

THE  
THEORY AND DESIGN  
OF  
INDUCTANCE COILS

V. G. WELSBY

# THE THEORY AND DESIGN OF INDUCTANCE COILS

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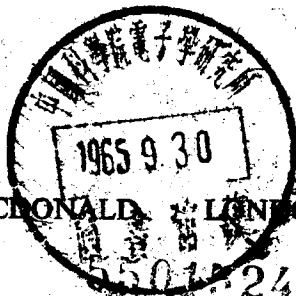
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WITH A FOREWORD BY

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## FOREWORD

The subject of inductor design is one which I know has interested Dr. Welsby for many years and to which he has devoted much thought. It is of considerable importance to telecommunication engineers. Whereas inductance coils may be designed for many purposes, by "rough and ready" methods, such methods will not suffice when the coils are to be included in telephone transmission equipment, particularly as filter elements or in measuring apparatus. For such applications, the coils must be designed and constructed to give a more closely specified performance; this is especially true with the modern methods of filter synthesis which are now coming into use.

To those whose task it is to design such coils, the information collected in this book should prove very valuable. Much of it may not be strictly new, but hitherto it had not been readily accessible in a form suitable for the coil designer, who is usually an engineer rather than a physicist. The author's personal contributions to the subject include much of Chapter IV and the work leading to the graphical methods of design described in Chapter XI.

W. G. RADLEY



## AUTHOR'S PREFACE TO FIRST EDITION

When the flow of electric current through a coil of wire is varied, the resulting change in the magnetic field surrounding the coil causes a voltage to be induced in the coil. The term "inductance coil" or "inductor" can be used to describe many practical applications of this principle, including, for example, the tuning coils used in radio apparatus, smoothing chokes for d.c. power supplies, the coupling and decoupling chokes used in valve amplifiers, control chokes for fluorescent lighting equipment, etc. The designer, requiring a coil to meet some particular specification, is faced with a bewildering array of possible types, from which he must select the one which is most suitable for the purpose, and then decide the correct size of wire and the number of turns to be used. In addition, if the coil is to have an iron core, the right material has to be chosen, and such details as lamination thickness, and air-gap length, fixed. It is to aid in the solution of problems of this nature that the present book has been prepared. Its object is to explain, in general terms, the underlying theory on which the design of inductance coils is based, and then to show how the theory can be simplified by the use of certain approximations and also to indicate to what extent these approximations are justified.

In this way it is possible to show how the performance of a given coil is influenced by its shape, size, core material, etc., and thus to show how its design should be changed to obtain any desired modification in its performance. The methods of designing various types of inductance coils are illustrated by numerical examples.

It should be pointed out that the terms "Mumetal", "Radiometal", etc., used to describe certain magnetic materials, are trade names which are used by courtesy of the manufacturers. The inclusion of curves showing some magnetic properties of these materials is intended mainly to illustrate theoretical points. Up-to-date values of permeability, loss coefficients, etc., obtainable with laminated, dust and ferrite cores, should be available from the manufacturers who supply the various materials.

In conclusion, the author would like to thank the Post Office Engineering Department for permission to use certain material which has been included in the book.

## AUTHOR'S PREFACE TO SECOND EDITION

The term "inductance coil" or inductor may be applied to a wide variety of coil types ranging for example from the large l.f. smoothing choke with many thousands of turns and a laminated iron core to the tiny h.f. tuning inductor consisting merely of one or two self-supporting loops formed in a piece of wire. One of the aims of this book is to explain the principles underlying the design of all types of inductors in terms which will be readily understood by engineers who are likely to be interested primarily in practical results rather than in a rigorous mathematical treatment of the subject. In the first edition, therefore, references to electromagnetic field theory were deliberately avoided because it was felt that it might be unfamiliar to some readers and might cause further confusion rather than a simplification of ideas. As a result of the development of micro-wave radio and of high-frequency techniques generally during the last decade, however, there is nowadays an increasing interest among electrical engineers in the principles of electromagnetic wave propagation. What is perhaps not so generally realized is that these principles can often be usefully applied not only to a solution of problems involving wave-guides and microwave aerials, but to enable a better understanding to be obtained of problems which have usually been approached only by means of classical circuit theory. In the present edition, therefore, an attempt has been made to show how electromagnetic field theory can sometimes be used to obtain a clearer picture of effects which are difficult to explain by simple circuit theory alone. Another change in this new edition is the use of the rationalized metre-kilogramme-second system of units. Some of the units used in the past were an unfortunate legacy from the days when electricity and magnetism were treated as two completely separate branches of science, each of which quite naturally had its own system of units defined in terms of the observed properties of electric and magnetic fields respectively. The rationalized M.K.S. system not only has the advantage that it removes awkward numerical factors from the basic equations, but the adoption of a comprehensive and consistent unit system also makes it easier to appreciate the essential interdependence of electric and magnetic field effects.

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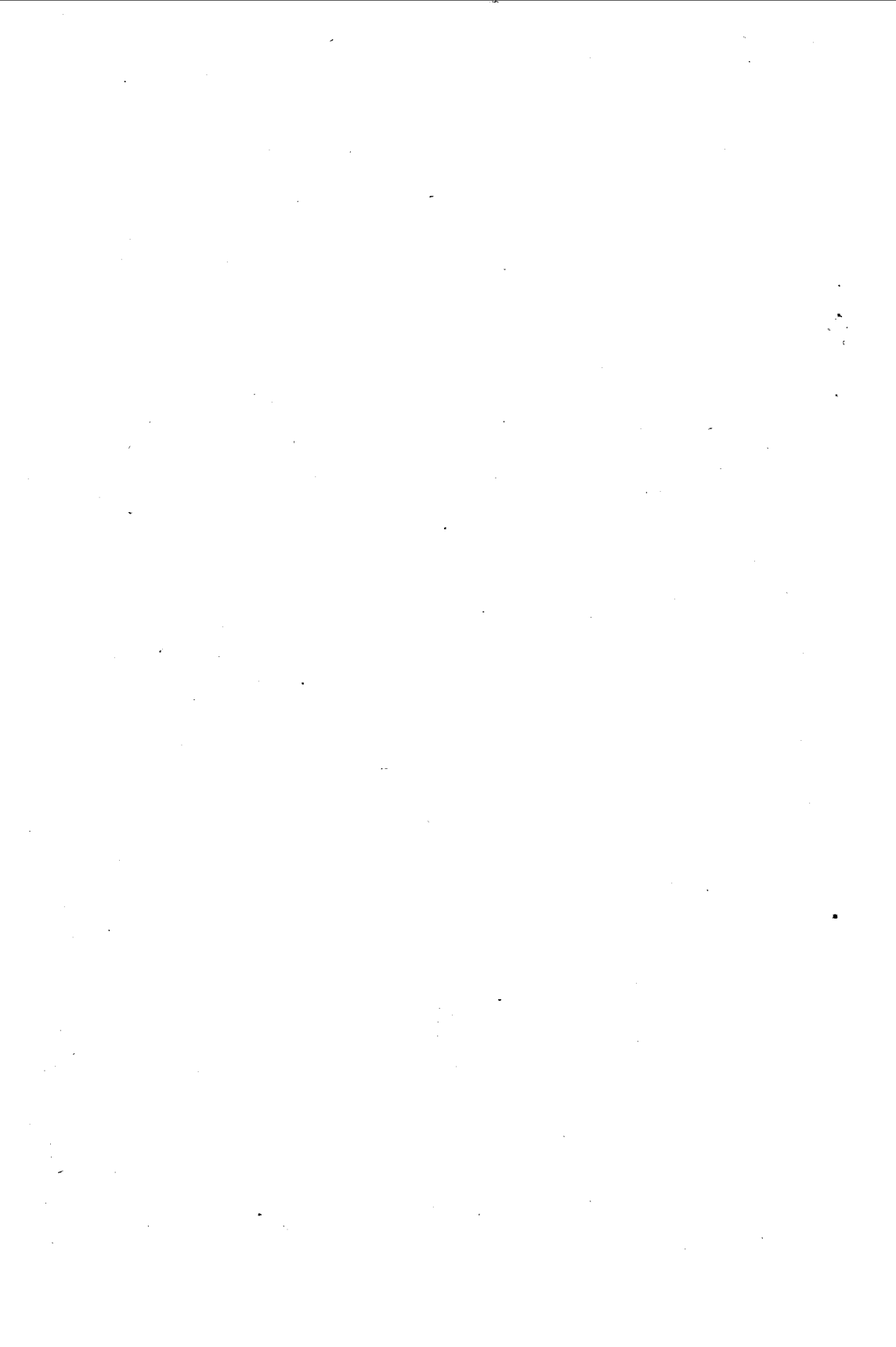
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## LIST OF SYMBOLS

$\eta$	Universal magnetic constant.
$\varepsilon$	Universal electric constant.
$\mu$	Permeability.
$\mu_0$	Initial permeability.
$\mu_d$	Incremental permeability.
$\bar{\mu}$	Complex permeability.
$\bar{\mu} = \mu' + j\mu''$	
$\mu_d$	Permeability of dust-core material.
$\kappa$	Permittivity.
$\Phi$	Magnetic flux.
$B$	Magnetic flux density.
$H$	Magnetic field intensity (magnetizing force).
$E$	Electric field intensity.
$L, R$	Inductance, resistance.
$L', R'$	"Shunt" values of inductance, resistance.
$L_0$	Value of $L$ corresponding to $\mu_0$ .
$Q = \frac{\omega L}{R} = \frac{R'}{\omega L}$	
$Z = \frac{V}{I}$	(circuit impedance).
$Z = \frac{E}{H}$	(field impedance).
$l$	Length (in metres unless stated).
$r$	Radius (in metres unless stated).
$t$	Thickness (in metres unless stated).
$A$	Area (in sq. metres unless stated).
$K$	D.C. resistance factor.
$k$	Eddy current factor of winding.
$F_e$	Eddy current factor of core.
$F_c$	Residual loss factor of core.
$F_h$	Hysteresis factor of core.
$h$	Hysteresis coefficient of material.
$k_n$	Harmonic distortion factor.



$P$	Field propagation coefficient (also used for power).
$\gamma$	Factor in d.c. resistance formulae (also propagation coefficient in section 53 only).
$\beta$	Factor in eddy current loss formulae.
$\alpha$	Coefficient in hysteresis loop equations (also attenuation coefficient).
$\sigma$	Conductivity.
$\delta$	Depth of field penetration (also loss angle of ferrite material).
$\theta$	Ratio of depth of penetration to lamination thickness.
$\lambda$	Space factor of winding.

Suffix *c* relates to self-capacitance, residual loss or core.

„	$r$	„	„	self-resonant frequency.
„	$e$	„	„	eddy currents.
„	$h$	„	„	hysteresis.
„	$dc$	„	„	d.c. resistance.
„	$g$	„	„	air-gap.
„	$p$	„	„	polarizing field.
„	$\Delta$	„	„	incremental quantity.
„	$d$	„	„	dust-core.
„	$a$	„	„	air-core.