Direct Current Machines

M G SAY

E OPENSHAW TAYLOR

(4)

Direct Current Machines

M G SAY
PHD, MSC, DIC, FIEE, FRSE

E OPENSHAW TAYLOR
BSC, DIC, FIEE, SEN MIEEE, FRSE

Professors Emeriti of Electrical Engineering
Heriot-Watt University, Edinburgh

Pitman

PITMAN PUBLISHING LIMITED
39 Parker Street, London WC2B 5PB

Associated Companies

Copp Clark Pitman, Toronto
Pitman Publishing New Zealand Ltd, Wellington
Pitman Publishing Pty Ltd, Melbourne

First published in Great Britain 1980

Text in IBM Press Roman by Composer Typesetting, Bath
Printed and bound in Great Britain
at The Pitman Press, Bath

ISBN 0 273 01219 3

Preface

Dr A. E. Clayton of Manchester published *The Performance and Design of Direct Current Machines* in 1927. The book, several times revised, was a standard and widely read text. The present authors have undertaken the task of carrying on the tradition.

The half-century since 1927 has seen radical changes in the technology of d.c. machines. When Dr Clayton wrote, d.c. mains supplies were common: now they are virtually extinct. Electronics meant little more than radio: power electronics now dominates the supply and control of d.c. machines. Permanent magnets then were usually very hard metallic alloys: mouldable ferrites are now in common use for the fields of small production-run machines. Run-of-the-mill d.c. motors were heteropolar and cylindrical, with copper-sector commutators and carbon brushes: the range now includes unconventional shapes, brushless commutation, homopolar structure and even superconducting field windings. Simple devices were adequate to provide starting, braking and speed control: automation in engineering production now demands sophisticated and precision control of speed and load, sometimes with very high rates of acceleration and reversal, with dynamic performance in a dominant role.

There has been also a change in the methods of machine analysis with the introduction by R. H. Park and Gabriel Kron of the 'generalized theory'. The conventional d.c. machine fits elegantly into this concept because it has, at least in idealized form, all the essential elements including the quasi- (or pseudo-) stationary rotor windings which, although they physically rotate, preserve a fixed m.m.f. axis. Generalized theory expresses machine behaviour, both steady-state and transient, as a matrix of simultaneous differential equations. But a machine is not merely an impedance matrix.

Design by computer has displaced the earlier methods based largely on empiricism and 'know-how'. The ubiquitous digital computer solves numerical equations in design, but numerical results are meaningful only in relation to the underlying physical principles. A machine is not just a digital print-out: it is a highly complex electromagnetic-mechanical-thermal device.

Clearly there has to be a reassessment of the content of a textbook devoted to the d.c. machine. The present authors are at one with Dr Clayton in the belief that elementary machine behaviour can best be grasped through application of flux-current interaction, which is so easily demonstrated experimentally. The treatment therefore starts with this principle, the prototype forms in which it can be realized, and the essential function of the commutator. Ideal cases are followed by progressive departures from the ideal that affect practical performance, and commutation is discussed in both conventional and novel forms. Generator and motor operation on traditional d.c. supply is included for completeness, but considerable attention is given to performance of motors fed from a.c. supplies through converters. Motors for use in control systems are described in some detail, together with the features of a number of 'special' machines and devices. The final chapter deals with typical constructional features and with the basics of design.

As both control engineering and semiconductor technology already have a voluminous literature, the authors have deemed it preferable to deal with machines themselves rather than with the multifarious control systems in which they find application.

All formulae, whether concerned with electrical, magnetic, mechanical or thermal phenomena, are couched consistently in SI base units. Only in numerical examples have decimal multiples and submultiples been employed for convenience, in conformity with IEC recommendations. Guidance, if needed, can be gained from a booklet, 'Symbols and Abbreviations for Electrical and Electronic Engineering' (Institution of Electrical Engineers, 1979).

Our thanks are due to good friends in the industry and in the Universities, abroad as well as in the U.K. We are particularly indebted to a number of anonymous critics who have made valuable suggestions for the improvement of the text. We hope that readers, too, will help by their comments.

Guestling, Sussex 1980

M.G. Say
E. Openshaw Taylor

Contents

1 Introductory

1.1 DIRECT CURRENT MACHINES

Michael Faraday enunciated in 1831 the principle of electromagnetic induction. A year later he demonstrated the first *homopolar* electromagnetic generator – a copper disc rotated by a spindle in an axially directed permanent-magnetic field, with sliding contacts ('brushes') at the edge and centre of the disc to pick off the generated e.m.f. The *heteropolar* version, in which a winding is rotated in the magnetic field of successive N–S pole-pairs, soon followed. The first rotating machines embodying Ampere's *commutator* appeared in 1833. These generators were soon found to be reversible and to act equally well as motors. Modern electromagnetic machines, notwithstanding a century and a half of technological progress, still exploit the basic Faraday principle.

D.C. machines may work as generators or motors or brakes. In the *generator mode* the machine is driven by a prime mover and develops electrical power. In the *motor mode*, power supplied to the machine electrically is converted into mechanical power as output. The *brake mode* is a generator action but with the electrical power either regenerated or dissipated within the machine system, developing in consequence a mechanical braking effect.

Almost all land-based electrical power-supply networks are a.c. systems of generation, transformation, transmission and distribution; and as d.c. supplies are readily derived therefrom by rectification, there is little need for large d.c. generators. Further, it is accepted practice in industry to employ a.c. motors wherever they are inherently suitable or can be given appropriate characteristics by means of power-electronic devices. Yet there remain important fields of application in which d.c. machines offer economic and technical advantage, for the outstanding property of the d.c. motor is its versatility. It can be designed for wide ranges of voltage/current or speed/torque relations for both steady-state and transient operation. The advent of the controlled power rectifier has made it possible to link a d.c. machine to an a.c. mains supply, and speed-control 'packages' are readily obtainable for industrial application.

Power plants isolated from land-based supply networks (as on road vehicles, ships and aircraft) may embody d.c. machinery. Normally a secondary battery provides a central supply and requires a means for charging it; and although the trend is towards a.c. generators and rectifiers for the battery-charging duty, many wholly d.c. equipments are still in wide use. Small back-up and standby generating plants providing useful energy from windmills and mountain streams often use d.c. generators because the fluctuating and intermittent drive would, with an a.c. generator, produce unacceptable variation in the generated frequency.

D.C. series motors, well adapted to the requirements of traction, drive intercity and rapid-transit trains. In some process drives and mine-winding engines it may be found advantageous to employ d.c. machinery for effective speed control. Battery vehicles for fork-lift trucks and delivery vans are likely to become more common. Portable drills, sewing machines and hand tools are well served by the 'universal' series motor on single-phase a.c. supply, but work at higher efficiency on d.c. Miniature d.c. motors working from dry cells operate razors, cameras, tape-recorders and similar small loads.

Automation has brought about a resurgence of interest in precision d.c. machines. High-energy permanent magnets, epoxy-resin winding encapsulation and advanced brush or electronic commutation have made possible a range of control machines that are smaller, cheaper and more reliable than their a.c. rivals, particularly in ratings of a few watts. With power transistors such machines can fulfil basic control functions in feedback systems. At higher ratings the 'crossfield' amplifier machine has been applied. A class exemplified by the 'stepper' motor is operated by d.c. pulses to act as a precision transfer device between the digital information presented by the input driver and the corresponding mechanical motion imparted to the load. While having some resemblance to an a.c. machine, the supply is undeniably unidirectional, so that the question 'Is it a d.c. or an a.c. machine?' has a justifiably equivocal answer.

When a simple conductor (typically a disc) is moved through a permanent-magnetic field, circulating currents flow that develop heat and impose a braking force on the conductor. The effect is put to use in eddy-current brakes and couplings, and particularly for providing electrical-energy meters with a braking torque proportional to the speed of the meter movement.

Most industrial motors have heteropolar field systems and a cylindrical shape. There are however, several other possible forms such as the disc, the linear and the tubular. Further, the homopolar field arrangement has been developed to provide large current at low voltage for electrochemical processes. The homopolar shape lends itself to the use of superconducting field windings, by means of which very high working flux densities can be achieved. But whatever the topology of the machine, and be it homo- or heteropolar, the essential requirement is that a current flow in a direction at right-angles to that of the flux of a permanent or electromagnet in such a way as to utilize most effectively the mechanical force developed by flux-current interaction.

Classification

D.C. machines are built for ratings from megawatts to milliwatts. For convenient reference they may be roughly classified as follows:

Industrial Large generators; and motors for mills, cranes, machine tools and general industrial drives of ratings above a few kilowatts, and usually supplied from a.c. mains through rectifiers.

Small Motors for hand-tools, domestic equipments and similar mains-fed applications; and automobile starter motors. Their construction is a simplification of that for conventional industrial machines.

Traction Motors for railways and battery-fed road vehicles.

Miniature Motors of a few watts output for intermittent loads not requiring precision control.

Control Machines associated with open- and closed-loop control systems; they are commonly provided with permanent-magnet fields.

Special Linear machines, actuators, eddy-current devices, motors with superconducting field windings, and unconventional designs.

1.2 MODELS

Electrical science is concerned, on the microscopic scale, with the properties and phenomena that stem from the sub-atomic electrical nature of matter; and on the macroscopic scale, with the concept of electromagnetic-field theory applied to matter in bulk, as codified in the Maxwell electromagnetic-field equations. For the present purpose we adopt a technological viewpoint. If the magnetic field is taken as associated with a current, and the current itself as a continuous flow rather than as a stream of discrete particles, a more direct approach to machines is achieved on a basis of simple concepts. What we seek is a *model* on which analysis can be based. One model, that of flux–current interaction, gives a physical picture of the mechanism of electromechanical energy conversion; a second is the dynamic-circuit representation which gives a concise and powerful analysis of overall machine behaviour under steady-state and, particularly, transient operating conditions. Both models are related to the concept of magnetic flux.

Magnetic flux

A magnetic field is a region of space in which useful physical effects occur. A pictorial model of the field can be made by drawing closed loops of flux such that their direction and spacing are measures of the direction and concentration of the flux, i.e. the vector flux density. In present context the magnetic circuit is composed partly of ferromagnetic material, partly of an airgap. The magnetic material serves to 'guide' the flux in a desired path; the airgap is necessary to make the useful magnetic-field effects readily accessible.

The lines drawn in a flux plot have no real existence, and must not be regarded as like the bristles on a brush which move bodily with the brush-head. In a given magnetic field the flux density may change direction, become weaker in some regions and stronger in others: but the lines do not 'move' into new positions. All we can say is that, if we map the field at various instants, the patterns differ.

Engineers regard a magnetic flux Φ as 'produced' by electric current, explicit in the case of an electromagnet, implicit for a permanent magnet. A current i develops, around any path that links it, a magnetomotive force $F = i$ [ampere]. The effect of a current can be multiplied by coiling the electric circuit into N turns so that around a path linking all the turns the m.m.f. is $F = Ni$ [ampere or ampere-turn]. The m.m.f. is distributed along the path to give a path element of length dx the magnetic field strength H [ampere (-turn) per metre]. The summation of $H \cdot dx$ around a single-loop closed path is F. At any point, H gives rise to a flux density $B = \mu H$ [tesla], where μ is the absolute permeability of the ambient medium. Summation of B over the area available to the flux path gives the total flux Φ [weber].

The 'law' of the magnetic circuit relates the total flux Φ to the total m.m.f. F through the expressions

$$\Phi = F/S = F\Lambda \tag{1.1}$$

where S is the total reluctance [ampere (-turn) per weber] and $\Lambda = 1/S$ is the total permeance [weber per ampere (-turn)] of the magnetic circuit. For a part-length x [metre] of material of absolute permeability μ and having a uniform cross-section a over which the flux density B is everywhere the same, the m.m.f. is $F_x = H \cdot x$. We may therefore write $F_x = \Phi S_x = \Phi/\Lambda_x$ where

$$S_x = F_x/\Phi = Hx/\mu Ha = x/\mu a \quad \text{and} \quad \Lambda_x = \mu a/x$$

for the part-reluctance and part-permeance. For a succession of parts $x, y, z \ldots$ in series, through which the same flux passes,

$$F = F_x + F_y + F_z + \ldots \quad \text{and} \quad S = S_x + S_y + S_z + \ldots$$

For parts in parallel that share the flux,

$$F = F_x = F_y = F_z = \ldots \quad \text{and} \quad \Lambda = \Lambda_x + \Lambda_y + \Lambda_z + \ldots$$

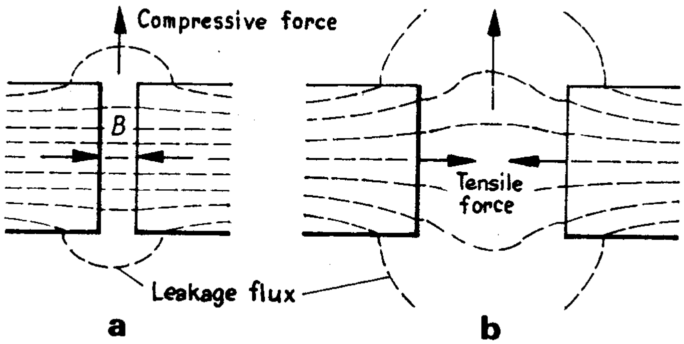
For fields in free space or in air, the absolute permeability is the magnetic constant $\mu = \mu_0 = 4\pi/10^7 \cong 1/800\,000$, which means that $H \cong 800\,000\,B$. For fields in ferromagnetic material, μ is very much greater, the ratio $\mu/\mu_0 = \mu_r$, the relative permeability, ranging from a few hundreds to a half-million. Consequently H lies between $2B$ and $2000\,B$. But in such materials μ_r depends on B and varies with the degree of magnetic saturation. Nevertheless, it is sometimes permissible to ignore the requirements of ferromagnetic parts of a magnetic circuit and to assume that the whole m.m.f. is impressed on the airgap: this in effect assumes that the relative permeability of the material is infinite.

Flux Plots In a practical device it is necessary to know the distribution of the magnetic field. As this is three-dimensional, an exact solution is difficult;

but a two-dimensional sketch can in many cases adequately represent the field in the most significant region – the airgap. The plot can be drawn by applying a few simple rules relating to the space between the bounding surfaces of an otherwise ferromagnetic magnetic circuit:

(i) The permeability of the ferromagnetic material ('iron') is assumed to be infinite, so that the gap surfaces become magnetic equipotentials. Intermediate equipotential lines may be added to aid the plotting process.
(ii) The flux is mapped by lines between the surfaces, drawn always orthogonal to the equipotentials and making with them a network of curvilinear squares. The plot can then be used for quantitative assessments.
(iii) Only one flux can occupy the gap, but it may be derived as the vector superposition of component flux densities produced by individual currents.

Maxwell Stress A general principle formulated by Maxwell is that forces on magnetic bodies are transmitted across the gap between them by a system of two stresses: (i) a tensile stress of value $\frac{1}{2}BH$ along the line of action of the flux (i.e. a flux line), and (ii) a compression stress of the same value in all directions perpendicular to the flux lines. In an elementary way, the flux lines can be considered to have an 'elastic thread' property, with the tendency to shorten lengthwise and thicken laterally. With practice (and with due regard to the fictive nature of the flux lines) an immediate impression of magnetic force can be gained from a flux plot.



1.1 Maxwell stresses

Fig. 1.1 shows two iron bars forming part of a magnetic circuit. In (*a*) the polar surfaces are close together and the flux is mainly concentrated between the surfaces. The density B is large, and so also is H, so that $\frac{1}{2}BH$ represents a strong 'tensile' attraction force between the pole-faces. Not all the flux contributes to axial attraction: some, the leakage flux, exists at the flanks of each pole. Flux crossing the boundary between air and a highly permeable ferromagnetic material must enter or leave the boundary almost at right-angles to the surface, so that tensile stress due to leakage flux cannot augment the pole-face attraction. All the 'compressive' stresses balance out by symmetry.

Now consider case (*b*). The greater reluctance of the long airgap reduces the total flux and the pole-face flux density, and there is more leakage because there is little difference in the reluctance of useful and leakage paths. The force of attraction is consequently much less than in case (*a*). The comparison brings out the importance of ascertaining the flux distribution if reliable estimates of mechanical force are to be made. It might be good enough in (*a*) to assume the flux to be uniform over the pole-faces and to pass from face to face directly with negligible leakage. But this will not do for (*b*), which can be dealt with only by aid of a flux plot.

Magnetic Field Energy The Maxwell stress concept implies that energy to the value $\frac{1}{2}BH$ is stored in a unit cube of the space occupied by a magnetic field: thus $\frac{1}{2}BH$ is the magnetic energy density. The complete energy storage in a system is consequently given by $w_f = \frac{1}{2}\Phi F$ [joule]. If an elemental relative displacement dx of parts in a magnetic field results in a change dw_f of the stored energy, the work done in making the displacement is given by

$$dw_f = f_e \cdot dx \quad \text{or} \quad f_e = dw_f/dx \qquad (1.2)$$

by the principle of virtual work, [1] where f_e is the force that the magnetic field engenders between the parts. If the elemental movement is angular, $d\theta$, then

$$dw_f = M_e \cdot d\theta \quad \text{or} \quad M_e = dw_f/d\theta \qquad (1.3)$$

where M_e is the magnetically produced torque.

Leakage and Saturation Two important aspects of field distribution have practical significance. One, already mentioned, is *leakage.* There is no magnetic insulator, and although flux can be encouraged to follow a ferromagnetic path and to cross a working airgap, it cannot be confined completely thereto. The second is *saturation.* Ferromagnetic materials are essential components of the magnetic circuits of almost all electrical machines, but their permeability varies widely over the range of economically usable flux density. In consequence, flux varies in a nonlinear way with current, a condition that is occasionally crucial to machine behaviour.

Flux–current interaction model

Important and useful effects occur in magnetic fields in which ferromagnetic parts can move under magneto-mechanical forces by translation or rotation, where mechanical forces are produced on currents, and where electromotive forces are generated by changes of the flux linking an electric circuit. These are set out in Sect. 1.3, using a direct application of the Maxwell stress concept. Although limited to simple cases, the approach gives a direct physical appreciation of the operation of a machine system.

Dynamic-circuit model

Here the flux Φ due to a current i is described in terms of an inductance parameter L. The flux linked with an electric circuit is then expressed as $N\Phi = \Psi = Li$, and the associated field energy is $w_f = \frac{1}{2}Li^2$. The analysis is thus couched in electric-circuit terms, and what is a highly complex electromagnetic system associated with a defined conducting path can be handled with the aid of network theory. Lumped parameters of resistance and inductance are set up to represent electric, thermal and magnetic-field properties, associated with the circuit properties of voltage, current and e.m.f., to provide a concise shorthand of the system that they model. Account is no longer taken directly of the magnetic field distribution, although flux plotting may sometimes be necessary in establishing the parametric values.

In machines, an essential property is the disturbance of fields by displacement of parts in the process of electrical/mechanical energy conversion, so that the inductances are functions of position. Each electric circuit of the machine has a behaviour expressed by a differential equation, and the device is modelled by as many such equations as there are distinct electric circuits.

As a motor or a generator has mechanical attachments in the form of a load or a driver, both dynamic, the mechanical system reacts on the electrical system to affect acceleration, speed and position. A complete analysis therefore requires consideration of (i) the electric circuit equations, (ii) the energy-conversion process, and (iii) the equation of motion; that is, the whole electromagnetic/mechanical linked system.

Other models

A machine is as much an electromagnetic-field device as is a wave-guide, and must internally satisfy the Maxwell equations. The field pattern could be expressed in Laplacian or Poissonian terms, and energy transfer across the gap found from application of the Poynting vector. Alternatively, the wave-impedance concept could be applied to unify field and circuit theory. An outline of such models is given by Nasar [2]. Rigorous field theory is elegant and satisfying, but difficult to solve because the topology introduces very complicated boundary conditions. As we have simpler ways, less general but more tractable, it is helpful to employ them.

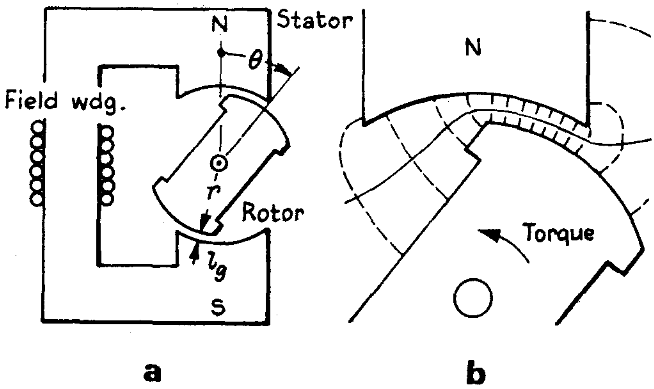
1.3 PRINCIPLES

In common with other electromagnetic/mechanical devices, the operation of the d.c. machine can be explained in terms of basic principles concerned (i) with the development of magneto-mechanical forces and (ii) with the induction of e.m.f.s by the rate of change of flux linkage.

Alignment force

When a magnetic field is established in an ambient low-permeability medium (such as air), pieces of high-permeability material (such as iron) experience mechanical forces tending to align them with the direction of the field in such a way as to reduce the reluctance of the system.

In the elementary machine of Fig. 1.2(*a*), a fixed ferromagnetic member ('stator') is excited by the current in a 'field' winding to produce a flux in the magnetic circuit. A structurally polarized (but unexcited) ferromagnetic 'rotor' member is placed in the gap between the stator poles N and S. A flux plot, drawn in accordance with the rules set out in Sect. 1.2, is shown in (*b*) for the upper gap region (with the radial gap length exaggerated for clarity).



1.2 Alignment principle

The stator and rotor facing gap surfaces are taken as equipotentials and one intermediate equipotential has been added. Over the uniform and short gap of length l_g the flux density is high: but it is radial, and from the Maxwell stress concept can result only in a radial attraction between rotor and stator. This force is balanced by an opposite attraction at the lower pole. In the lateral regions of the gap outside the short-gap region, the flux lines terminate on either stator or rotor pole flanks, and the Maxwell tension forces act in a direction to turn the rotor counterclockwise into alignment with the stator poles, increasing the area of the short-gap region and reducing the magnetic-circuit reluctance.

The torque can be deduced from eq. (1.3). If θ, the angle between the stator and rotor magnetic axes, is such as to give a relatively large polar overlap, the flux and gap energy can be taken as wholly contained in the short-gap regions, where the field energy density is $\frac{1}{2}BH = \frac{1}{2}B^2/\mu_0$. With a rotor of axial length l, the active gap volume is $2ll_g(r\theta)$, and the gap energy is $w_f = B^2 l l_g r\theta/\mu_0$. An elemental increase $d\theta$ in the displacement angle reduces the gap energy, and equating this to the mechanical work done, $M_f \cdot d\theta$, gives the torque on the rotor

$$M_f = dw_f/d\theta = -B^2 lrl_g/\mu_0 \quad (1.4)$$

opposing the angular movement and acting to align the magnetic axes.

Now the flux across the short gap is radial and can produce only *radial*