

# FOR ALL PRACTICAL PURPOSES

*introduction to*  
**CONTEMPORARY  
MATHEMATICS**

**FOURTH EDITION**





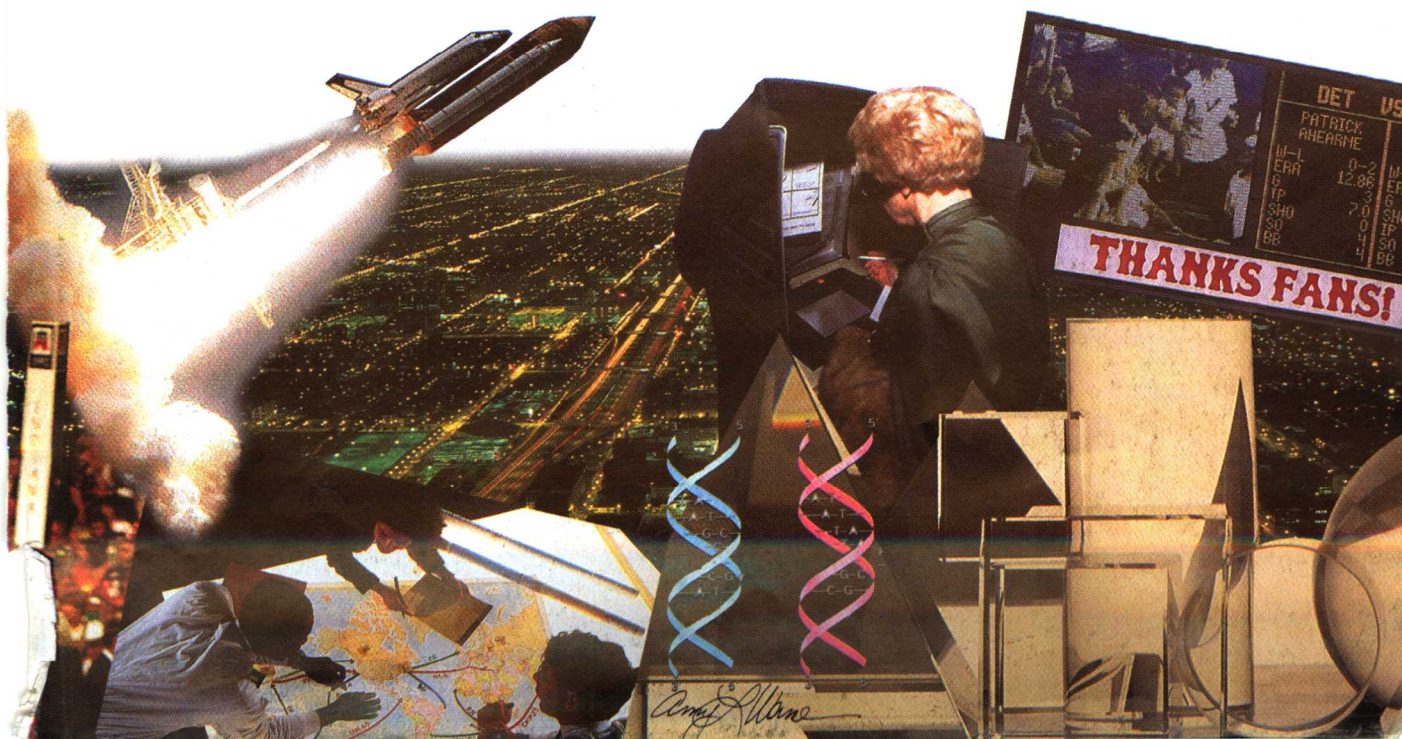
# FOR ALL PRACTICAL PURPOSES

INTRODUCTION TO CONTEMPORARY MATHEMATICS

*fourth edition*



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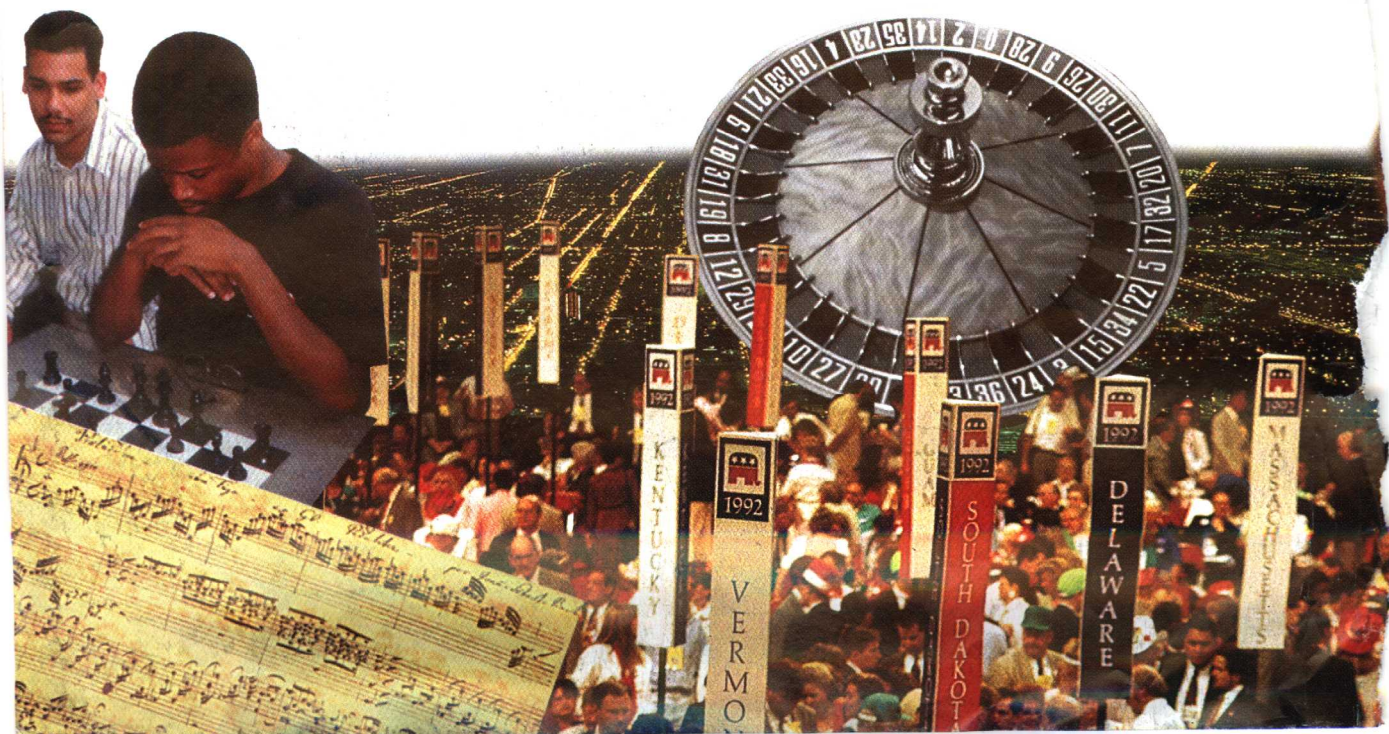
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# P R E F A C E

## A PRACTICAL APPROACH

It is difficult to believe that this is the fourth edition of *For All Practical Purposes*. *For All Practical Purposes* still seems like a new idea—an upstart textbook trying to change what we teach and how we teach it. But *FAPP* is an established textbook and is the leading mathematics for liberal arts text, having been used at over 500 colleges.

The Consortium for Mathematics and Its Applications continues to see its role as that of bringing a wide range of new contemporary applications of mathematics into the classroom. A new edition of *FAPP* is an opportunity to demonstrate to students and faculty alike the dynamic and ever-changing face of our subject and the new problems we can model and solve.

A new edition is also our opportunity to evolve with students' and teachers' needs. To that end, our mission for this revision was to make *FAPP* a textbook that is easier to use—both for instructors and students. Each sentence and paragraph has been scrutinized for clarity and overall consistency of style. Each new term and concept is carefully defined the first time it appears in the text. New format features, such as additional subheads, help readers follow the progress of the material. We have made these and other pedagogic improvements without sacrificing the integrity of the *FAPP* philosophy. A basic statement of this philosophy appeared in the preface to the first edition:

Every mathematician at some time has been called upon to answer the innocent question, “Just what is mathematics used for?” With understandable frequency, usually at social gatherings, the question is raised in similar ways: “What do mathematicians do, practice, or believe in?” At a time when success in our society depends heavily on satisfying the need for developing quantitative skills and reasoning ability, the mystique surrounding mathematics persists. *For All Practical Purposes: Introduction to Contemporary Mathematics* is our response to these questions and our attempt to fill this need.

*For All Practical Purposes* represents our effort to bring the excitement of contemporary mathematical thinking to the nonspecialist, as well as help him or her develop the capacity to engage in logical thinking and to read critically the technical information with which our contemporary society abounds. For the study of mathematics we attempt to implement Thomas Jefferson's notion of an “enlightened citizenry,” in which individuals having acquired a broad

knowledge of topics exercise sound judgment in making personal and political decisions. Environmental and economic issues dominate modern life, and behind these issues are complex matters of science, technology, and mathematics that call for an awareness of fundamental principles.

To achieve these goals, *For All Practical Purposes* stresses the connections between contemporary mathematics and modern society. Since the technological explosion that followed World War II, a cluster of mathematical sciences has emerged. This encompasses statistics, computer science, operations research, and decision science. At the same time, the more traditional areas of mathematics have enjoyed spectacular growth both in theoretical power and in their applications. In science and industry, mathematical models are the tools par excellence for solving complex problems. In this book our goal is to convey the power of mathematics, as illustrated by the great variety of problems that can be modeled and solved by analytic and quantitative means.

## NEW CONTENT FEATURES OF THE FOURTH EDITION

Every chapter in this new edition has been carefully revised, with attention to greater clarity of exposition and to contemporary issues and examples. Major changes and additions to this edition have come in Part IV, Social Choice and Decision Making. We gratefully acknowledge the efforts of Steven Brams, Bruce Conrad, and Alan Taylor for the inclusion of exciting new material that has never appeared before in any undergraduate text. For example, this section offers a new procedure for divorce settlements called the adjusted winner procedure, a whole chapter on the theory of moves in game theory, and the first publication of the Banzhaf power index for the electoral college for the 1990s.

Content changes make a point. Mathematics is dynamic; new mathematics is being invented and applied in new ways everyday. Our goal and our text must be flexible enough to vividly demonstrate the contemporary nature of our subject and its applications to our daily lives. Some of the more important content changes are as follows:

- **Part I, Management Science**, features a greatly simplified discussion of linear programming (Chapter 4). We responded to the advice of students and instructors and now begin the chapter with a manufacturing problem involving only one resource. Minimum quantities are introduced for the first time in this edition, making the problems more realistic while retaining algebraic simplicity. All the algebraic steps are now shown and described. After the student has confidence in solving one-resource problems, two-resource problems are introduced.

► **Part II, Statistics: The Science of Data**, is better integrated. In Chapter 5, *Producing Data*, a new section on statistical estimation carefully introduces the concepts of sampling variability and margin of error, which are fortified with examples of Gallup poll surveys on crime. This section prepares the reader for the more formal treatment of probability in Chapter 7, *Probability: The Mathematics of Chance*, and inference in Chapter 8. Chapter 6, *Describing Data*, more fully outlines the strategy of data analysis. The discussion of variance and standard deviation is clearer and supported by examples comparing metabolic rates for men and women.

► **Part III, Coding Information**, is also more focused and accessible. For instance, we now discuss only the important bar codes: ZIP codes and UPC codes.

► **Part IV, Social Choice and Decision Making**, has evolved considerably to reflect changes in contemporary society *and* to make the material more accessible to students. Chapter 11, *Social Choice: The Impossible Dream*, is reorganized so that the chapter begins with elections involving only two alternatives and then moves to elections involving three or more alternatives. This latter treatment is more focused than in previous editions, with specific voting systems, such as the Hare system currently used in Ireland and Australia, tied to specific defects, such as a failure of the Condorcet winner criterion. The chapter's concluding section, *Insurmountable Difficulties: From Paradox to Impossibility*, now includes a comprehensive discussion of Arrow's impossibility theorem.

Chapter 12, *Weighted Voting Systems*, breaks ground with the inclusion of the Banzhaf power index for the electoral college.

Chapter 13, *Fair Division*, opens with a new section on a recently developed scheme for handling divorces (called the adjusted winner procedure), which uses the Donald and Ivana Trump divorce to illustrate the procedure. This is followed by a discussion of the Knaster inheritance procedure for dividing an inheritance if there are more than two heirs. An engaging new section, *Divide-and-Choose*, sets the stage for the remainder of the chapter, which is devoted to cake cutting.

A refocused Chapter 14, *Apportionment*, more clearly compares and contrasts the U.S. House of Representatives apportionment methods proposed by Hamilton, Jefferson, and Webster by showing specifically the apportionment that each method gives and the paradoxes they create. The chapter ends with a new section on the method currently used to apportion the House (called the Hill–Huntington method) and a summary section discussing which apportionment method is best. In response to questions about the relevance of the Alabama paradox, which occurs only if the House size is allowed to change, a discussion of the population paradox has been included.

The ideas presented in Chapter 15, *Game Theory: The Mathematics of Competition*, are brought up to date in a new Chapter 16, called *Theory of Moves*. *Theory of moves* shows how competitors from Samson and Delilah to Leno and Letterman can resolve conflicts using a farsighted strategy and, perhaps, deception.

► **For Part V, *On Size and Shape***, we have honed our coverage to include only those topics that are most popular and appropriate to the book. For instance, to illustrate hyperbolic geometry (Chapter 19), Beltrami's pseudosphere has been replaced with an example on the geometry of political alliances. Chapter 18, *Geometric Growth*, features many new financial growth models, including certificates of deposit, money market accounts, savings plans, sinking funds, inflation, and depreciation. Part V concludes with an invitation to readers to attempt their own Escher-like tilings (Chapter 21).

## IMPROVED PEDAGOGY

All chapters have been carefully reviewed and revised to make learning the material easier for students. Some new pedagogic features are as follows:

► Key concepts and algorithms are now highlighted in color to reinforce their importance. This new feature will reassure students about the chapters' "take home messages."

► Chapter-ending exercises are now categorized by subject matter, which reflects the subject-matter divisions in the chapter. This new feature will help instructors with their assignments and help students succeed at solving problems.

► More than 150 new exercises and writing projects have been included in this edition. For instance, new statistics exercises present attractive data on such issues as whether drinking wine helps prevent heart attacks (6-48) and whether recent years have improved the plight of the endangered Florida manatee (6-21 and 6-25). New Exercise 14-3 highlights one of the weirdest paradoxes of apportionment: the population paradox. Also particularly interesting are those exercises that ask students to use free software programs to create models for hyperbolic geometry (see Chapter 19).

## SUPPLEMENTS

The supplements package for *FAPP* has been extensively revamped for the fourth edition. Existing supplements have been updated and improved and new supplements have been added for both instructors and students. The



components of this package are described below. For more information about any of the supplements, please contact your local W. H. Freeman sales representative.

► An exciting new CD-ROM/study guide learning package, called *FAPP Interactive*, for students combines printed and electronic materials that provide innovative approaches to the topics treated in the textbook. Each chapter of *FAPP Interactive* is divided into three parts. One part focuses on summaries of the topics within each chapter. These summaries are paired with questions and answers designed to determine whether or not the student sufficiently understands the material being presented. The second part uses an impressive array of tools—easy-to-use software, video clips, animations, and original examples—that are applied to more questions. Exploring these tools and questions will give students a deeper understanding of the topics in the textbook. Each chapter also contains an electronic multiple-choice quiz that is designed to help students learn the material and at the same time become more comfortable taking multiple-choice tests. Each option of every question provides instant feedback for the student: choosing an incorrect option reveals a hint; choosing a correct option reveals positive reinforcement. Students at all levels will benefit a great deal from this exciting and functional student supplement.

► The *Instructor's Guide* has long been a useful tool for instructors switching from a more traditional teaching approach to the *FAPP* approach. It has been thoroughly revised to reflect the changes in the new edition of the text. Popular features retained from earlier editions include extensive classroom-tested teaching hints, solutions to even-numbered text exercises, detailed summaries, and suggested skill objectives. New to this edition will be strategies for using the collaborative learning approach in teaching most chapters.

► For the first time, a full *Test Bank* has been developed to accompany *FAPP*. Each chapter will contain approximately 50 multiple-choice and 25 short-answer questions, for a total of approximately 1500 questions. The *Test Bank* will be available in printed, Macintosh, and IBM versions.

► A set of *Transparency Masters* that contain approximately 110 key figures from the textbook is also being offered.

There is also an Annenberg video series of 26 half-hour programs called *For All Practical Purposes Telecourse*. For more information on telecourse preview, purchase, or rental, please call 1-800-LEARNER, or write The Annenberg CPB Project, P.O. Box 2345, South Burlington, VT 05407-2345.

We thank the individuals who have contributed to the supplements package, including the authors of supplements to previous editions. Following is a list of

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Solomon Garfunkel, COMAP  
September 1996



# TO THE STUDENT

Have fun. Enjoy. That is what this book is all about. You may at one time or another have wondered why people go into mathematics. Do they have different genes, that they like this stuff? This text is one attempt to show some of the payoffs for working with mathematics. The areas we discuss and the problems we tackle are real. Most of the people you will meet in these pages are alive and much of their work has been done in the last twenty or thirty years. We have tried to show you mathematics as we see it—solving problems we care about and have fun doing.

We don't expect you to become a mathematician. We understand that you won't remember much about the techniques presented here in a year or two. That's not the point. What we hope is that you will do some mathematics, solve some problems, and gain an appreciation for what we do and why. To a large extent mathematics is not about formulas and equations; it is about CDs and CAT scans, and parking meters, and presidential polls, and computer graphics. Mathematics is about looking at our world and creating representations we can work with to solve problems that matter—and we hope it is in this spirit that you use this book. Welcome to the world of contemporary mathematics.

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