Computer Aided Design in Mechanical Engineering

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Respectfully Dedicated
to the Memory
of My Beloved Parents

BHAGIRATHI AND N VISWANATHAN

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PREFACE

The primary aim of writing this book is to indicate the enormous amount of numerical work involved in solving large-size problems of interest to practising engineers. It is imperative to get to know the most efficient algorithm to solve these problems. Throughout the course of the book, emphasis has been on the core and time needed to solve any given problem by different methods. It is presumed that the reader has the basic knowledge of strength of materials, theory of machines and computer programming.

The study has been restricted to the behaviour in the linear range in order to reduce the size of the book. Transient and steady-state vibration problems as well as static problems have been considered. Use of finite difference and finite element methods of formulation have been indicated. A number of computer programmes (classroom-tested) have been given. Example problems have been worked out and an adequate number of additional exercises have been included. The last chapter is on "case studies" using the subject matter covered in the five chapters. The study of cyclic symmetric objects has been given importance in Chapter 4.

The material reported in this book will be useful to practising engineers in industries having digital computers and to college seniors and research students in the field of machine design.

At the outset I would like to wholeheartedly thank the authorities of the Indian Institute of Technology, Madras for the congenial atmosphere provided for writing this book. I gratefully acknowledge the enormous amount of work put in by my students Mr. M. Ananda Rao, Mr. P. Balasubramanyam, Mr. G. Natarajan, Mr. V. Om Prakash, Mr. K. Ramesh, Mr. P. Srinivasan and Mr. P. Seshu in writing the computer programmes and running sample problems. A major portion of Chapter 4 is the Ph D work of Mr. P. Balasubramanyam. The enthusiasm shown by him in preparing this chapter is appreciated. The support given by the Computer Centre at IIT Madras in preparing the computer listings is acknowledged. The information shown in Tables 1.1 to 1.5 has been collec-

ted from the *Handbook on Finite Element Systems* edited by C.A. Brebbia. I am grateful to him.

Appreciation is also due to my wife and children without whose cooperation and help it would have been impossible for me to complete this assignment.

V RAMAMURTI

CONTENTS

1. INTRODUCTION

The large size problems handled by modern digital computers connected with static and dynamic analysis of complicated machines or structures are generally of the form

$$[M]\ddot{u} + [C]\dot{u} + [K]u = \{F(t)\} \qquad (1.1)$$

where $[M]$ is the global mass matrix, $[C]$ the global damping matrix and $[K]$ the global stiffness matrix. $\{F(t)\}$ is a given forcing function vector in time, $\{u\}$ is the resultant displacement vector, $\{\dot{u}\}$ and $\{\ddot{u}\}$ represent its velocity and acceleration respectively. Generally, $[M]$, $[C]$ and $[K]$ are banded. Depending upon the nature of these coefficients, the problems are classified as static, dynamic, linear or non-linear. The following are some of the specific classifications:

(i) When $[C] = 0$, $[M] = 0$ $[K]$ and $\{F(t)\}$ are constants, the result is a static linear problem.

(ii) When $[M]$ and $[C]$ are absent, and $[K]$ is a function of $\{u\}$ and $\{F(t)\}$ a constant the result is a non-linear static problem.

(iii) If $\{F(t)\}$ and $[C]$ are absent, and $[M]$ and $[K]$ are constants, one gets an eigenvalue problem.

(iv) If $[M]$, $[C]$ and $[K]$ are constants and $\{F(t)\}$ is a periodic forcing function, the result is a multi-degree of freedom steady state vibration problem.

(v) If $[M]$, $[C]$ and $[K]$ are constants and $\{F(t)\}$ is a transient function of time, the result is a transient vibration problem.

A considerable amount of effort has gone into the solution of such problems. The use of the finite element or finite difference method for analysing varieties of problems leads us ultimately to Eq. (1.1).

1.1 Finite Element Method

In this approach the unit under consideration has been treated as if it is made up of basic elements like trusses, beams, plates, shells, pipes, etc. and stiffness and mass matrices are computed depending on the nature of behaviour. Excellent treatment of the subject can be found in Ref. [1.1, 1.4]. Besides programmes incorporating these features are available in standard packages [1.5]. Some of the familiar ones are ADINA, ANSYS, APPLESAP, ASKA, MARC, MSC NASTRAN and SAP 7. They are available at the following addresses:

ADINA	: Mr G. Larsen ADINA Engineering A B Stanj.rnsgatan 227 S 72473 Vasteras, Sweden
ANSYS	: Mr K P. Kohnke Swanson Analysis System Inc. Houston PA 15342, USA
APPLESAP	: ITALIM PIANTI SPA CAD Systems Department P22 a Piccapietra 9 16121 Genova, Italy
ASKA	: IKO Software Service GmbH Albstadtweg 10 D7000 Stuttgart, 80, West Germany
MARC	: Marc Analysis Research Corporation Verrijn Stuartlaan 29 2288 Ekrijswijk, Netherlands
MSC NASTRAN	: The Mac Neal Schwendler Corporation 7442 North Fignersa Street Los Angles, CA 90041, USA
SAP7	: Structural Mechanics Computer Lab DRC 394 University of South California LA, CA 90007, USA

Some of the special features of these programmes are as follows:

ADINA	: Non-linear analysis of reinforced and prestressed concrete structures, thermoelastic, plastic and creep analysis of heat treatment processes, non-linear dynamic analysis of large fluid structure systems and analysis of problems in fracture mechanics.
ANSYS	: Capabilities include bilinear elements, heat transfer analysis, fluid flow, electric flow, graphic package and extensive pre- and post-processing.
APPLESAP	: Developed from two original programmes SAP 4 and DOT, its special features include roof life load, foundation settlement and wind actions.
ASKA	: Applications in nuclear engineering, metal forming processes, car body design and rocket structures.
MARC	: Viscoelastic behaviour, creep behaviour, crack behaviour, fluid structure interaction, and large displacement analysis.
MSC NASTRAN	: Heat transfer, aeroelasticity, acoustics, electro magnetism, random response, cyclic symmetry and graphics.
SAP 7	: Extension of SAP 4 and NONSAP, geometric and material non-linearity, included linear viscoelasticity, random vibration, layered sandwich material.

The existing information on these six packages has been compiled in Tables 1.1 to 1.5.

TABLE 1.1

Element Type

Programme	Truss Beam	2D Solid	3D Solid	Axi Solid	Plate Bending	Shell	Crack Element	Fluid Mech.
ADINA	√	√	√	√	√	√	√	√
ANSYS	√	√	√	√	√	√	√	√
APPLE SAP	√	√	√	√	√	√		
ASKA	√	√	√	√	√	√	√	√
MARC	√	√	√	√	√	√		√
MSC NASTRAN	√	√	√	√		√	√	√
SAP 7	√	√	√	√	√	√		√

TABLE 1.2

Material Properties

Programme	Linear Elastic Isotropic Anisotropic	Non-linear Elastic	Visco-elastic	Plastic	Large Strain	Soil Mechanics Materials
ADINA	√	√	√	√	√	√
ANSYS	√	√	√	√		
APPLE SAP	√					
ASKA	√	√	√	√		√
MARC	√	√	√	√	√	√
MSC NASTRAN	√	√	√	√		√
SAP 7	√	√		√		√

TABLE 1.3

Analysis Capabilities

Programme	Static Analysis	Transient	Harmonic response	Buckling	Post Buckling	Fracture Mech
ADINA	√	√	√	√	√	√
ANSYS	√	√	√	√	√	√
APPLE SAP	√	√	√			
ASKA	√	√	√	√	√	√
MARC	√	√	√	√	√	√
MSC NASTRAN	√	√	√	√	√	
SAP 7	√	√	√	√	√	

TABLE 1.4
Other Capabilities

Programme	Automatic Mash Generation	Automatic Node Numbering	Plot Routine	Inter-active graphics	Fre Format input
ADINA			√	√	
ANSYS	√		√	√	√
APPLE SAP	√		√		√
ASKA	√	√	√	√	√
MARC	√		√	√	√
MSC NASTRAN	√	√	√	√	√
SAP 7	√	√	√	√	

TABLE 1.5
Operating Systems

Programme	CDC	IBM	UNIVAC	DEC	ICL	CRAY
ADINA	√	√	√	√		
ANSYS	√	√	√	√	√	√
APPLE SAP						
ASKA	√	√	√	√	√	
MARC	√					
MSC NASTRAN	√	√	√	√		√
SAP 7	√	√	√	√		

1.1.1 Finite Element Procedure

The structure is idealised by just subdividing the original object into an assembly of discrete elements such that the resulting structure will simulate the original one. The elements are connected to each other at points known as nodes. After making a reasonable assumption on the behaviour of an element, the kinetic energy T and strain energy U are calculated as a function of nodal point displacements.

If the structure is composed of N elements, we can write

$$T = \sum_{i=1}^{N} T_i$$

$$U = \sum_{i=1}^{N} U_i \tag{1.2}$$

The application of Lagrange's equation then results in the governing equation of type (1.1) when damping is ignored.

1.1.2 Discretisation

Discretising the structure requires experience and complete understanding of the behaviour of the structure. The structure can behave like a beam, truss, plate, shell, etc. Having chosen one of the models, one can compute the following:

$$\{f\} = [N]\{\delta\} \tag{1.3}$$

$\{f\}$ is the displacement vector at an arbitrary point inside the element, $\{\delta\}$ the nodal displacement vector and $[N]$ the shape function. Likewise, components of strain in an arbitrary point can also be written as

$$\{\epsilon\} = [B]\{\delta\} \tag{1.4}$$

where $[B]$ is the strain displacement vector.
The stress component $\{\sigma\}$ can be expressed as

$$\{\sigma\} = [D]\{\epsilon\} \tag{1.5}$$

where $[D]$ is the elasticity matrix.
The strain energy U for the element can be written as

$$U = \frac{1}{2}\int_{\text{vol}} \epsilon^T\sigma d \text{ vol} \tag{1.6}$$

The velocity vector of an arbitrary point can be written as

$$\{f'\} = [N]\{\delta\}' \tag{1.7}$$

where $\{\delta\}'$ is the time derivative of $\{\delta\}$.

Hence the kinetic energy T can be written as

$$T = \frac{1}{2}\int_{\text{vol}} \rho[N]^T\delta'^T[N]\delta' d \text{ vol} \tag{1.8}$$

where ρ is the density of the material.
Equation (1.6) can be recast as

$$U = \frac{1}{2}\{\delta\}^T[K_e]\{\delta\} \tag{(1.9)}$$

$$[K_e] = \int_{\text{vol}} [B]^T [D][B] \, d(\text{vol}) \tag{1.10}$$

Here $[K_e]$ is known as the element stiffness matrix.

Similarly Eq. (1.8) can be recast as

$$T = \frac{1}{2}\{\delta'\}^T[M_e]\{\delta'\} \tag{1.11}$$

where

$$[M_e] = \int_{\text{vol}} \rho[N]^T[N] \, d(\text{vol}) \tag{1.12}$$

$[M_e]$ is known as the mass matrix.

As a typical example let us consider a beam as shown in Fig. 1.1. The typical beam element is of length l, of uniform cross-section A, moment of inertia I and Young's modulus E. Using the right hand coordinate system and assuming two displacements (along the length and at right angles to the length), and rotation in the plane of the displacements at either end of the beam, this beam has six degrees of freedom (three at each node).

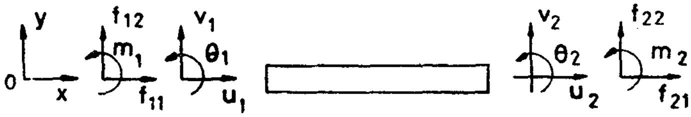

FIG. 1.1
Beam Element

The nodal displacement vector for this beam is

$$\{\delta\} = \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} \tag{1.13}$$

One can write the displacement field inside the element as

$$\{f\} = \begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} a_1 & 0 & 0 & a_2 & 0 & 0 \\ 0 & a_3 & a_4 & 0 & a_5 & a_6 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} \tag{1.14}$$

The constants a_1 to a_6 can be evaluated from the boundary conditions at $x = 0$ and at $x = l$.
Hence

$$\begin{aligned} a_1 &= \left(\frac{l-x}{l}\right), \quad a_2 = x/l \\ a_3 &= 1 - (3x^2/l^2) + (2x^3/l^3) \\ a_4 &= x - (2x^2/l) + \frac{x^3}{l^2} \\ a_5 &= (3x^2/l^2) - (2x^3/l^3) \\ a_6 &= -(x^2/l) + (x^3/l^2) \end{aligned} \tag{1.15}$$

For this problem, the longitudinal strain

$$\epsilon = \frac{du}{dx} - y\frac{d^2v}{dx^2} \tag{1.16}$$

Following the procedure underlined in Eq. (1.10) the stiffness matrix can be expressed as [1.6]

$$[K_e] = \begin{bmatrix} \frac{AE}{l} & 0 & 0 & -\frac{AE}{l} & 0 & 0 \\ & \frac{12EI}{l^3} & +\frac{6EI}{l^2} & 0 & -\frac{12EI}{l^3} & +\frac{6EI}{l^2} \\ & & \frac{4EI}{l} & 0 & \frac{6EI}{l^2} & \frac{2EI}{l} \\ & & & \frac{AE}{l} & 0 & 0 \\ & \text{Symmetric} & & & \frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ & & & & & \frac{4EI}{l} \end{bmatrix} \quad (1.17)$$

The mass matrix can also be expressed in a similar fashion using Eq. (1.12). This can be written as [1.6]

$$[M]_e = \frac{\rho Al}{420} \begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22l & 0 & 54 & -13l \\ 0 & 22l & 4l^2 & 0 & 13l & -3l^2 \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 & 13l & 0 & 156 & -22l \\ 0 & -13l & -3l^2 & 0 & -22l & 4l^2 \end{bmatrix} \quad (1.18)$$

When a series of elements are involved the overall stiffness and mass matrices can be obtained by combining the matrices of the elements. The logic of this assembly is explained later in Example 2.11.

1.2 Finite Difference Method

There are occasions when the behaviour of the object under load can be mathematically formulated. If this leads to a governing differential equation whose closed form solution is not easily available, approximate methods of solution must be employed. Initial value (for transient vibration) and boundary value (for static problems) problems involving either ordinary or partial differential equations may be solved by such methods. The derivatives of functions appearing in the differential equation are approximated by Taylor expansion of the unknown function.

Taylor series expansion of f_j is given by

$$f_j = f_{i\pm1} = f(x_i \pm h) = f_i \pm hf_i' + \frac{h^2}{\underline{|2}}f_i'' \pm \frac{h^3}{\underline{|3}}f_i''' + \ldots \tag{1.19}$$

where ( )′ stands for df/dx and h is the spacing.

From Eq. (1.19) it is seen that

$$f_i' = \frac{f_{i+1} - f_{i-1}}{2h} \tag{1.20}$$

with an error of the order of $\frac{h^2}{6}f'''$.

Similarly,

$$f_i'' = \frac{1}{h^2}(f_{i+1} - 2f_i + f_{i-1}) \tag{1.21}$$

with an error of the order $\frac{h^2}{12}f_i^{IV}$.

When this is extended to partial differential equations in two variables x and y, the same argument can be extended. If h is the constant spacing in x as well as in y directions, and if (i, j) denotes any location, one can write

$$\left(\frac{\partial^2 f}{\partial x^2}\right)_{\text{at } i,j} = \frac{1}{h^2}(f_{i+1,j} - 2f_{i,j} + f_{i-1,j}) \tag{1.22}$$

with an error of the order of $\frac{h^2}{12}\cdot\frac{\partial^4 f}{\partial x^4}$.

Similarly,

$$\left(\frac{\partial^2 f}{\partial y^2}\right)_{\text{at } i,j} = \frac{1}{h^2}(f_{i,j+1} - 2f_{i,j} + f_{i,j-1}) \tag{1.23}$$

with an error of the order of $\frac{h^2}{12}\cdot\frac{\partial^4 f}{\partial y^4}$.

All other combinations of interest have been discussed in Ref. [1.7].

1.2.1 Beam Problem

The governing differential equation can be written in the following form

$$EI\frac{d^2W}{dX^2} = -Mx$$

$$\frac{d^2Mx}{dX^2} = P \tag{1.24}$$

where EI = Flexural rigidity of the beam

W = Lateral deflection

X = Axial location

P = Intensity of loading

Non-dimensionalising of the variables can be done as follows:

$X = a\xi$ where a is a reference length

$W = aw$

$$M_X = \frac{EI}{a}\cdot(m_\xi)$$

ξ, w and m_ξ are now dimensionless

$$\frac{d}{dX} = \frac{d}{ad_\xi}, \qquad \frac{d^2}{dX^2} = \frac{1}{a^2}\frac{d^2}{d\xi^2}$$

Equations (1.24) get recast in the following form

$$\left.\begin{aligned} w'' + m_\xi &= 0 \\ m_\xi'' &= \frac{Pa^3}{EI} \end{aligned}\right\} \tag{1.25}$$

where $(\ )' = d/d\xi$

If w and m_ξ are treated as dependent variables and ξ as the independent variable,

$$w'' = \frac{w_{i+1} - 2w_i + w_{i-1}}{h^2}$$

and

$$m_\xi'' = \frac{(m_\xi)_{i+1} - 2(m_\xi)_i + (m_\xi)_{i-1}}{h^2} \tag{1.26}$$

where h is the spacing.

If L is the total length of the beam and if there are totally n intervals, h can be expressed as

$$h = \frac{1}{a}\cdot\frac{L}{n} \tag{1.27}$$

a can be so chosen as to make h much smaller than 1. This will lead to an error of order h^2.

The governing differential equation at the number of locations on the beam together with the boundary conditions will result in a set of simultaneous equations when the differential equations are recast as difference equations.

1.2.2 Shell Problem

For a general shell as shown in Fig. 1.2 the governing differential equation of equilibrium can be written as [1.8]

$$\begin{aligned} &a_1u'' + a_2u + a_3u^{\cdot} + a_4v'^{\cdot} + a_5v^{\cdot} + a_6w' \\ &\quad + a_7w + a_8m_\xi' + a_9m_\xi = -p_\xi \\ &a_{10}u'^{\cdot} + a_{11}u^{\cdot} + a_{12}v'' + a_{13}v' + a_{14}v'^{\cdot\cdot} \\ &\quad + a_{15}w'^{\cdot\cdot} + a_{16}w^{\cdot\prime} + a_{17}w^{\cdot} + a_{18}m_\xi' = -p_\theta \end{aligned}$$

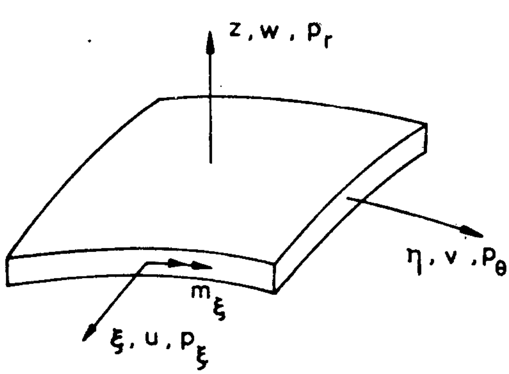

FIG. 1.2
Element of a Shell

$$\begin{aligned} &a_{19}u' + a_{20}u + a_{21}v''^{\cdot} + a_{22}v'^{\cdot} + a_{23}v^{\cdot} \\ &+ a_{24}w''^{\cdot\cdot} + a_{25}w'^{\cdot\cdot} + a_{26}w^{\cdot} \\ &+ a_{27}m_\xi'' + a_{28}m_\xi' + a_{29}\dot{m}_\xi = -p_r \end{aligned}$$

$$a_{30}u' + a_{31}u + a_{32}v^{\cdot} + a_{33}w'' + a_{34}w' + a_{35}w^{\cdot\cdot} + a_{36}m_\xi = 0 \qquad (1.28)$$

Here u, v, w are the three non-dimensional displacements and m_ξ is the non-dimensional bending moment.

$$(\ )' = \frac{\partial}{\partial \xi}, (\ )^{\cdot} = \frac{\partial}{\partial \eta} \qquad (1\ 29)$$

where ξ and η are non-dimensional x and y coordinates a_1 to a are coefficients. These equations can also be recast as difference equations using equations of the type (1.20) and (1.21). These together with the boundary conditions will again lead to a set of simultaneous equations. When the two curvatures of the shell are zero and when u and v are absent, they will lead to equations of plates. In these simplified equations, if differentiation with respect to y (or η) are also ignored, they will reduce to the beam equations (1.25) discussed earlier (only coefficients a_{27}, a_{33} and a_{36} will remain). To summarise Eqs. (1.11) are the most general government differential equations for beams, plates and shells.

Shells of revolution assume special importance in the field of mechanical engineering since units like pressure vessels, boilers, tube mills kiln shells and many process equipment are shells of revolution. In all these cases, the displacements and stress resultants can be expressed as trigonometric functions in the circumferential direction thereby eliminating the differentiation in y (or η) direction. The governing partial differential equations reduce to ordinary differential equations in x(or ξ) only. These have been derived in Ref. [1.8] and used in [1.9]. They are of the following form:

$$a_1u'' + a_2u' + a_3u + a_4v' + a_5v + a_6w' + a_7w + a_8m_\xi' + a_9m_\xi = -p_\xi$$