ADAPTIVE SYSTEMS IN CONTROL & SIGNAL PROCESSING 1983

Edited by
I. D. LANDAU
M. TOMIZUKA
and
D. M. AUSLANDER

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FOREWORD

The first IFAC workshop dedicated to the field of adaptive systems was held in San Francisco, USA on June 20-22, 1983. It was initiated by the Working Group on Adaptive Systems, which is part of the IFAC Technical Committee on Theory. The organization of this workshop was motivated by the important developments that have taken place in this field in the last few years. We should note that besides the theoretical aspects of the research (and at least in part because of the intense theoretical activity) the number of applications of adaptive control is growing, and this attracts more people from the general community to this field. On the other hand, the connections between adaptive signal processing and adaptive control have also been emphasized recently. For this reason, the workshop has hosted a number of contributions in the area of adaptive signal processing.

The workshop was organized around five main topics:

- New adaptive control algorithms
- Multivariable adaptive control
- Robustness of adaptive control
- Adaptive signal processing
- Applications of adaptive control

Ten contributions addressing topics of general interest were presented in the plenary sessions, and three round tables were organized. Summaries of the round table discussions are included in these Proceedings.

The Editors

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ROBUSTNESS OF ADAPTIVE CONTROL ALGORITHMS 2

ADAPTIVE CONTROL OF A CLASS OF LINEAR TIME VARYING SYSTEMS

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Abstract. The key contribution of the paper is to develop a new and explicit characterisation of the concept of persistency of excitation for time invariant systems in the presence of possibly unbounded signals. The implication of this result in the adaptive control of a class of linear time varying systems is also investigated. Simulation results are presented comparing alternative algorithms for the adaptive control of time varying systems.

Keywords. Adaptive control, time varying systems, identifiability, least-squares estimation.

1. INTRODUCTION

One of the prime motivations for adaptive control is to provide a mechanism for dealing with time varying systems. However, to-date, most of the literature deals with time invariant systems, see for example, Feuer and Morse (1978), Narenda and Valavani (1978), Goodwin, Ramadge and Caines (1980, 1981), Morse (1980), Narenda and Lin (1980), Egardt (1980), Landau (1981), Goodwin and Sin (1981), Elliott and Wolovich (1978), Kreisselmeir (1980, 1982).

Some of the algorithms with proven convergence properties for the time invariant case e.g. gradient type algorithms, are suitable, in principle, for slowly timevarying systems. However, other algorithms, e.g. recursive least squares, are unsuitable for the time varying case since the algorithm gain asymptotically approaches zero. For the latter class of algorithms various ad-hoc modifications have been proposed so that parameter time variations can be accommodated. One approach (Astrom, et. al., 1977, Goodwin and Payne, 1977) is to use recursive least squares with exponential data weighting. Various refinements (Astrom (1981) and Wittenmark and Astrom (1982)) of this approach have also been proposed to avoid burst phenomena e.g. by making the weighting factor a function of the observed prediction error (Fortescue, Kershenbaum and Ydstie, 1981).

The basic consequence of using exponential data weighting is that the gain of the least squares algorithm is prevented from going to zero. A similar end result can be achieved in other ways, for example, by resetting the covariance matrix (Goodwin et. al., 1983); by adding an extra term to the covariance update (Vogel and Edgar, 1982); or, by using a finite or oscillating length data window (Goodwin and Payne, 1977).

Another formulation that has been suggested by several authors (Weislander and Wittenmark, 1979) is to model the parameter time variations by a state-space model and then to use the corresponding Kalman filter for estimation purposes. This again corresponds to adding a term to the covariance update. It has also been suggested that some of the algorithms can be combined (Wittenmark (1979)).

Many of the above algorithms, tailored for the time varying case, have been analyzed in the time invariant situation. This is a reasonable first step since one would have little confidence in an algorithm that was not upwards compatible to the latter case. For example, Cordero and Mayne (1981), have shown that the variable forgetting factor one-step-ahead algorithm of Fortescue et. al. (1981) is globally covergent in the time invariant case provided the weighting factor is set to one when the covariance exceeds some prespecified bound. Similar results have been established by Lozano (1982, 1983) (who uses exponential data weighting where the weighting is made a function of the eigenvalues of the covariance matrix) and by Goodwin, Elliott and Teoh (1983) (who use covariance resetting).

With robustness considerations in mind, Anderson and Johnson (1982) and Johnstone and Anderson (1982b) have established exponential convergence, subject to a persistent excitation condition, of various adaptive control algorithms of the model reference type. These results depend explicitly on the stability properties proved elsewhere (e.g. Goodwin, Ramadge and Caines (1980)) for these algorithms in the time invariant case. The additional property of exponential convergence has implications for time varying systems since it has been shown (Anderson and Johnstone (1983)) that exponential convergence implies tracking error and parameter error boundedness when the plant parameters are actually slowly time varying.

For stochastic systems, Caines and Chen (1982) have presented a counterexample showing no stable control law exists when the parameter variations are an independent process. However, if one restricts the class of allowable parameter variations, then it is possible to design stable controllers for example, Caines and Dorer (1980) and Caines (1981) have established global convergence for a stochastic approximation adaptive control algorithm when the parameter variations are modelled as a (convergent) martingale process having bounded variance. Some very preliminary results have also been described (Hersh and Zarrop (1982) for cases when the parameters undergo jump changes at prespecified instants.

In the current paper we make a distinction between jump and drift parameters. "Jump parameters" refers to the case where the parameters undergo large variations infrequently whereas "drift parameters" refers to the case where the parameters undergo small variations frequently.

In section 2, we will develop a new "persistent excitation" condition for systems having possibly unbounded signals. An important aspect of this result is that it does not rely upon first establishing boundedness of the system variables as has been the case with previous results on persistent excitation (see for example Anderson and Johnson (1982)). The result uses a different proof technique but was inspired by a recent proof of global stability for a direct hybrid pole assignment adaptive control algorithm (Elliott, Cristi and Das (1982)).

In the latter work a two-time-frame estimation scheme is employed such that the parameters are updated at every sample point but the control law parameters are updated only every N samples. A similar idea is explicit in Goodwin, Teoh and McInnis (1982) and implicit in Johnstone and Anderson (1982a). We shall also use two-time-frame estimation here and show that this leads to a relatively simple new result on persistency of excitation with possibly unbounded feedback signals.

We will show in section 3 that the new persistent excitation condition allows one to establish global exponential convergence of standard indirect adaptive pole-assignment algorithms in the time invariant case. In section 4 and 5 we discuss the qualitative interpretation of these results for jump and drift parameters respectively. In section 6, we present some simulation studies and give comparisons of different algorithms for time varying adaptive control.

2. A NEW PERSISTENCY OF EXCITATION CONDITION

We shall consider a single input, single output system described as follows:

$$y(t) = -a_1(t)y(t-1)-a_2(t)y(t-2) \dots -a_n(t)y(t-n) + b_1(t)u(t-1)+\dots+b_n(t)u(t-n) \qquad (2.1)$$

Note that in the above model the parameters depend upon time. In the time invariant case, the model simplifies to the standard deterministic autoregressive moving average model of the form:

$$A(q^{-1})y(t) = B(q^{-1})u(t) \qquad (2.2)$$

where q^{-1} denotes the unit delay operator, and $A(q^{-1})$, $B(q^{-1})$ are polynomials of order n.

The model (2.1) can also be expressed in various equivalent forms. For example we can write

$$A(t,q^{-1})y(t) = B(t,q^{-1})u(t) \qquad (2.3)$$

The model (2.1) can also be expressed in regression form as

$$y(t) = \phi(t-1)^T\theta(t) \qquad (2.4)$$

where

$$\phi(t-1)^T = [-y(t-1),\dots,-y(t-n),u(t-1),\dots u(t-n)] \qquad (2.5)$$

$$\theta(t)^T = [a_1(t),\dots,a_n(t),b_1(t),\dots,b_n(t)] \qquad (2.6)$$

For the moment, we will restrict attention to the time invariant case and state a key controllability result. We shall subsequently use this controllability result to develop a persistency of excitation condition for use in adaptive control.

We first note that in the time invariant case, the regression vector $\phi(t)$ defined in equation (2.5) satisfies the following state space model:

$$\phi(t) = \left[\begin{array}{cccc|cccc} -a_1 & \dots & \dots & -a_n & b_1 & \dots & \dots & b_n \\ 1 & & & & & & & \\ & \ddots & & & & & 0 & \\ & & 1 & 0 & & & & \\ \hline 0 & \dots & \dots & 0 & 0 & \dots & \dots & 0 \\ & & & & 1 & & & \\ & 0 & & & & \ddots & & \\ & & & & & & 1 & 0 \end{array}\right] \phi(t-1) + \left[\begin{array}{c} 0 \\ 0 \\ \vdots \\ 0 \\ \hline 1 \\ 0 \\ \vdots \\ 0 \end{array}\right] u \qquad (2.7)$$

$$\triangleq F\,\phi(t-1) + G\,u(t) \qquad (2.8)$$

If we define $x(t)$ as $\phi(t-1)$, then we note that we can use the model (2.8) to construct the following non-minimal 2n dimensional state space model for $y(t)$:

$$x(t+1) = F\,x(t) + G\,u(t) \qquad (2.9)$$

$$y(t) = H\,x(t) \qquad (2.10)$$

where $H = [1\ 0 \dots 0]F = [-a_1,\dots -a_n, b_1,\dots b_n]$ (2.11)

It can be verified that the model (2.9), (2.10) is not completely observable. However, the following new result shows that the model (2.9) is completely reachable provided $A(q^{-1})$, $B(q^{-1})$ are relatively prime.

Lemma 2.1 (Key Controllability Lemma). The 2n dimensional state space model (2.7) for the vector $\phi(t)$ is completely reachable if and only if $A(q^{-1})$, $B(q^{-1})$ are relatively prime.

Proof: For details see Goodwin and Teoh (1983). ∇∇∇

The importance of the above lemma in the context of persistent excitation is that it shows that the vector $\{\phi(t)\}$ is 'controllable' from u(t) and thus one might expect that $\{u(t)\}$ can be chosen so that $\{\phi(t)\}$ spans the whole space. This is in accordance with one's intuitive notion of the concept of persistency of excitation. Concrete results of this nature will be presented below.

When the parameters of the system are known and time invariant, then the closed loop poles can be arbitrarily assigned by determining the input from (see for example (Kailath (1980), Goodwin and Sin (1983)):

$$L(q^{-1})\, u(t) = -P(q^{-1})y(t) + v(t) \qquad (2.12)$$

where $L(q^{-1})$, $P(q^{-1})$ are unique polynomials of order (n-1) and $\{v(t)\}$ is an arbitrary external input.

The feedback control law (2.12) can equivalently be written in terms of the vector $\phi(t)$ as

$$u(t) = -K\phi(t-1) + v(t) \qquad (2.13)$$

where

$$K = [p_1-a_1p_0,\ldots,p_{n-1}-a_{n-1}p_0,-a_np_0,\ell_1+b_1p_0,\ldots,\ell_{n-1}+b_{n-1}p_0,b_np_0] \qquad (2.14)$$

With two-time frame estimation in mind, we shall assume that the feedback law (2.13) is held constant over an interval $I(t_0) = [t_0,t_0+N-1]$ and analyze the minimum eigenvalue of $X(t_0)^TX(t_0)$ where

$$X(t_0+1)^T = [\phi(t_0+1),\ \phi(t_0+2),\ldots,\ \phi(t_0+N)] \qquad (2.15)$$

We now have the following new result on persistency of excitation:

Theorem 2.1 (Persistency of Excitation) Consider the system (2.7) and the feedback control law (2.13), then provided

(i) $A(q^{-1})$, $B(q^{-1})$ are relatively prime

(ii) the feedback law (2.13) is constant over the interval $I(t_0) = [t_0,t_0+N-1]$

(iii) the external input, v(t), is of the form:

$$v(t) = \sum_{k=1}^{s} \Gamma_k \sin(\omega_k t+\sigma_k) \qquad (2.16)$$

where $\omega_k \in (0,\pi)$; $\Gamma_k \neq 0$ and $\omega_j \neq \omega_k$; $k=1,\ldots,s$; $j=1,\ldots,s$

(iv) The length of the interval, N, and the number of sinusoids, s, satisfy

(a) $N \geq 10n$ (2.17)

(b) $s \geq 4n$ (2.18)

where n is the order of the system we have

$$\lambda_{min}[X(t_0+1)X(t_0+1)^T] \geq \varepsilon_1 > 0 \qquad (2.19)$$

where ε_1 is independent of t_0 and the initial conditions $\phi(t_0)$.

Proof: See Goodwin & Teoh (1983) for details. ∇∇∇

The above theorem makes precise the intuitive notion of persistency of excitation introduced earlier. Note that the theorem depends upon the Key Controllability Lemma (Lemma 2.1). As far as the authors are aware, this is the first general persistency of excitation result which does not depend upon an a-priori uniform boundedness condition on the system response. In the next section we show how the above result can be used in a straightforward fashion to establish convergence of an indirect pole-assignment adaptive control algorithm in the time invariant case.

3. CONVERGENCE OF A POLE ASSIGNMENT ALGORITHM IN THE TIME INVARIANT CASE

Here we shall consider an indirect pole-assignment adaptive control law using a two-time frame estimator in the linear time invariant case.

The system will be assumed to satisfy (2.2) subject to the following assumptions:

Assumption A: $A(q^{-1})$, $B(q^{-1})$ are relatively prime.

Assumption B: The order n is known.

Let N and s be chosen as to satisfy equations (2.17), (2.18) and let ξ be a prespecified arbitrary integer. Then, the two-time frame adaptive control algorithm is:

(i) Parameter Estimation Update (Least Squares)

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{P(t-2)\phi(t-1)e(t)}{1+\phi(t-1)^TP(t-2)\phi(t-1)}$$

$$e(t) = y(t) - \phi(t-1)^T\hat{\theta}(t-1); \quad t = 1,2\ldots \text{ and } \hat{\theta}(0) \text{ given.} \qquad (3.1)$$

(ii) Covariance Update with Resetting

$$P'(t-1) = P(t-2) - \frac{P(t-2)\phi(t-1)\phi(t-1)^TP(t-2)}{1+\phi(t-1)^TP(t-2)\phi(t-1)} \qquad (3.2)$$

If $\frac{t}{\xi N}$ is an integer

Then resetting occurs as follows:

$$P(t-1) = \frac{1}{k_0}I \quad ; \quad 0 < k_0 < \infty \qquad (3.3)$$

Else

$$P(t-1) = P'(t-1) \qquad (3.4)$$

(iii) Control Law Update (in the Second Time Frame)

(a) If $\frac{t}{N}$ is an integer
Then evaluate

$$\hat{A}(t,q^{-1}) = 1 + \hat{\theta}_1(t)q^{-1} + \ldots \hat{\theta}_n(t)q^{-n} \quad (3.5)$$

$$\hat{B}(t,q^{-1}) = \hat{\theta}_{n+1}(t)q^{-1} + \ldots + \hat{\theta}_{2n}(t)q^{-n} \quad (3.6)$$

Solve the following equation for $\hat{L}(t,q^{-1})$, $\hat{P}(t,q^{-1})$, each of order (n-1):

$$\hat{A}(t,q^{-1})\hat{L}(t,q^{-1}) + \hat{B}(t,q^{-1})\hat{P}(t,q^{-1}) = A^*(q^{-1}) \quad (3.7)$$

where $A^*(q^{-1})$ is an arbitrary stable polynomial.

[In the event that $\hat{A}(t,q^{-1})$, $\hat{B}(t,q^{-1})$ are not relatively prime, then $\hat{L}(t,q^{-1})$ and $\hat{P}(t,q^{-1})$ can be chosen arbitrarily].

(b) Else put

$$\hat{L}(t,q^{-1}) = \hat{L}(t-1,q^{-1}); \ \hat{P}(t,q^{-1}) = \hat{P}(t-1,q^{-1}) \quad (3.8)$$

(iv) Evaluation of the input

$$\hat{L}(t,q^{-1})u(t) = -\hat{P}(t,q^{-1})y(t) + v(t) \quad (3.9)$$

where $\{v(t)\}$ is as in (2.16)

We now have the following covergence result.

Theorem 3.1 Consider the algorithm (3.1) to (3.9) applied to the system (2.2) subject to assumption (A) and (B), then $\hat{\theta}(t)$ approaches the true value, θ_0, exponentially fast and $\{u(t)\}$, $\{y(t)\}$ remain bounded for all time.

Proof: Straightforward using the results of Theorem 2.1 and the Small Gain Theorem (Desoer and Vidyasagar (1975)). See Goodwin & Teoh (1983) for full details. ∇∇∇

The above algorithm uses iterative least squares with covariance resetting. Three points can be made about this procedure:

(i) If resetting is not used, then the algorithm reduces to ordinary recursive least squares. In this case and for time invariant problems, it can still be shown that $\hat{\theta}(t)$ converges to θ_0 but not exponentially fast.

(ii) It is essential to note that ordinary least squares can not be used in the time varying case since the gain of the algorithm goes to zero. However, our experience is that, even for time invariant problems, resetting is helpful since it captures the rapid initial convergence of least squares without having the slow asymptotic convergence that is well known for ordinary least squares.

(iii) In the above analysis, we have reset to a scaled value of the identity matrix. However, it can be seen that an identical result is achieved if the resetting is made to any matrix, P, satisfying:

$$\lambda_{max}(P^{-1}) < \lambda_{min}(P^{-1}) + \varepsilon_1\xi \quad (3.10)$$

In particular, one could reset to $\frac{\varepsilon_1\xi}{2\mathrm{trace}P'(t-1)^{-1}}[P'(t-1)^{-1}]$.

This satisfies (3.10) and has the advantage (Lozano (1982)) that the directional information built up in $P(t-1)^{-1}$ is retained.

4. JUMP PARAMETERS

In the literature (see for example Wittenmark, 1979) two types of parameter variation have been considered, namely, strongly time varying (or jump parameters) and slowly time varying (or drift parameters). This classification is helpful in discussing the convergence properties. We shall treat the former case in this section and the latter in the next section.

For our purposes we shall define jump parameters as follows:

Definition 4.1: The parameters, $\theta(t)$, in the model (2.1) are jump parameters (having jumps at $\{t_i : t_i > t_{i-1}, i=0,1,\ldots,\infty\}$) if

(a) $\theta(t) = \theta_i$ for $t_i \le t < t_{i+1}$ (4.1)

(b) $\underset{i}{\mathrm{Min}}|t_i - t_{i-1}| = t_{min}$ (4.2)

(c) $\theta_i \in M$ a bounded set. (4.3) ∇∇∇

Jump parameters are often a realistic model in practical cases especially when nonlinear systems are approximated by linear models at different operating points. Then an abrupt change in operating point gives a jump change to the parameters in the linear model. This type of time varying model has been the subject of several recent papers (Wittenmark, 1979; Wieslander and Wittenmark, 1971; Fortescue et. al., 1981 and Vogel and Edgar, 1982).

For the purpose of adaptive control, we shall further constrain the set of possible parameter values as follows:

Assumption C: For all possible parameter values, θ_i, the corresponding pair $A(q^{-1})$, $B(q^{-1})$ are relatively prime and the magnitude of the determinant of the associated eliminant matrix is bounded below by a constant independent of t. ∇∇∇

We now discuss the qualitative performance characteristics of the adaptive control algorithm (3.1) to (3.9) when applied to systems having jump parameters. Our key purpose is to indicate the kind of information necessary to ensure that the system input and outputs remain bounded.

When a jump occurs, the system response may begin to diverge. However, there is a maximum rate at which this can occur in view of (4.3). Moreover, we know from section 3, that since the parameters are constant between jumps, there exists a finite time N_ϵ such that $\hat{\theta}(t)$ will be within an ϵ neighbourhood of $\theta_0(t)$ and hence one can re-establish stabilizing control . Now, provided a sufficiently long period passes before the next jump occurs, then the response will be brought back to its original magnitude. (If insufficient time is allowed between jumps then it is easy to construct examples such that the response builds up even though a stabilizing controller is found between the jumps).

It is possible to compute an expression for the minimum time between jumps, t_{min}, in terms of the following quantities so that $\{u(t)\}$, $\{y(t)\}$ remain bounded

(i) The diameter of the set M.
(ii) The lower bound on the elimenant matrix in assumption C.
(iii) The constants k_o, N, s, ξ in the algorithm of section 3.
(iv) The precise nature of $A^*(q^{-1})$.

The explicit expression for t_{min} is complicated (Teoh (1983)) and offers little extra insight.

One practical point worth noting is that it is not necessary to apply the external input for all time, instead it suffices to add this signal for a period N_ϵ after a jump has occurred. The idea of adding an external signal for a finite period when changes in the plant are perceived has been suggested by other authors, e.g. Vogel and Edgar (1982). For chemical plants, etc., it is generally not desirable to impose additional inputs continuously during steady conditions. However, the procedure suggested here only injects the external signal when unsteady conditions arise from other sources, e.g. plant time variations.

Note that we also have assumed that the order of the system remains unchanged during jumps. This assumption is certainly restrictive but to handle more general situations would require an on-line order determination as part of the algorithm. This would lead to additional considerations well beyond the scope of the current paper.

5. DRIFT PARAMETERS

For our purposes we shall define drift parameters as follows:

Defintion 5.1 The parameters, $\theta(t)$, in the model (2.1) are drift parameters if

(a) $||\theta(t) - \theta(t-1)|| < \delta$ (5.1)

(b) $\theta(t) \in M$ a bounded set (5.2)

We shall also require the following additional assumption:

Assumption D: For each fixed t, $A(q^{-1},\theta(t))$, $B(q^{-1}, \theta(t))$ are relatively prime ∇∇∇

Note that assumption D is necessary to ensure that the system does not drift into a region where the order changes. As pointed out in the previous section, the more general situation, though interesting, involves considerably more complexity. A similar assumption to D appears in other papers in this general area (see for example Anderson and Johnstone (1983)).

We now investigate the qualitative behaviour of the algorithm of section 3 when applied to the drift parameter case. Since, we have established exponential convergence in the time invariant case then we can argue as in Anderson and Johnstone (1983) to conclude that stablity is retained in the time varying case provided δ in (5.1) is smaller than some fixed number depending on the size of the initial parameter error. Note the role played by exponential convergence in making this claim.

6. SIMULATION STUDIES

Extensive simulation studies of the adaptive control algorithm described above have been carried out together with comparisons with exponentially weighted least squares and gradient algorithms. In this section, we present a summary of the results obtained.

(i) The best algorithm overall appears to be recursive least squares with covariance resetting as described in section 3.
(ii) The algorithm of section 3 is relatively insensitive to the resetting period, though we have found that in the case of jump parameters it is helpful to monitor the prediction error and reset when this value exceeds some threshold. In the case of drift parameters we have found that there exists an optimal resetting interval.
(iii) Recursive least squares with exponential data weighting is highly sensitive to the choice of the weighting factor λ and performs extremely poorly for all λ in the case of drift parameters.
(iv) Gradient schemes are simple but converge extremely slowly and are therefore unsuitable for all but very slowly varying systems.

Typical simulation results are as shown in Figures 6.1 and 6.2 for a system having sinusoidally varying parameters and set point variation as in Fig. 6.1a. Figure 6.1 shows the excellent performance of the covariance resetting scheme. Fig. 6.2 shows the poor performance of the exponential weighted least squares algorithm for the same problem. (Note that for the results in Fig 6.2 the best value of λ was chosen!)

7. CONCLUSIONS

This paper has presented results in the adaptive control of linear time invariant and time varying systems. The key result is a new persistent excitation condition for systems having non-uniformly bounded signals. The implication of this result