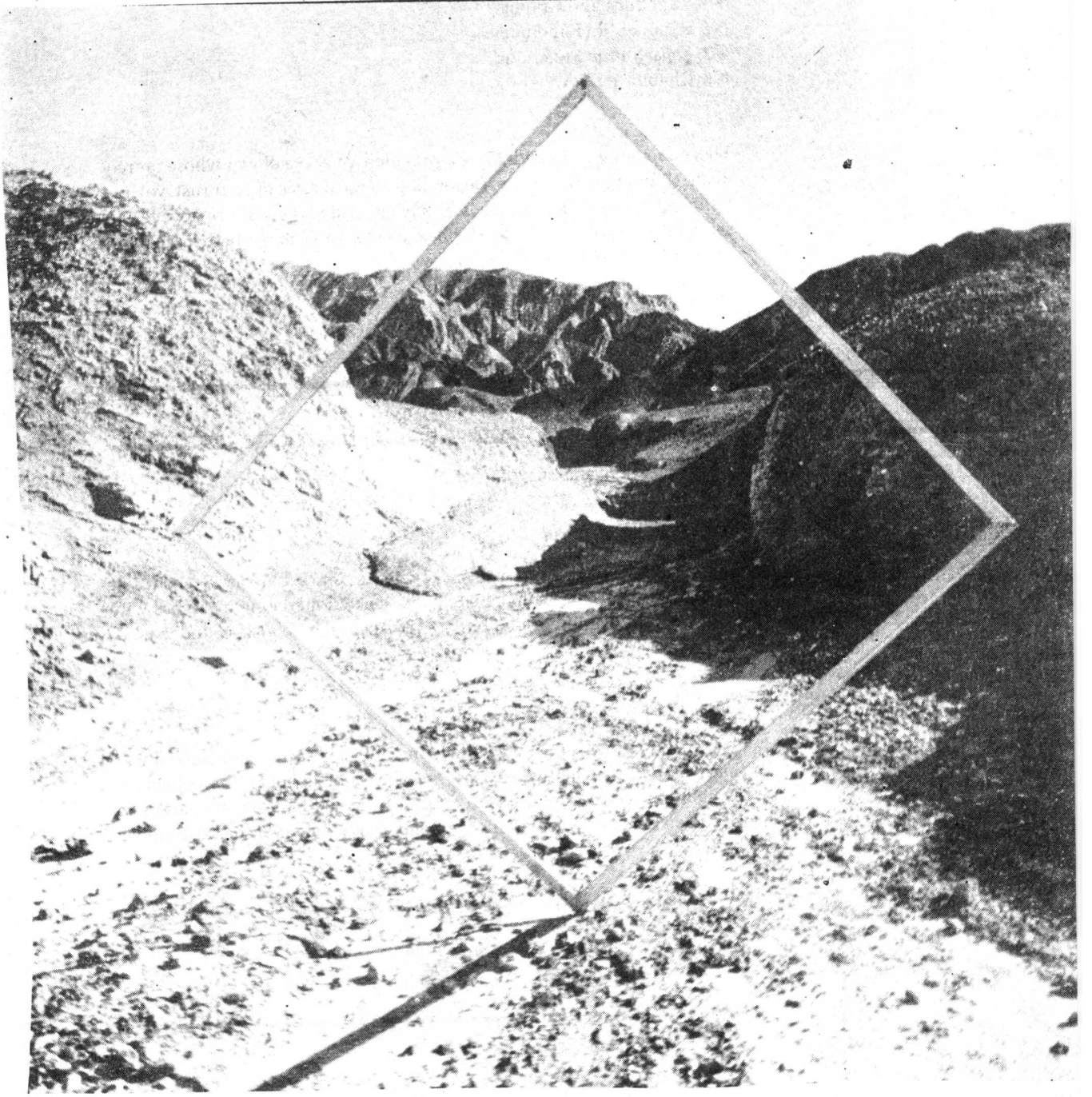

CHAPTER 0 Contents

This chapter and Appendix A are provided for those of you whose prerequisite skills are a little rusty. Depending on the degree of your rust, you can either refer to selected sections briefly as needed, you can review certain sections in depth at the start of the course, or you can study them in depth at appropriate points of the course.

0-1 Sets

- Set Properties and Set Notation
- Set Operations
- Application

In this section we will review a few key ideas from set theory. Set concepts and notation not only help us talk about certain mathematical ideas with greater clarity and precision, but are indispensable to a clear understanding of probability.

■ Set Properties and Set Notation

We can think of a **set** as any collection of objects specified in such a way that we can tell whether any given object is or is not in the collection. Capital letters, such as A, B, and C, are often used to designate particular sets. Each object in a set is called a **member** or **element** of the set. Symbolically,

$a \in A$	means	"a is an element of set A"
$a \notin A$	means	"a is not an element of set A"

A set without any elements is called the **empty** or **null set.** For example, the set of all people over 10 feet tall is an empty set. Symbolically,

$\varnothing$	represents	"the empty or null set"

A set is usually described either by listing all its elements between braces { } or by enclosing a rule within braces that determines the elements of the set. Thus, if $P(x)$ is a statement about x, then

$S = \{x \mid P(x)\}$	means	"S is the set of all x such that $P(x)$ is true"

Recall that the vertical bar in the symbolic form is read "such that." The following example illustrates the rule and listing methods of representing sets.

Example 1

Rule	*Listing*
{x|x is a weekend day}	= {Saturday, Sunday}
$\{x \mid x^2 = 4\}$	$= \{-2, 2\}$
{x|x is an odd positive counting number}	= {1, 3, 5, . . .}

The three dots . . . in the last set in Example 1 indicate that the pattern established by the first three entries continues indefinitely. The first two sets in Example 1 are **finite sets** (we intuitively know that the elements can be counted); the last set is an **infinite set** (we intuitively know that there is no end in counting the elements). When listing the elements in a set, we do not list an element more than once.

Problem 1 Let G be the set of all numbers such that $x^2 = 9$.*

(A) Denote G by the rule method.
(B) Denote G by the listing method.
(C) Indicate whether the following are true or false: $3 \in G$, $9 \notin G$.

If each element of a set A is also an element of set B, we say that A is a **subset** of B. For example, the set of all women students in a class is a subset of the whole class. Note that the definition allows a set to be a subset of itself. If set A and set B have exactly the same elements, then the two sets are said to be **equal.** Symbolically,

* Answers to matched problems are found near the end of each section just before the exercise set.