waves

berkeley physics course – volume 3

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Frank S. Crawford, Jr.

Professor of Physics
University of California, Berkeley

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WAVES

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Preface to the Berkeley Physics Course

This is a two-year elementary college physics course for students majoring in science and engineering. The intention of the writers has been to present elementary physics as far as possible in the way in which it is used by physicists working on the forefront of their field. We have sought to make a course which would vigorously emphasize the foundations of physics. Our specific objectives were to introduce coherently into an elementary curriculum the ideas of special relativity, of quantum physics, and of statistical physics.

This course is intended for any student who has had a physics course in high school. A mathematics course including the calculus should be taken at the same time as this course.

There are several new college physics courses under development in the United States at this time. The idea of making a new course has come to many physicists, affected by the needs both of the advancement of science and engineering and of the increasing emphasis on science in elementary schools and in high schools. Our own course was conceived in a conversation between Philip Morrison of Cornell University and C. Kittel late in 1961. We were encouraged by John Mays and his colleagues of the National Science Foundation and by Walter C. Michels, then the Chairman of the Commission on College Physics. An informal committee was formed to guide the course through the initial stages. The committee consisted originally of Luis Alvarez, William B. Fretter, Charles Kittel, Walter D. Knight, Philip Morrison, Edward M. Purcell, Malvin A. Ruderman, and Jerrold R. Zacharias. The committee met first in May 1962, in Berkeley; at that time it drew up a provisional outline of an entirely new physics course. Because of heavy obligations of several of the original members, the committee was partially reconstituted in January 1964, and now consists of the undersigned. Contributions of others are acknowledged in the prefaces to the individual volumes.

The provisional outline and its associated spirit were a powerful influence on the course material finally produced. The outline covered in detail the topics and attitudes which we believed should and could be taught to beginning college students of science and engineering. It was never our intention to develop a course limited to honors students or to students with advanced standing. We have sought to present the principles of physics from fresh and unified viewpoints, and parts of the course may therefore seem almost as new to the instructor as to the students.

The five volumes of the course as planned will include:

I. Mechanics (Kittel, Knight, Ruderman) III. Waves (Crawford)
II. Electricity and Magnetism (Purcell) IV. Quantum Physics (Wichmann)
V. Statistical Physics (Reif)

The authors of each volume have been free to choose that style and method of presentation which seemed to them appropriate to their subject.

The initial course activity led Alan M. Portis to devise a new elementary physics laboratory, now known as the Berkeley Physics Laboratory. Because the course emphasizes the principles of physics, some teachers may feel that it does not deal sufficiently with experimental physics. The laboratory is rich in important experiments, and is designed to balance the course.

The financial support of the course development was provided by the National Science Foundation, with considerable indirect support by the University of California. The funds were administered by Educational Services Incorporated, a nonprofit organization established to administer curriculum improvement programs. We are particularly indebted to Gilbert Oakley, James Aldrich, and William Jones, all of ESI, for their sympathetic and vigorous support. ESI established in Berkeley an office under the very competent direction of Mrs. Mary R. Maloney to assist the development of the course and the laboratory. The University of California has no official connection with our program, but it has aided us in important ways. For this help we thank in particular two successive chairmen of the Department of Physics, August C. Helmholz and Burton J. Moyer; the faculty and nonacademic staff of the Department; Donald Coney, and many others in the University. Abraham Olshen gave much help with the early organizational problems.

January, 1965

Eugene D. Commins
Frank S. Crawford, Jr.
Walter D. Knight
Philip Morrison
Alan M. Portis
Edward M. Purcell
Frederick Reif
Malvin A. Ruderman
Eyvind H. Wichmann
Charles Kittel, *Chairman*

A Further Note

Volumes I, II, and V were published in final form in the period from January 1965 to June 1967. During the preparation of Volumes III and IV for final publication some organizational changes occurred. Education Development Center succeeded Educational Services Incorporated as the administering organization. There were also some changes in the committee itself and some redistribution of responsibilities. The committee is particularly grateful to those of our colleagues who have tried this course in the classroom and who, on the basis of their experience, have offered criticism and suggestions for improvements.

As with the previously published volumes, your corrections and suggestions will always be welcome.

June, 1968
Berkeley, California

Frank S. Crawford, Jr.
Charles Kittel
Walter D. Knight
Alan M. Portis
Frederick Reif
Malvin A. Ruderman
Eyvind H. Wichmann
A. Carl Helmholz, Edward M. Purcell } *Chairmen*

Preface to Volume III

This volume is devoted to the study of waves. That is a broad subject. Everyone knows many natural phenomena that involve waves—there are water waves, sound waves, light waves, radio waves, seismic waves, de Broglie waves, as well as other waves. Furthermore, perusal of the shelves of any physics library reveals that the study of a single facet of wave phenomena—for example, *supersonic sound waves in water*—may occupy whole books or periodicals and may even absorb the complete attention of individual scientists. Amazingly, a professional "specialist" in one of these narrow fields of study can usually communicate fairly easily with other supposedly narrow specialists in other supposedly unrelated fields. He has first to learn their slang, their units (like what a parsec is), and what numbers are important. Indeed, when he experiences a change of interest, he may become a "narrow specialist" in a new field surprisingly quickly. This is possible because scientists share a common language due to the remarkable fact that many entirely different and apparently unrelated physical phenomena can be described in terms of a common set of concepts. Many of these shared concepts are implicit in the word *wave*.

The principal objective of this book is to develop an understanding of basic wave concepts and of their relations with one another. To that end the book is organized in terms of these concepts rather than in terms of such observable natural phenomena as sound, light, and so on.

A complementary goal is to acquire familiarity with many interesting and important examples of waves, and thus to arrive at a concrete realization of the wide applicability and generality of the concepts. After each new concept is introduced, therefore, it is illustrated by immediate application to many different physical systems: strings, slinkies, transmission lines, mailing tubes, light beams, and so forth. This may be contrasted with the different approach of first developing the useful concepts using one simple example (the stretched string) and then considering other interesting physical systems.

By choosing illustrative examples having geometric "similitude" with one another I hope to encourage the student to search for similarities and analogies between different wave phenomena. I also hope to stimulate him to develop the courage to *use* such analogies in "hazarding a guess" when confronted with new phenomena. The use of analogy has well-known dangers and pitfalls, but so does everything. (The guess that light waves might be "just like" mechanical waves, in a sort of jelly-like "ether" was very fruitful; it helped guide Maxwell in his attempts to guess his famous equations. It yielded interesting predictions. When experiments—especially those of Michelson and Morley—indicated that this mechanical model could not be entirely correct, Einstein showed how to discard the model yet keep Maxwell's equations. Einstein preferred to guess the equations directly—what might be called "pure" guesswork. Nowadays, although most physicists still use analogies and models to help them guess new equations, they usually publish only the equations.)

The home experiments form an important part of this volume. They can provide pleasure—and insight—of a kind not to be acquired through the ordinary lecture demonstrations and laboratory experiments, important as these are. The home experiments are all of the "kitchen physics" type, requiring little or no special equipment. (An optics kit is provided. Tuning forks, slinkies, and mailing tubes are not provided, but they are cheap and thus not "special.") These experiments really *are* meant to be done at home, not at the lab. Many would be better termed *demonstrations* rather than experiments.

Every major concept discussed in the text is demonstrated in at least one home experiment. Besides illustrating concepts, the home experiments give the student a chance to experience close personal "contact" with phenomena. Because of the "home" aspect of the experiments, the contact is intimate and leisurely. This is important. There is no lab partner who may pick up the ball and run with it while you are still reading the rules of the game (or sit on it when you want to pick it up); no instructor, explaining the meaning of *his* demonstration, when what you really need is to perform *your* demonstration, with your own hands, at your own speed, and as often as you wish.

A very valuable feature of the home experiment is that, upon discovering at 10 P.M. that one has misunderstood an experiment done last week, by 10:15 P.M. one can have set it up once again and repeated it. This is important. For one thing, in real experimental work no one ever "gets it right" the first time. Afterthoughts are a secret of success. (There are others.) Nothing is more frustrating or more inhibiting to learning than inability to pursue an experimental afterthought because "the equipment is torn down," or "it is after 5 P.M.," or some other stupid reason.

Finally, through the home experiments I hope to nurture what I may call "an appreciation of phenomena." I would like to beguile the student into creating with his own hands a scene that simultaneously surprises and delights his eyes, his ears, and his brain . . .

Clear-colored stones
are vibrating in the brook-bed . . .
or the water is.

—SOSEKI†

† Reprinted from *The Four Seasons* (tr. Peter Beilenson), copyright © 1958, by The Peter Pauper Press, Mount Vernon, N.Y., and used by permission of the publisher.

Acknowledgments

In its preliminary versions, Vol. III was used in several classes at Berkeley. Valuable criticisms and comments on the preliminary editions came from Berkeley students; from Berkeley professors L. Alvarez, S. Parker, A. Portis, and especially from C. Kittel; from J. C. Thompson and his students at the University of Texas; and from W. Walker and his students at the University of California at Santa Barbara. Extremely useful specific criticism was provided by S. Pasternack's attentive reading of the preliminary edition. Of particular help and influence were the detailed criticisms of W. Walker, who read the almost-final version.

Luis Alvarez also contributed his first published experiment, "A Simplified Method for Determination of the Wavelength of Light," *School Science and Mathematics* **32**, 89 (1932), which is the basis for Home Exp. 9.10.

I am especially grateful to Joseph Doyle, who read the entire final manuscript. His considered criticisms and suggestions led to many important changes. He also introduced me to the Japanese haiku that ends the preface. He and another graduate student, Robert Fisher, contributed many fine ideas for home experiments. My daughter Sarah (age $4\frac{1}{2}$) and son Matthew ($2\frac{1}{2}$) not only contributed their slinkies but also demonstrated that systems may have degrees of freedom nobody ever thought of. My wife Bevalyn contributed her kitchen and very much more.

Publication of early preliminary versions was supervised by Mrs. Mary R. Maloney. Mrs. Lila Lowell supervised the last preliminary edition and typed most of the final manuscript. The illustrations owe their final form to Felix Cooper.

I acknowledge gratefully the contributions others have made, but final responsibility for the manuscript rests with me. I shall welcome any further corrections, complaints, compliments, suggestions for revision, and ideas for new home experiments, which may be sent to me at the Physics Department, University of California, Berkeley, California, 94720. Any home experiment used in the next edition will show the contributor's name, even though it may first have been done by Lord Rayleigh or somebody.

F. S. Crawford, Jr.

Teaching Notes

Organization of the Course

Traveling waves have great aesthetic appeal, and it would be tempting to begin with them. In spite of their aesthetic and mathematical beauty, however, waves are physically rather complicated because they involve interactions between large numbers of particles. Since I want to emphasize physical systems rather than mathematics, I begin with the simplest physical *system*, rather than with the simplest *wave*.

Chapter 1 Free Oscillations of Simple Systems: We first review the free oscillations of a one-dimensional harmonic oscillator, emphasizing the physical aspects of inertia and return force, the physical meaning of ω^2, and the fact that for a real system the oscillation amplitude must not be too large if we are to get simple harmonic motion. Next, we consider free oscillations of two coupled oscillators and introduce the concept of normal mode. We emphasize that the mode is like a single "extended" harmonic oscillator, with all parts throbbing at the same frequency and all in phase, and that, for a given mode, ω^2 has the same physical meaning as it does for a one-dimensional oscillator.

What to omit: Throughout the book, several physical systems recur repeatedly. The teacher should not discuss all of them, nor the student study all of them. Examples 2 and 8 are longitudinal oscillations of mass and springs for one (Ex. 2) and two (Ex. 8) degrees of freedom. In later chapters this system is extended to many degrees of freedom, to continuous systems (rubber rope and slinky undergoing longitudinal oscillations) and is used as a model to assist comprehension of sound waves. A teacher who wishes to omit sound may also wish to omit all longitudinal oscillations from the beginning. Similarly, Examples 4 and 10 are *LC* circuits for one and two degrees of freedom. In later chapters they are extended to *LC* networks and then to continuous transmission lines. A teacher who wishes to omit the study of electromagnetic waves in transmission lines, therefore, can omit all *LC* circuit examples from the very beginning. (He can do this and still give a thorough discussion of electromagnetic waves, starting in Chap. 7 with Maxwell's equations.) Do *not* omit transverse oscillations (Examples 3 and 9).

Home experiments: We strongly advocate Home Exp. 1.24 (Sloshing mode in a pan of water) and the related Prob. 1.25 (Seiches), to get the student started "doing it himself." Home Exp. 1.8 (Coupled cans of soup) makes a good class demonstration. Of course, you may already have available such a demonstration (coupled pendulums). Nevertheless, I advocate slinky and soup cans for its crudity, even as a class demonstration, since it may encourage the student to get his own slinky and soup.

Chapter 2 Free Oscillations of Systems with Many Degrees of Freedom: We extend the number of degrees of freedom from two to a very large number and find the transverse modes—the standing waves—of a continuous string. We define k and introduce the concept of a dispersion relation, giving ω as a function of k. We use the modes of the string to introduce Fourier analysis of periodic functions in Sec. 2.3. The exact dispersion relation for beaded springs is given in Sec. 2.4.

What to omit: Sec. 2.3 is optional—especially if the students already know some Fourier analysis. Example 5 (Sec. 2.4) is a linear array of coupled pendulums, the simplest system having a low-frequency cutoff. They are used later to help explain the behavior of other systems that have a low-frequency cutoff. A teacher who does not intend to discuss at a later time systems driven below cutoff (waveguide, ionosphere, total reflection of light in glass, barrier penetration of de Broglie waves, high-pass filters, etc.) need not consider Example 5.

Chapter 3 Forced Oscillations: Chapters 1 and 2 started with free oscillations of a harmonic oscillator and ended with free standing waves of closed systems. In Chaps. 3 and 4 we consider forced oscillations, first of *closed* sytems (Chap. 3) where we find "resonances," and then in *open* systems (Chap. 4) where we find traveling waves. In Sec. 3.2 we review the damped driven one-dimensional oscillator, considering its transient behavior as well as its steady-state behavior. Then we go to two or more degrees of freedom, and discover that there is a resonance corresponding to every mode of free oscillation. We also consider closed systems driven below their lowest (or above their highest) mode frequency and discover exponential waves and "filtering" action.

What to omit: Transients (in Sec. 3.2) can be omitted. Some teachers may also wish to omit everything about systems driven beyond cutoff.

Home experiments: Home Exps. 3.8 (Forced oscillations in a system of two coupled cans of soup) and 3.16 (Mechanical bandpass filter) require phonograph turntables. They make excellent class demonstrations, especially of exponential waves for systems driven beyond cutoff.

Chapter 4 Traveling Waves: Here we introduce ***traveling*** waves resulting from forced oscillations of an *open* system (contrasted with the ***standing*** waves resulting from forced oscillations of a *closed* system that we found in Chap. 3). The remainder of Chap. 4 is devoted to studying phase velocity (including dispersion) and impedance in traveling waves. We contrast the two "traveling wave concepts," ***phase velocity*** and ***impedance,*** with the "standing wave concepts," ***inertia*** and ***return force,*** and also contrast the fundamental difference in phase relationships for standing versus traveling waves.

Home experiments: We recommend Home Exp. 4.12 (Water prism). This is the first optics kit experiment; it uses the purple filter (which passes red and blue but cuts out green). We strongly recommend Home Exp. 4.18 (Measuring the solar constant at the earth's surface) with your face as detector.

Chapter 5 Reflection: By the end of Chap. 4 we have at our disposal both standing and traveling waves (in one dimension). In Chap. 5 we consider general superpositions of standing and traveling waves. In deriving reflection coefficients we make a very "physical" use of the superposition principle, rather than emphasizing boundary conditions. (Use of boundary conditions is emphasized in the problems.)

What to omit: There are many examples, involving sound, transmission lines, and light. Don't do them all! Chapter 5 is essentially the "application" of what we have acquired in Chaps. 1–4. Any or all of it can be omitted.

Home experiments: Everyone should do Home Exp. 5.3 (Transitory standing waves on a slinky). Home Exps. 5.17 and 5.18 are especially interesting.

Chapter 6 Modulations, Pulses, and Wave Packets: In Chaps. 1–5 we work mainly with a single frequency ω (except for Sec. 2.3 on Fourier analysis). In Chap. 6 we consider superpositions, involving different frequencies, to form pulses and wave packets and to extend the concepts of Fourier analysis (developed in Chap. 2 for periodic functions) so as to include nonperiodic functions.

What to omit: Most of the physics is in the first three sections. A teacher who has omitted Fourier analysis in Sec. 2.3 will undoubtedly want to omit Secs. 6.4 and 6.5, where Fourier integrals are introduced and applied.

Home experiments: No one believes in group velocity until they have watched water wave packets (see Home Exp. 6.11). Everyone should also do Home Exps. 6.12 and 6.13.

Problems: Frequency and phase modulation are discussed in problems rather than in the text. So are such interesting recent developments as Mode-locking of a laser (Prob. 6.23), Frequency multiplexing (Prob. 6.32), and Multiplex Interferometric Fourier Spectroscopy (Prob. 6.33).

Chapter 7 Waves in Two and Three Dimensions: In Chaps. 1–6 the waves are all one-dimensional. In Chap. 7 we go to three dimensions. The propagation vector **k** is introduced. Electromagnetic waves are studied using Maxwell's equations as the starting point. (In earlier chapters there are many examples of electromagnetic waves in transmission lines, evolving from the *LC*-circuit example.) Water waves are also studied.

What to omit: Sec. 7.3 (Water Waves) can be omitted, but we recommend the home experiments on water waves whether or not Sec. 7.3 is studied. A teacher mainly interested in optics could actually start his course at Sec. 7.4 (Electromagnetic Waves) and continue on through Chaps. 7, 8, and 9.

Chapter 8 Polarization: This chapter is devoted to study of polarization of electromagnetic waves and of waves on slinkies, with emphasis on the physical relation between partial polarization and coherence.

Home experiments: Everyone should do at least Home Exps. 8.12, 8.14, 8.16, and 8.18 (Exp. 8.14 requiring slinky; the others, the optics kit).

Chapter 9 Interference and Diffraction: Here we consider superpositions of waves that have traveled different paths from source to detector. We emphasize the physical meaning of coherence. Geometrical optics is treated as a wave phenomenon—the behavior of a diffraction-limited beam impinging on various reflecting and refracting surfaces.

Home experiments: Everyone should do at least one each of the many home experiments on interference, diffraction, coherence, and geometrical optics. We also strongly recommend 9.50 (Quadrupole radiation from a tuning fork.)

Problems: Some topics are developed in the problems: Stellar interferometers, including the recently developed "long-base-line interferometry" (Prob. 9.57); the analogy between the phase-contrast microscope and the conversion of AM radio waves to FM is discussed in Prob. 9.59.

Home Experiments

General remarks: At least one home experiment should be assigned per week. For your convenience we list here all experiments involving water waves, waves in slinkies, and sound waves. We also later describe the optics kit.

Water waves: Discussed in Chap. 7; in addition they form a recurring theme developed in the following series of easy home experiments:

1.24 Sloshing modes in pan of water
1.25 Seiches
2.31 Sawtooth shallow-water standing waves
2.33 Surface tension modes
3.33 Sawtooth shallow-water standing waves
3.34 Rectangular two-dimensional standing surface waves on water
3.35 Standing waves in water
6.11 Water wave packets
6.12 Shallow-water wave packets—tidal waves
6.19 Phase and group velocities for deep-water waves
6.25 Resonance in tidal waves
7.11 Dispersion law for water waves
9.29 Diffraction of water waves.

Slinkies: Every student should have a Slinky (about $1 in any toy store). Four of the following experiments require a record-player turntable and are therefore outside the "kitchen physics" cost range. However, many students already have record players. (The experiments involving record players make good lecture demonstrations.)

1.8 Coupled cans of soup
2.1 Slinky—dependence of frequency on length
2.2 Slinky as a continuous system
2.4 "Tone quality" of a slinky
3.7 Resonance in a damped slinky (turntable required)
3.8 Forced oscillations in a system of two coupled cans of soup (turntable required)
3.16 Mechanical bandpass filter (turntable required)
3.23 Exponential penetration into reactive region (turntable required)
4.4 Phase velocity for waves on a slinky
5.3 Transitory standing waves on a slinky
8.14 Slinky polarization

Sound: Many home experiments on sound involve use of two identical tuning forks, preferably C523.3 or A440. The cheapest kind (about $1.25 each), which are perfectly adequate, are available in music stores. Mailing tubes can be purchased for about 25¢ each in stationery or art-supply stores. The following home experiments involve sound:

1.4 Measuring the frequency of vibrations
1.7 Coupled hacksaw blades
1.12 Beats from two tuning forks
1.13 Nonlinearities in your ear—combination tones
1.18 Beats between weakly coupled nonidentical guitar strings
2.4 "Tone quality" of a slinky
2.5 Piano as Fourier-analyzing machine—insensitivity of ear to phase
2.6 Piano harmonics—equal-temperament scale
3.27 Resonant frequency width of a mailing tube

4.6 Measuring the velocity of sound with wave packets
4.15 Whiskey-bottle resonator (Helmholtz resonator)
4.16 Sound velocity in air, helium, and natural gas
4.26 Sound impedance
5.15 Effective length of open-ended tube for standing waves
5.16 Resonance in cardboard tubes
5.17 Is your sound-detecting system (eardrum, nerves, brain) a phase-sensitive detector?
5.18 Measuring the relative phase at the two ends of an open tube
5.19 Overtones in tuning fork
5.31 Resonances in toy balloons
6.13 Musical trills and bandwidth
9.50 Radiation pattern of tuning fork—quadrupole radiation

Optics Kit

Components: Four linear polarizers, a circular polarizer, a quarter-wave plate, a half-wave plate, a diffraction grating, and four color filters (red, green, blue, and purple). The components are described in the text (linear polarizer on p. 411; circular polarizer, p. 433; quarter- and half-wave retardation plates, p. 435; diffraction grating, p. 496). Some experiments also require microscope slides, a showcase-lamp line source, or a flashlight-bulb point source as described in Home Exp. 4.12, p. 217. Aside from Exp. 4.12, all experiments requiring the optics kit are in Chaps. 8 and 9. They are too numerous to list here.

Home experiment

The first experiment involving the complete optics kit should be identification of all the components by the student. (Components are listed on the envelope container glued to the inside back cover.) Label the components in some way for future reference. For example, use scissors to round off slightly the four corners of the circular polarizer, and then scratch "IN" near one edge of the input face or stick a tiny piece of tape on that face. Clip *one* corner of the one-quarter-wave retarder; clip *two* corners of the two-quarter- (half-) wave retarder. Scratch a line along the axis of easy transmission on the linear polarizers. (This axis is parallel to one of the edges of the polarizer.)

We should remark that the "quarter-wave plate" gives a spatial retardation of 1400 ± 200 Å, nearly independent of wavelength (for visible light). Thus the wavelength for which it is a quarter-wave retarder is 5600 ± 800 Å. The ±200 Å is the manufacturer's tolerance. A manufactured batch that gives retardation 1400 Å is a quarter-wave retarder for green (5600 Å), but it retards by less than one quarter-wave for longer wavelengths (red) and more for shorter (blue). Another batch that happens to give retardation 1400 + 200 = 1600 Å is a quarter-wave retarder only for red (6400 Å). One that retards by 1400 − 200 is a quarter-wave plate only for blue (4800 Å). Similar remarks apply to the circular polarizer, since it consists of a sandwich of quarter-wave plate and linear polarizer at 45 deg, and the quarter-wave plate is a retarder of 1400 ± 200 Å. Thus there may be slightly distracting color effects when using white light. The student must be warned that in any experiment where he is supposed to get "black," i.e., extinction, he will always have some "non-extinguished" light of the "wrong" color leaking through. For example, I was naive when I wrote Home Exp. 8.12. You should perhaps strike out everything after the word "band" in the sentences "Do you see the dark band at green? That is the color of 5600 Å!"

Use of Complex Numbers

Complex numbers simplify algebra when sinusoidal oscillations or waves are to be superposed. They may also obscure the physics. For that reason I have avoided their use, especially in the first part of the book. All the trigonometric identities that are needed will be found inside the front cover. In Chap. 6 I do make use of the complex representation exp $i\omega t$, so as to use the well-known graphical or "phasor diagram" method of superposing vibrations. In Chap. 8 (Polarization) I use complex quantities extensively. In Chap. 9 (Interference and Diffraction) I do not make much use of complex quantities, even though it would sometimes simplify the algebra. Many teachers may wish to make much more extensive use of complex numbers than I do, especially in Chap. 9. In the sections on Fourier series (Sec. 2.3) and Fourier integrals (Secs. 6.4 and 6.5), I make no use of complex quantities. (I especially wanted to avoid Fourier integrals involving "negative frequencies"!)

A Note on the MKS System of Electrical Units†

Most textbooks in electrical engineering, and many elementary physics texts, use a system of electrical units called the *rationalized MKS system.* This system employs the MKS mechanical units based on the *meter*, the *kilogram*, and the *second*. The MKS unit of force is the *newton*, defined as the force which causes a 1-kilogram mass to accelerate at a rate of 1 meter/sec^2. Thus a newton is equivalent to exactly 10^5 dynes. The corresponding unit of energy, the newton-meter, or *joule*, is equivalent to 10^7 ergs.

The electrical units in the MKS system include our familiar "practical" units—coulomb, volt, ampere, and ohm—along with some new ones. Someone noticed that it was possible to assimilate the long-used practical units into a complete system devised as follows. Write Coulomb's law as we did in Eq. 1.1:

$$\mathbf{F}_2 = k\,\frac{q_1 q_2 \hat{\mathbf{r}}_{21}}{r_{21}{}^2} \tag{1}$$

Instead of setting k equal to 1, give it a value such that F_2 will be given in newtons if q_1 and q_2 are expressed in coulombs and r_{21} in meters. Knowing the relation between the newton and the dyne, between the coulomb and the esu, and between the meter and the centimeter, you can easily calculate that k must have the value 0.8988×10^{10}. (Two 1-coulomb charges a meter apart produce quite a force—around a million tons!) It makes no difference if we write $1/(4\pi\epsilon_0)$ instead of k, where the constant ϵ_0 is a number such that $1/(4\pi\epsilon_0) = k = 0.8988 \times 10^{10}$. Coulomb's law now reads:

$$F = \frac{1}{4\pi\epsilon_0}\,\frac{q_1 q_2}{r^2} \tag{2}$$

with the constant ϵ_0 specified as

$$\epsilon_0 = 8.854 \times 10^{-12}\ \text{coulomb}^2/\text{newton-m}^2 \tag{3}$$

† Reprinted from Berkeley Physics Course, Vol. II, *Electricity and Magnetism*, by Edward M. Purcell, © 1963, 1964, 1965 by Education Development Center, Inc., successor by merger to Educational Services Incorporated.

Separating out a factor $1/4\pi$ was an arbitrary move, which will have the effect of removing the 4π that would appear in many of the electrical formulas, at the price of introducing it into some others, as here in Coulomb's law. That is all that "rationalized" means. The constant ϵ_0 is called the dielectric constant (or "permittivity") of free space.

Electric potential is to be measured in volts, and electric field strength E in volts/meter. The force on a charge q, in field $\mathbf{E}$, is:

$$\mathbf{F}\text{ (newtons)} = q\mathbf{E}\text{ (coulombs} \times \text{volts/meter)} \tag{4}$$

An ampere is 1 coulomb/sec, of course. The force per meter of length on each of two parallel wires, r meters apart, carrying current I measured in amperes, is:

$$f\text{ (newtons/meter)} = \left(\frac{\mu_0}{4\pi}\right) \frac{2I^2}{r} \frac{\text{(amp}^2)}{\text{(meters)}} \tag{5}$$

Recalling our CGS formula for the same situation,

$$f\text{ (dynes/cm)} = \frac{2I^2}{rc^2} \frac{\text{(esu/sec)}^2}{\text{(cm}^3/\text{sec}^2)} \tag{6}$$

we compute that $(\mu_0/4\pi)$ must have the value 10^{-7}. Thus the constant μ_0, called the permeability of free space, must be:

$$\mu_0 = 4\pi \times 10^{-7}\text{ newtons/amp}^2 \quad \text{(exactly)} \tag{7}$$

The magnetic field $\mathbf{B}$ is defined by writing the Lorentz force law as follows:

$$\mathbf{F}\text{ (newtons)} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} \tag{8}$$

where $\mathbf{v}$ is the velocity of a particle in meters/sec, q its charge in coulombs. This requires a new unit for $\mathbf{B}$. The unit is called a *tesla*, or a *weber/m²*. One tesla is equivalent to precisely 10^4 gauss. In this system the auxiliary field $\mathbf{H}$ is expressed in different units, and is related to $\mathbf{B}$, in free space, in this way:

$$\mathbf{B} = \mu_0\mathbf{H} \quad \text{(in free space)} \tag{9}$$

The relation of $\mathbf{H}$ to the free current is

$$\int \mathbf{H} \cdot d\mathbf{s} = I_{\text{free}} \tag{10}$$

I_{free} being the free current, in amperes, enclosed by the loop around which the line integral on the left is taken. Since ds is to be measured in meters, the unit for H is called simply, *ampere/meter*.

Maxwell's equations for the fields in free space look like this, in the rationalized MKS system:

$$\begin{aligned} \text{div}\,\mathbf{E} &= \rho & \text{curl}\,\mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \text{div}\,\mathbf{B} &= 0 & \text{curl}\,\mathbf{B} &= \mu_0\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0\mathbf{J} \end{aligned} \tag{11}$$

If you will compare this with our Gaussian CGS version, in which c appears out in the open, you will see that Eqs. 11 imply a wave velocity $1/\sqrt{\epsilon_0\mu_0}$ (in meters/sec, of course). That is:

$$\epsilon_0\mu_0 = \frac{1}{c^2} \tag{12}$$

In our Gaussian CGS system the unit of charge, esu, was established by Coulomb's law, with $k \equiv 1$. In this MKS system the coulomb is defined, basically, not by Eq. 1 but by Eq. 5, that is, by the force between currents rather than the force between charges. For in Eq. 5 we have $\mu_0 \equiv 4\pi \times 10^{-7}$. In other words, if a new experimental measurement of the speed of light were to change the accepted value of c, we should have to revise the value of the constant ϵ_0, not that of μ_0.

A partial list of the MKS units is given below, with their equivalents in Gaussian CGS units.

Quantity	*Symbol*	*Unit, in rationalized MKS system*	*Equivalent, in Gaussian CGS units*
Distance	s	meter	10^2 cm
Force	$\mathbf{F}$	newton	10^5 dynes
Work energy	W	joule	10^7 ergs
Charge	q	coulomb	2.998×10^9 esu
Current	I	ampere	2.998×10^9 esu/sec
Electric potential	φ	volt	(1/299.8) statvolts
Electric field	$\mathbf{E}$	volts/meter	(1/29980) statvolts/cm
Resistance	R	ohm	1.139×10^{-12} sec/cm
Magnetic field	$\mathbf{B}$	tesla	10^4 gauss
Magnetic flux	Φ	weber	10^8 gauss-cm^2
Auxiliary field H	$\mathbf{H}$	amperes/meter	$4\pi \times 10^{-3}$ oersted

This MKS system is convenient in engineering. For a treatment of the fundamental physics of fields and matter, it has one basic defect. Maxwell's equations for the vacuum fields, in this system, are symmetrical in the electric and magnetic field only if **H**, not **B**, appears in the role of the magnetic field. (Notice that Eqs. 11 above are not symmetrical, even in the absence of **J**.) On the other hand, as we showed in Chapter 10, **B**, not **H**, is the fundamental magnetic field inside matter. That is not a matter of definition or of units, but a fact of nature, reflecting the absence of magnetic charge. Thus the MKS system, as it has been constructed, tends to obscure either the fundamental electromagnetic symmetry of the vacuum, or the essential asymmetry of the sources. That was one of our reasons for preferring the Gaussian CGS system in this book. The other reason is that Gaussian CGS units, augmented by practical units on occasion, are still the units used by most working physicists.

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