

DIGITAL SIGNAL PROCESSING

**A. Smirnov**

# Processing of Multidimensional Signals



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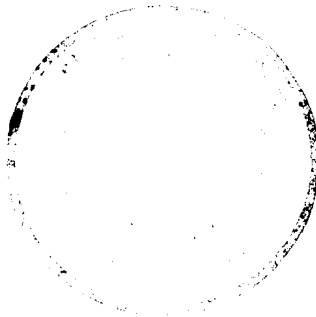
Alexandre Smirnov

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**Processing of  
Multidimensional Signals**

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### *Series Editors*

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Johann-Wolfgang-Goethe-Universität  
Institut für angewandte Physik  
Robert-Mayer-Str. 2-4  
D-60325 Frankfurt

Prof. Dr.-Ing. Anastasios Venetsanopoulos  
University of Toronto  
Dept. of Electrical and Computer Engineering  
10 King's College Road  
M5S 3G4 Toronto, Ontario  
Canada

### *Author*

Professor Alexandre Smirnov  
Departamento de Fisica  
Universidade da Beira Interior  
Covilha 6200  
Portugal

#### Library of Congress Cataloging-in-Publication Data

Smirnov, Alexandre, Processing of multidimensional signals / Alexandre Smirnov (Digital signal processing) Includes bibliographical references and index  
ISBN 3-540-65449-6

1. Image processing - Digital techniques. 2. Signal processing- Mathematics. 3. Fourier transformations. 4. System analysis. I. Title. II. Series: Digital signal processing (Springer-Verlag)  
TK 1637.S65 1999  
621.367--dc21 98-55109

ISBN 3-540-65449-6 Springer-Verlag Berlin Heidelberg New York

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Printed in Germany

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Typesetting: Camera-ready copy from author  
Cover-Design: de'blik, Berlin  
SPIN 10693677 62/3020 5 4 3 2 1 0 Printed on acid-free paper

# *Digital Signal Processing*

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Alexandre Smirnov  
Processing of Multidimensional Signals

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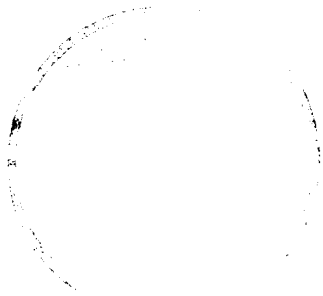
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*To My Wife*  
*and*  
*To My Friends in Portugal*



# PREFACE

The author received his Master Degree (1961) and PhD Degree (1967) in Radio Engineering from the University of Telecommunications, St.Petersburg, Russia.

He was a lecturer of the University and a senior scientist of Vavilov State Institute of Optics, dealing with methods and systems of image processing.

Since 1994 he is a professor of the University of Beira Interior, Covilha, Portugal, his pedagogical and scientific activity being linked with optics and remote sensing.

This book is an attempt to summarize his experience in all these fields.

The author loves Covilha and takes an opportunity of thanking the rectors of the University, Prof. Candido Manuel Passos Morgado and Prof. Manuel José dos Santos Silva.

The author is most grateful to his colleagues, D. Manuela Ferreira, Dr. Afonso Mesquita, Dr. Antonio Matos, Prof. José Vitorio and Prof. Anvar Meirmanov, who helped him when he needed the help, and thanks Prof. Andrew Utkin and Prof. Vadim Yurinsky, who participated in the discussions on the book.

He also thanks the authority of National Foundation for Science and Technology for kind willingness to assist in realization of the research projects in the frameworks of the programs INTERREG and PRAXIS.

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# INTRODUCTION

## I.1 Signals as Physical Objects and Carriers of Information

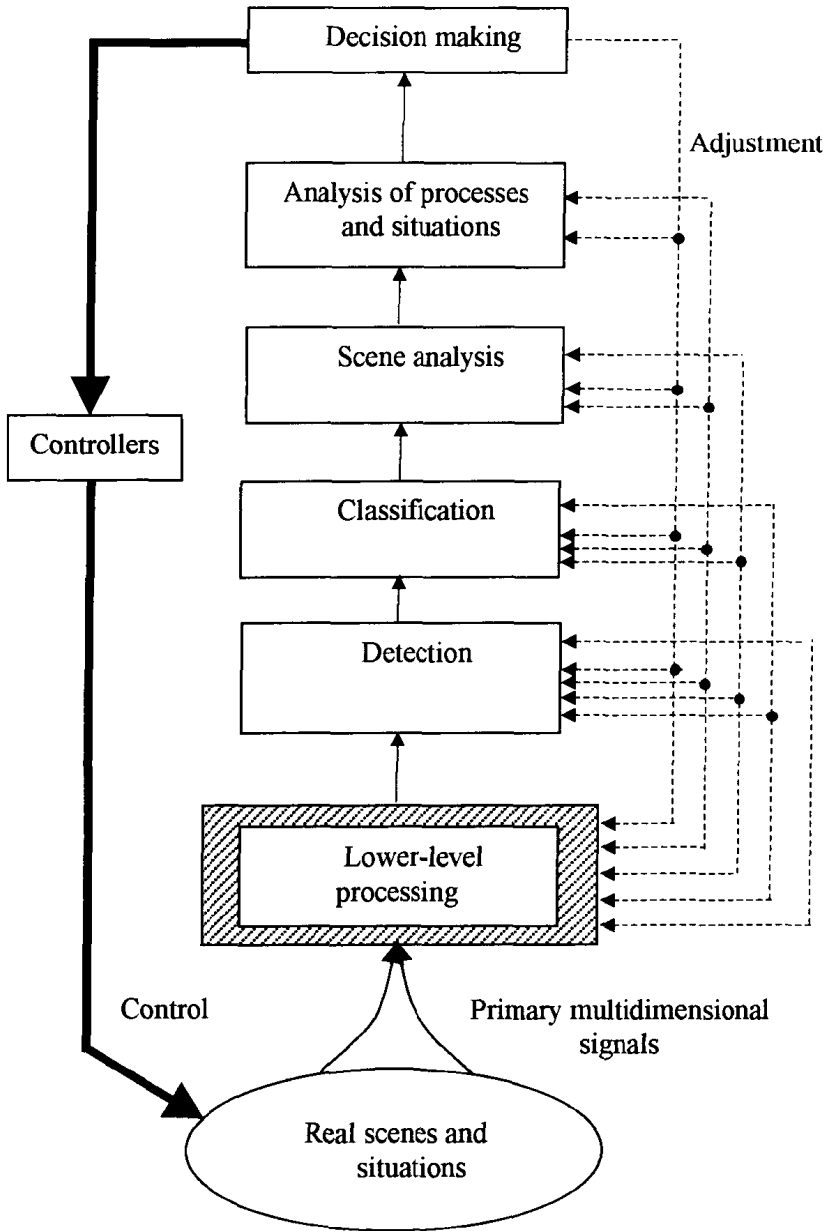
Our interaction with our natural and artificial environment is based on perception, transmission, storage and processing of *signals* of different kind. Through the eyes and ears we perceive optical (visual) and acoustical (audible) signals which are transformed into nerve signals to be transmitted, stored and processed in our neural network and brain. In reply, the brain generates other nerve signals which pass through the neural network and put our muscles into operation. In such a way we adapt ourselves to the environment or/and change (control) the environment. In technical information systems (which help us to solve the same tasks of adaptation to or transformation of the environment) we encounter electrical and radio-signals, optical and acoustical signals (including signals outside the boundaries of our perception), mechanical, pneumatic, hydraulic and many other signals.

In spite of differences between all those signals we can define, at least, their two common properties:

- All signals are *physical objects* existing in three-dimensional space and in the time;
- All signals are *carriers of information*.

As physical objects signals have some physical properties, including a specific type of *energy* and space-time *structure*. As carriers of information signals have some *semantic properties*. In information systems signals are undergone to energetic, structural (spatio-temporal) and semantic *transformations* resulting in alteration of their properties, and we can imagine a general multilevel hierarchical system of signal processing and control (Fig. I.1).

According to Fig. I.1, primary signals containing some information about real objects, scenes and situations are preliminary processed (pre-processed) at lower levels of the system. Algorithms of signal processing on higher levels, possibly, include detection and classification of the objects as sources of signals, analysis of scenes composed by interconnected objects, analysis of properties of the objects and analysis of processes and situations in the scenes.



**Fig. 1.1.** One among possible interpretations of the process of our interaction with the environment, including signal acquisition, signal processing and control of the sources of signals

Possibly, some *knowledge (model of the world)* is used at the higher levels and, possibly, the hierarchical organization is supplemented by parallel processing in several signal pathways.

After making decisions on objects, scenes and situations a process of their control is realized with the help of some controllers in a feedback control loop.

Local feedback loops (dotted lines in Fig. 1.1) can be used to adjust lower levels by higher levels providing the best (in a sense) signal processing and, eventually, the best (in a sense) control of the environment.

This hierarchical system can be:

- a human realizing signal acquisition, all levels of signal processing, and control;
- a technical automatic system which performs the same task instead of the human;
- a semi-automatic system composed by a human and a technical subsystem operating in so called interactive mode.

In the latter case the most widely used interaction scheme is the following: the technical system pre-processes primary signals, while the human (system user) uses the output signals of the system to provide higher-level processing and control.

It is this case we consider in the book. We deal with the technical systems of pre-processing of multidimensional signals, supposing that the system user (observer of output images or/and the listener of output sound signals of the system) uses the signals for this or that higher-level processing, for making decisions and for control (Fig. 1.2).

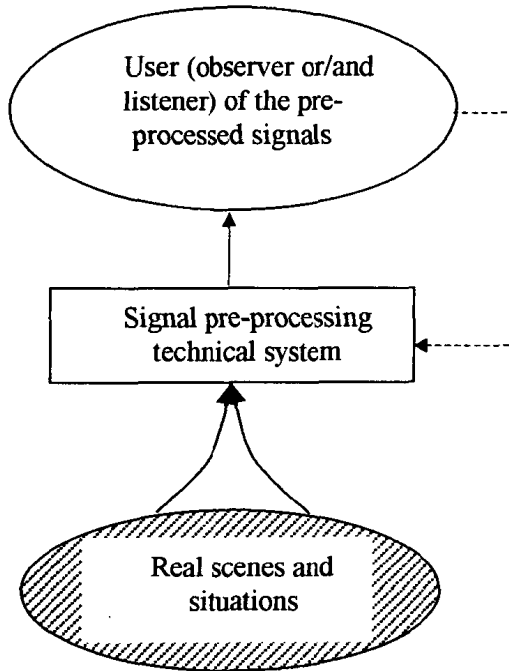
The set of signals to be considered includes temporal electrical and radio-signals with different types of modulation, acoustical signals, and optical, electrooptical, photographic, television, radiolocation and other images. Transformations to be considered mostly cover the processes of *signal transmission* and *signal storage* in pre-processing systems and their subsystems.

In the frameworks of this paradigm we have a chance to consider a great set of practical information systems and to investigate diverse multidimensional signal transformations at the physical and structural level. Its disadvantage reveals in the tasks of system optimization. If we try to create not just a system, but the best (optimum) system we should understand how the pre-processed signals are transformed at the higher levels and what control process is eventually realized according to Fig. 1.1. Thus investigations of pre-processing technique as such appear to be incompleting, and the omitted higher-level signal processing in such or other form penetrate into the problem of substantiation of *criteria of system optimality*. We are going to discuss this problem in Chapter 4.

It should be mentioned that the same problems are being considered by several specialized sciences, including theory of electrical circuits, information theory, physical and technical acoustics, physical and applied optics, theory of photographic process, theory of radiolocation, etc., and each of them investigates some specific signals and specific signal transformations.

Two opposite tendencies can be traced in development of these sciences: further *specialization* and *generalization*.

This tutorial text is a step in the direction of *generalization*. In spite of different physical properties of signals as physical objects we emphasize their common features as *signals* and emphasize common features of their physical transformations as *transformations of signals*. We try to put in order the experience accumulated in this field and to supplement it with physical and mathematical reasoning.



**Fig. I.2.** Basic conception of this book: supposedly, primary multidimensional signals are perceived and pre-processed by a technical information system; output signals of the system are perceived, processed at higher levels and practically used by the human (system user)

## I.2 Black Box Paradigm

Our investigation of physical signal transformations is based on fundamental *black box* paradigm. Any signal pre-processing system, or a subsystem of more complicated system, or a smaller part of the subsystem can be represented as a