Stable Adaptive Systems

Kumpati S. Narendra

Center for Systems Science
Yale University
New Haven, Connecticut

Anuradha M. Annaswamy

Department of Aerospace and Mechanical Engineering
Boston University
Boston, Massachusetts

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Stable Adaptive Systems

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Preface

This is an exciting time to be working in the field of adaptive control. Research in recent years has led to the emergence of a wide spectrum of problems and the field is sufficiently mature to attract theoreticians looking for an area in which nonlinear systems arise naturally. In addition, the adaptive algorithms being studied complement current computing technology resulting in a powerful approach with great potential impact on the world of applications. Sophisticated, yet practical, adaptive controllers are now feasible. For these reasons, it is not surprising that adaptive control has found a large following in all segments of the control community as a whole.

Three decades after the term *adaptation* was introduced into the engineering literature, it is now generally realized that adaptive systems are special classes of nonlinear systems and hence capable of exhibiting behavior not encountered in linear systems. The difficulty experienced in coming up with a universally acceptable definition of adaptive systems, as well as in generating techniques for their analysis and synthesis, may be traced ultimately to this fact. It is well known that design techniques for dynamical systems are closely related to their stability properties. Since necessary and sufficient conditions for the stability of linear systems have been developed over the past century, it is not surprising that well known design methods have been established for such systems. In contrast to this, general methods for the analysis and synthesis of nonlinear systems do not exist since conditions for their stability can be established only on a system by system basis. To design tractable synthesis procedures, adaptive systems are structured in such a fashion that their behavior asymptotically approaches that of linear systems. The central theme of this book is stability of adaptive systems, and it is based on the conviction that adaptive systems can be designed with confidence only when their global stability properties are well understood

In 1980, the stability problem of an idealized version of an adaptive system was resolved and it has come to represent a landmark in the development of adaptive systems. Following this, in recent years, a multiplicity of ideas have been generated in the field. The flood of new information is so great that the beginner tends to be overwhelmed by the numerous techniques and perspectives adopted. The time appears to be appropriate to attempt a unified presentation of results that are currently well known and to establish the close connections that exist between seemingly independent developments in the field.

The entire book is written in a self-contained fashion and is meant to serve as a text book on adaptive systems at the senior undergraduate or first-year graduate level. A knowledge of linear algebra and differential equations as well as an acquaintance with basic concepts in linear systems theory is assumed. The book can be used for an intensive one-semester course, or a two-semester course with emphasis on recent research in the second semester. The problems included at the end of the chapters should be considered as an integral part of the book. Following the approach used by the authors while teaching this course at Yale, the problems are divided into three categories I, II, and III. Problems in Part I are relatively easy and are meant primarily to test the student's knowledge of mathematical prerequisites and systems concepts. Part II contains problems that can be solved by the application of results derived in the chapters. Problems in Part III are substantially more difficult and occasionally include open questions in adaptive control for which solutions are not currently available.

The book is arranged in such a manner that Chapters 1-7 are accessible to the beginner while Chapters 8-11 are meant for the more advanced, research-oriented student. A fairly extensive first chapter sets the tone for the entire book. Besides introducing basic concepts, it also attempts to trace the evolution of the field to its present state from early approaches that were popular in the 1960s. In particular, an effort has been made to delineate clearly the basis for many of the assumptions that are made in the following chapters which are generally scattered in the technical literature. The authors believe that the importance of these ideas for a broad understanding of the field justifies the unusual length of the introduction. Chapter 2 is devoted to a discussion of results in stability theory with emphasis on those results which are directly relevant to the study of adaptive systems. While Chapters 3-5 deal with the stability properties of adaptive observers and controllers, Chapter 6 introduces the important concept of persistent excitation. In Chapter 7 it is shown that all the systems discussed in Chapters 3-6 can be analyzed in a unified fashion using error models. Chapters 8-10 deal with areas where there has been intense research activity in the last eight years and the final chapter contains five detailed case studies of systems where adaptive control has proved successful.

Developments in the field of adaptive control have proceeded in a parallel fashion in both discrete and continuous systems, and results in one area usually have counterparts in the other. Many of the problems formulated using discrete or continuous time models can also be studied in the presence of stochastic disturbances. This book deals with continuous time finite dimensional deterministic systems. We believe that a thorough understanding of this class will provide the essential foundation for establishing analogous results in both the discrete time and stochastic cases.

It is our privilege to thank a long list of friends and colleagues who have helped us in the preparation of the book in different ways. We are especially grateful to Petar Kokotovic for his careful scrutiny of the first seven chapters and numerous valuable suggestions. We would also like to thank Peter Dorato, Petros Ioannou, Robert Kosut, Gerhard Kreisselmeier, Steve Morse, and R.P. Singh for their comments on portions of the manuscript and to Job van Amerongen, Dan Koditschek, Heinz Unbehauen, and Eric Ydstie for critically evaluating parts of Chapter 11. Manuel Duarte and Jovan Boskovic took part in numerous technical discussions and Jovan's help in collecting the material for Chapter 11 is particularly appreciated. Both of them worked very hard toward proofreading the final manuscript. We are most grateful to them for their commitment to the completion of the book. Finally, we would like to thank Eileen Bernadette Moran for encouraging us to do this project with Prentice Hall, Tim Bozik, engineering editor, for his enthusiasm and help, and Sophie Papanikolaou, production editor, for her remarkable efficiency.

The first author would like to thank his doctoral students in the area of adaptive control during the period 1970-88. Many of the ideas in this book first appeared in papers that they coauthored with him. The time spent with them was instructive, productive, and thoroughly enjoyable.

The second author gratefully acknowledges the IBM Corporation for awarding a postdoctoral fellowship during her stay at Yale from 1985-87. This enabled her to devote her time fully to the book. She would also like to thank Rose Schulz for her hospitality during the entire time the book was in preparation.

Finally, without the patience and encouragement of Barbara Narendra and Mandayam Srinivasan, this book would not have been undertaken, nor completed.

Kumpati S. Narendra

Anuradha M. Annaswamy

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Introduction

1.1 INTRODUCTION

Questions of control in general and automatic control in particular, are assuming major importance in modern society even as social and technological systems are becoming increasingly complex and highly interconnected. By "control of a process" we mean, qualitatively, the ability to direct, alter, or improve its behavior, and a control system is one in which some quantities of interest are maintained more or less accurately around a prescribed value. Control becomes truly automatic when systems are made to be self-regulating. This is brought about by the simple concept of *feedback* which is one of the fundamental ideas of engineering. The essence of the concept consists of the triad: measurement, comparison, and correction. By measuring the quantity of interest, comparing it with the desired value, and using the error to correct the process, the familiar chain of cause and effect in the process is converted into a closed loop of interdependent events. This closed sequence of information transmission, referred to as feedback, underlies the entire technology of automatic control based on self-regulation. Although the existence of self-regulating mechanisms in living organisms has been recognized for a long time and the deliberate construction of self-regulating systems, such as float regulators and water clocks can be traced back to antiquity [30], the abstract concept of the closed causal loop is a distinct achievement of the twentieth century.

Up to the beginning of the twentieth century, automatic control remained a specialty of mechanical engineering. This was followed by a period when electrical regulators and controllers became quite common. About the 1940s, electrical, mechanical, and chemical engineers were designing automatic control devices in their respective fields using very similar methods arrived at by different routes and disguised under completely different terminologies. Although at first no connection between these developments was recognized, it gradually became clear that the concepts had a common basis, and at the end of World War II, a theory that was mathematically elegant and universal in its scope came into being. In 1948, Wiener named this newly founded discipline *Cybernetics* [43]. In the last forty years, feedback control has evolved from an art into a scientific discipline which cuts across boundaries extending from design, development, and production on one hand, to mathematics on the other. In fact, even about the early 1960s, Bellman [6] felt that, having spent its fledgling years in the shade of the engineering world, control theory had emerged as a mathematical discipline that could exist independent of its applications.

The history of automatic control has witnessed a constant striving toward increased speed and accuracy. World War II, with its need for fast and accurate military systems, imparted a large impetus to the growth of the field. Frequency response methods were developed based on the efforts of Black, Nyquist, and Bode in the design of electronic feedback amplifiers. Using these methods, which are now classified under the rubric of classical control, it was possible to carry out both analysis and synthesis of closed loop systems in a systematic fashion based on open-loop frequency responses. In the course of time, these methods formed the foundations of feedback control theory and became ideally suited for the design of linear time-invariant systems. In the 1950s and 1960s, with developments in space technology, the feedback control problem grew more complex. Stringent requirements of accuracy, weight, and cost of space applications spurred the growth of new techniques for the design of optimal control systems. Models with more than one dependent variable were common occurrences and ushered in the era of multivariable control. Finally, the inevitable presence of noise in both input and output variables called for statistical solutions for the estimation and control problems, and the field witnessed a merging of control techniques with those well established in communications theory. The solution of the linear quadratic gaussian (LQG) regulator problem using the separation principle and the development of the Kalman filter became landmarks in the field in the 1960s.

Even as greater efforts were made in the direction of precise control, linear models were often found to be no longer valid and more accurate descriptions of the processes were necessary. Simple models of the process had to be replaced by more complex ones and uncertainties regarding inputs, parameter values, and structure of the system increasingly entered the picture. Their importance in the design of fast and accurate controllers shifted attention in automatic control to new areas such as adaptive, self-optimizing, and self-organizing systems.

1.2 SYSTEMS THEORY

A *system* may be broadly defined as an aggregation of objects united by some form of interaction or interdependence. When one or more aspects of the system change with time, it is generally referred to as a *dynamical* system. The first step in the analysis of any system is to establish the quantities of interest and how they are interrelated.

The principal concern of systems theory is in the behavior of systems deduced from the properties of subsystems or elements of which they are composed and their interaction. Influences that originate outside the system and act on it so they are not directly affected by what happens in the system are called *inputs*. The quantities of interest that are affected by the action of these external influences are called *outputs* of the system. As a mathematical concept, a dynamical system can be considered a structure that receives an input $u(t)$ at each time instant t where t belongs to a time set $\mathcal{T}$ and emits an output $y(t)$. The values of the input are assumed to belong to some set $\mathcal{U}$ while those of the output belong to a set $\mathcal{Y}$. In most cases the output $y(t)$ depends not only on $u(t)$ but also on the past history of the inputs and hence that of the system. The concept of the *state* was introduced to predict the future behavior of the system based on the input from an initial time t_0.

The dynamical systems we will be concerned with in this book are described by ordinary vector differential equations of the form

$$\begin{aligned} \frac{dx(t)}{dt} \triangleq \dot{x}(t) &= f(x(t), u(t), \theta, t) \qquad t \in \mathbf{R}^+ \\ y(t) &= h(x(t), \theta, t) \end{aligned} \tag{1.1}$$

where

$$\begin{aligned} x(t) &\triangleq [x_1(t), \dots, x_n(t)]^T \in \mathbf{R}^n, \quad \theta \triangleq [\theta_1, \dots, \theta_r]^T \in \mathbf{R}^r, \\ u(t) &\triangleq [u_1(t), \dots, u_p(t)]^T \in \mathbf{R}^p, \quad y(t) \triangleq [y_1(t), \cdots, y_m(t)]^T \in \mathbf{R}^m. \end{aligned}$$

f and h are mappings defined as $f : \mathbf{R}^n \times \mathbf{R}^p \times \mathbf{R}^r \times \mathbf{R}^+ \to \mathbf{R}^n$ and $h : \mathbf{R}^n \times \mathbf{R}^r \times \mathbf{R}^+ \to \mathbf{R}^m$. The vector u is the input to the dynamical system and contains both elements that are under the control of the designer and those that are not. The former are referred to as *control inputs*. The vector $x(t)$ denotes the *state* of the system at time t and its elements $x_i(t) (i = 1, 2, \dots, n)$ are called *state-variables*. The state $x(t)$ at time t is determined by the state $x(t_0)$ at any time $t_0 < t$ and the input u defined over the interval $[t_0, t)$. The output $y(t)$ as defined by Eq. (1.1) is determined by the time t as well as the state of the system $x(t)$ at time t.

The state and output equations in Eq. (1.1), defining a given process, may be considered as an abstract summary of the data obtained by subjecting the process to different inputs and observing the corresponding outputs. Equation (1.1) is generally referred to as the mathematical model of the process. Once such a model is available, the emphasis shifts to the determination of a control function u which achieves the desired behavior of the process. Many of the major developments in control theory during the past two decades are related to this problem.

When the mappings f and h are linear, the system is said to be linear and may be represented in the form[1]

$$\begin{aligned} \dot{x} &= A(\theta, t)x + B(\theta, t)u \\ y &= H(\theta, t)x \end{aligned} \tag{1.2}$$

[1] The functional dependence of variables on t is sometimes suppressed for simplicity of notation.