BASIC CIRCUIT ANALYSIS

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PREFACE

The study of electric circuits or networks occupies a unique and important position in the curriculum. For the student, it is often the first true engineering course, in the sense that more emphasis is placed on practical applications than in such courses as physics and mathematics. The concepts, problem formulations, and solution techniques of circuit analysis apply in general to any lumped parameter dynamic system. Consequently, the mastery of these principles is a valuable asset to most applied scientists and engineers. It is particularly important for the student to realize that although the systems considered in this book happen to be electric circuits, the material does have other useful engineering applications. The circuit courses also provide an introduction to some of the properties of electronic and electromechanical devices and to the underlying electric and magnetic field effects that govern the operation of electrical apparatus.

All electrical phenomena involve either the dissipation or the storage of energy, concepts that are generally familiar to a beginning student from physics. Therefore, it seems natural to approach the development of the definitions and procedures associated with the formulation and solution of circuit problems in a logical and intuitive way by emphasizing the effects of energy. Accordingly, as a starting point, we define the ideal circuit elements in terms of the particular energy process they represent, and we introduce Kirchhoff's laws in relation to the principle of energy conservation.

The mathematical background required for this book includes a basic understanding of the differential and integral calculus of algebraic, trigonometric, and exponential functions, along with the geometric properties of these functions. Specifically, no prior competence in the solution of differential equations is assumed. The typical student will be studying mathematics concurrently. Our goal is to complement these efforts by relating mathematical techniques to physical applications. Personal experience has taught us that this greatly enhances the appreciation of mathematics by engineering students. Since the systems considered in this book are governed by differential equations, our efforts provide a natural introduction to the properties of differential equations in a setting that is highly motivating to the student.

We expect that a student using this book will have completed one or more physics courses in which the concepts of current and voltage have been discussed. However, we cover these quantities in Chapter 1, because a double exposure to such basic ideas is not excessive. Moreover, by not drawing directly on background acquired elsewhere, we are able to present a unified view and establish a common framework for the development of network principles.

We are fortunate to have at our disposal powerful computing aids such as the hand calculator, particularly the programmable type. These devices virtually eliminate the tedium associated with network calculations and free us to undertake elaborate analysis and design problems that will yield results of great accuracy with minimum effort. Consequently, detailed examples using practical element values are entirely appropriate and are used throughout to develop a realistic sense of magnitude. Although many of the problems and much of the numerical work in the book involve practical element values, it is sometimes desirable not to obscure the development of the theoretical principles with unnecessary computational details. In such circumstances we follow the well-established practice of using mathematically simple, though perhaps unrealistic, element values and frequencies. In most instances, students will be concurrently enrolled in a laboratory course, which will help them become even more familiar with practical element values.

We have found that the simple approach to solving differential equations (by assuming the solution in the form of e^{st}) has important advantages over the use of the Laplace transform technique. The exponential form of solution is a natural method that the beginning student may easily learn. It keeps the physical aspects of the problem in full view and makes it easy to proceed on the basis of a few fundamental principles. We take advantage of the exponential solution to provide a simple introduction to the transfer function and its properties, including the concepts of poles and zeros. These ideas are introduced at an early point and are used throughout the remainder of the book as an alternative to dealing with differential equations.

Throughout the text, examples provide immediate applications of concepts and techniques as they are developed. We consider these example problems and the end-of-chapter exercises to be of the utmost importance to students in their efforts to learn the material presented in the text. We have included exercises of varying difficulty intended both to illustrate basic ideas and techniques and to stimulate additional study. In some instances extensive justification of statements made in the text is delayed until the exercise problems, thereby providing the opportunity for the student to supply some of the omitted details. Occasionally, we use an exercise problem to suggest ways of extending ideas introduced in the chapter.

The order of topics is a reasonable compromise between the logical development of network principles and the immediate need for this material in other courses, such as electronic circuits. Chapter 1 introduces the Kirchhoff laws and the ideal circuit elements. The responses of the elements and their properties are illustrated by means of a series of worked-out examples. Chapter 2 is devoted to resistive and diode networks. We direct considerable attention to equivalence of networks and the theorems of Thévenin and Norton.

In Chapter 3 we introduce the natural response and define poles of a network. After a detailed study of first- and second-order systems, we discuss the impulse function and show that the response to an impulse source is a natural response. Chapter 4 brings us to the forced response and the transfer function concept. Here we illustrate the equivalences among the differential equation, the transfer function, and the impulse response.

In Chapter 5 we present a detailed study of the sinusoidal steady state in terms of phasor voltages and currents. We consider a variety of practical applications and techniques, bringing us to the end of the beginning. The remaining chapters are concerned with enlarged discussions and generalizations of material introduced in the first five. Chapter 6 covers power system topics, including three-phase circuits. Chapter 7 considers extensions of the concepts associated with the sinusoidal steady state including resonance, frequency response curves, and items of particular interest in communications and control systems.

In Chapter 8 we develop general formulations of the Kirchhoff law equations and introduce matrices and matrix algebra to facilitate this process. Chapter 9 contains a discussion of the major network theorems and a demonstration of various network properties. The last chapter is devoted to two-port networks, including the two-winding mutual inductor. We discuss various parameter systems and equivalent circuits. There are very few constraints on the order of presentation of material in Chapter 6 through Chapter 10, leaving instructors free to adapt the book to their particular needs.

We are indebted to many individuals for their cooperation in bringing this project to a conclusion. Our families have provided much patience, understanding, and encouragement. We are especially grateful to Professor Daniel O'Keefe, who read the entire manuscript and made many valuable suggestions. Our students have contributed more than they realize to this book by their struggles with various preliminary editions and by the many discussions provoked by their questions. In particular we thank Mr. Armando Garcia, who was especially helpful for his review of all the exercise problems, and Ms Mary Girard for her review of several chapters.

It is difficult to express adequately our gratitude to Ms Irene Asprelli who did all of the typing on this project from its inception through countless revisions and corrections to the final manuscript. Her good cheer, willing attitude, and professional competence were essential elements in this effort.

Finally, we acknowledge with gratitude the following reviewers: Professors Shlomo Karni, University of New Mexico; David R. Fannin, University of Missouri; and S. Louis Hakimi, Northwestern University.

G. J. K.
S. E. G.

CONTENTS

ONE

BASIC CONCEPTS — NETWORK ELEMENTS

There are three fundamental effects involving energy transformations, or exchanges, that occur in every electrical system: the dissipation of energy, the storage of energy in magnetic fields, and the storage of energy in electric fields. In the study of electric networks, these energy exchanges are examined through the use of idealized network elements that are defined to take into account the essential character of the effects.

All actual devices available to the electrical engineer, such as transistors, inductors, capacitors, batteries, and so forth, simultaneously involve all the energy exchanges to some extent. In other words, every real device in an electrical system will dissipate energy and store energy in an associated magnetic field, and an associated electric field. As a result, such a device may exhibit a complex behavior. To understand this behavior, the overall operation of the device must be determined by separating the entwined energy effects. This approach requires the introduction of certain network or circuit elements, each of which is defined to have only a single energy transformation. Since no particular device may exhibit a single energy effect to the total exclusion of all others, the network elements are idealizations and have no physical existence. We refer to them as ideal network elements; they remind us of other idealizations used in the study of physical phenomena, including a point mass, a rigid body, an ideal gas, and a linear spring. The ideal network elements will be defined in this chapter.

An electric network or circuit is an interconnection of ideal elements whose behavior is generally specified in terms of the network variables, namely, the voltages and currents associated with each element. The energy effects are readily determined from the network voltages and currents.

In this book we are concerned with the analysis of electric networks based on models or equivalent circuits of the actual devices available to electrical engineers. We begin by defining the ideal elements and relating them to the actual devices they model. It should be clear that accurate modeling of real devices is crucial to the success of this activity.

1-1 NETWORK VARIABLES—CURRENT, VOLTAGE, AND POWER

Electric charge occurs in nature in two forms: positive and negative charge. A basic physical particle is the electron, which has a negative charge and a small mass. All matter consists of vast numbers of elementary particles, including electrons. Generally, we find that a material body is electrically neutral; that is, the negative charge of the electrons in the body is balanced by an equal amount of positive charge on the other particles in the body. There are many ways of removing electrons from an electrically neutral body, which when done, leaves the body with a net positive charge equal in magnitude to the charge on the electrons that have been taken away.

There are many situations in which the distribution of charge throughout a region can be changed. Such an event is almost always the result of the motion of electrons that carry negative charges. On the other hand, positively charged particles are generally much more massive than electrons, so positive charges are relatively motionless. Practical devices derive their usefulness and properties from the motion of electrons.

On the scale of magnitudes appropriate to the study of electric networks, the amount of charge on the electron is inconveniently small. Consequently, the basic quantity of charge, the coulomb (in the SI unit system), is equivalent to the charge on 6.2×10^{18} electrons.[1] The student should remember, however, that electrons carry negative charge. Thus, 6.2×10^{18} electrons have an aggregate charge of -1 coulomb.

Electric charges interact with one another much as masses interact. Two charges in the same vicinity will exert a mutual force on one another, as indicated in Fig. 1-1. The main features of the force interaction between charged particles are contained in Coulomb's law, Eq. (1-1).

$$F = K\frac{Q_1Q_2}{r^2} \tag{1-1}$$

Note that the force varies with the inverse square of the distance separating the charges and that the force is proportional to the product of the charge

[1]All quantities in this text will be specified in Système International, or SI, units.

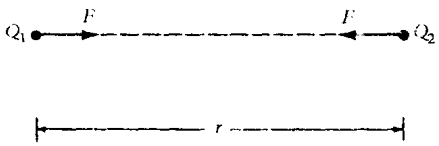

FIGURE 1-1

strengths Q_1Q_2. Finally, the force is one of attraction if Q_1 and Q_2 are of opposite sign; and it is the force of repulsion when the charges are of the same sign, that is, both are negative or both are positive.

Charges in motion constitute an electric current. A common way of establishing a current in a conductor such as a copper wire is to connect a chemical battery to the ends of a wire (Fig. 1-2). The strength of the current is defined as the time rate of transfer of charge along the conductor. Thus we have

$$i = \frac{dq}{dt} \tag{1-2}$$

where i is the current in amperes, q is the charge in coulombs, and t is the time in seconds. In other words, a current of one ampere will transfer charge at a rate of one coulomb per second.

The direction of the current is defined as being that of an equivalent displacement of positive charge. Thus, if all the charge carriers in the conduction process are positively charged, then the current direction is the same as the motion of these particles. On the other hand, if the charge carriers are negative, such as electrons, then the direction of the current is opposite to that of these particles. This last current direction coincides with that of an equivalent positive charge displacement. Finally, if there are just as many units of positive charge moving to the right as there are units of negative charge moving to the right, in the same interval of time, then the net

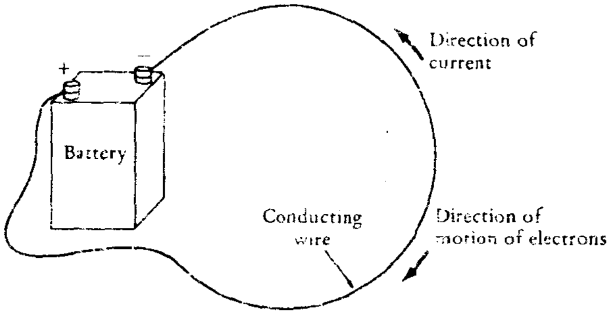

FIGURE 1-2

charge displacement is zero and so is the current. Throughout the rest of the book we will not be concerned with the motion of the actual charge carriers but only with the direction of the corresponding current.

Coulomb's law [Eq. (1-1)] describes the force interaction between two charges. There are other situations in which a charge will experience a force. When an electric current flows through a battery, the charges in the current experience a combination of forces partly owing to the fixed charge distribution on the electrodes of the battery. They are also subject to forces of chemical origin, a direct consequence of the chemical reaction occurring in the battery. It is through these latter forces that chemical energy is converted to electric energy. A charge moving through a magnetic field will experience a force related to the strength of the magnetic field, the amount of the charge, and its velocity. This effect is the basis of electric motor and generator operation.

The student will recall that work (or energy) is involved whenever a body in motion is subject to a force. From the previous discussion it is clear that electric currents, which are streams of charges in motion, will generally be accompanied by an expenditure of work when the moving charges are subject to forces. The work involved in moving a charge between two points is conveniently described in terms of the concept of voltage.

Consider the situation depicted in Fig. 1-3, in which a charge q is moved along the path indicated from point A to point B. Let us assume that as this charge moves along the path, it is subject to a force, indicated by f in Fig. 1-3. Our interest centers on the charge and the work expended by the force; we are not concerned at this point with the origin of the force. In general, the force may vary in direction and magnitude from point to point along the path. Usually the total work involved will be directly proportional to the amount of charge moving from A to B. The voltage associated with the path between A and B is defined as the work expended per unit of charge moved [Eq. (1-3)]. Thus we have

$$v = \frac{dW}{dq} \tag{1-3}$$

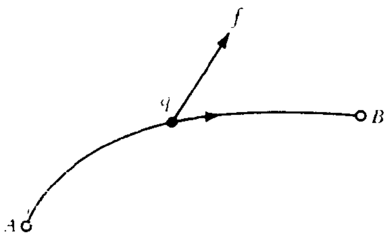

FIGURE 1-3

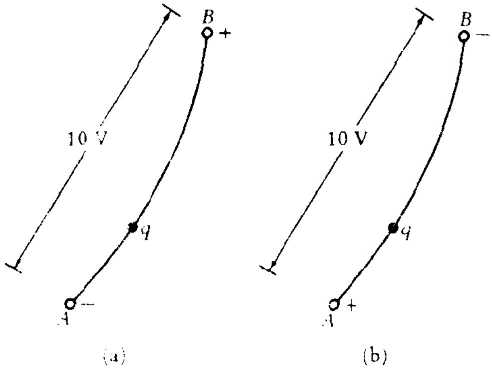

FIGURE 1-4

where v is the voltage in volts, W is the work in joules, and q is the charge in coulombs. In summary, the voltage along a path between points A and B is a measure of the work involved in moving charge over this path. Note that the concept of voltage involves two points and a particular path between these points.

In considering the voltage associated with a path, we must distinguish two possibilities. As a charge moves from A to B, the force exerted on the charge will have a net effect that either aids or opposes the motion of the charge along the path.[2] In this context, it is helpful to consider the analogy of a child in a wagon on a hill. If the wagon and child are near the top of the hill, then gravitational forces will, on the average, tend to pull the child downhill. However, if the child and wagon are at the bottom of the hill, then gravitational forces will oppose motion from the bottom to the top of the hill. An external agent, perhaps a willing parent, is required to overcome gravitational forces and pull the child up the hill. Clearly the child will gain energy when traveling from the bottom to the top, and conversely lose energy when going from top to bottom.

In general, in moving over a path from A to B, an electric charge will either gain or lose energy. To indicate that a charge gains energy in a displacement from A to B, we attach a plus sign (+) to B and a minus sign (−) to A, as in Fig. 1-4a. In other words, B is the "top of the hill." When the situation is reversed, that is, when a charge loses energy traveling from A to B, the plus sign is attached to A and the minus sign to B, and B is now the "bottom of the hill," as illustrated in Fig. 1-4b. In summary, in moving along a path toward the + or uphill end a positive charge will gain energy. It will lose energy in moving toward the − or downhill end of the path. Each of these conclusions is reversed for a negative charge.

[2]All charges in the present discussion are assumed to be positive unless explicitly stated otherwise.

TABLE 1-1

Abbreviations, Symbols, and Prefixes

Quantity	Unit
Admittance (Y)	mho (℧)
Angle (θ, α, and so forth)	degree or radian (° or rad)
Capacitance (C)	farad (F)
Charge (q, Q)	coulomb (C)
Conductance (G)	mho (℧)
Current (i, I)	ampere (A)
Cyclical frequency (f)	hertz (Hz)
Damping ratio (ζ)	none (dimensionless)
Dissipation factor (DF)	none (dimensionless)
Energy (W, W_m, W_e)	joule (J)
Force (f, F)	newton (N)
Impedance (Z)	ohm (Ω)
Inductance (L)	henry (H)
Logarithmic gain ratio	decibel (dB)
Magnetic flux (ϕ)	weber or volt-second (Wb or V · s)
Power (p, P)	watt (W)
Power factor (PF)	none (dimensionless)
Quality factor (Q, Q_0)	none (dimensionless)
Radian frequency (ω, ω_0)	radian/second (rad/s)
Reactance (X)	ohm (Ω)
Resistance (R)	ohm (Ω)
Susceptance (B)	mho (℧)
Time (t, T)	second (s)
Voltage (v, V)	volt (V)

Prefix	Abbreviation	Numerical Weight	Example
pico	p	10^{-12}	1 pF = 10^{-12} F
nano	n	10^{-9}	1 ns = 10^{-9} s
micro	μ	10^{-6}	1 μH = 10^{-6} H
milli	m	10^{-3}	1 ms = 10^{-3} s
kilo	k	10^{3}	1 kHz = 10^{3} Hz
mega	M	10^{6}	1 MHz = 10^{6} Hz

Note that a voltage of 10 volts is abbreviated 10 V, as in Fig. 1-4a. Similarly, a current of 10 amperes is designated 10 A, and a current of 10 milliamperes, that is, 0.01 amperes, as 10 mA. Table 1-1 lists the symbols and units of quantities and their abbreviations that will be used throughout the text.

It is instructive to consider the arrangement of Fig. 1-5, an automobile storage battery connected to a head lamp. We indicate a current of 5 A