STRESS AND STRAIN DATA HANDBOOK

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Preface

This book presents the general theories and principles of stress and strain for practical application. The primary intention is to provide a reference that meets the daily needs of the design engineer. The solutions and data required in engineering practice are often scattered through an extensive body of literature, and are not presented in a form that allows convenient application to the problem at hand. This handbook draws information from many sources into a useful and convenient single volume. Tedious derivations and detailed explanation of formulas are omitted. The data are presented in tables and graphs along with several examples to illustrate actual applications.

The scope of the book is indicated by the contents. It covers beams, frames, columns, beam-columns, plates, rings, torsion, shells, stability, and thermal stress. For each topic the general principles and theories are stated, followed by extensive tables and graphs for use in calculation of stress and strain. The data are arranged to provide a means of using general theories to solve practical engineering problems.

I express my gratitude to Mr. W. Yu and Dr. K. Pajouhi, who generously reviewed the manuscript; and to Mr. J. Garcia, who drew most of the graphs and illustrations. Finally, I would like to say that although every effort has been made to avoid errors, it is possible some could exist. I will be grateful for any suggestions you may have concerning needed corrections.

Teng H. Hsu

Notation

A	Cross-sectional area
A_r, A_u, A_x	Coefficients of deflection
B_x, B_y	Bending stress coefficients
B_i, B_j, B_u	Coefficients of slope
B_r	Bending stress coefficient for circular plates
C	Coefficient of constraint in elastic stability, numerical factor
C_c	Torsional stiffness of circular section
C_m	Reduction factor applied to bending term in interaction formula and dependent upon curvature caused by applied moments
C_o	Torsional stiffness of non-circular section
C_w	Warping constant of a section
D	Numerical factor, flexural rigidity of plate or shell
E	Modulus of elasticity in tension and compression, numerical factor
E_t	Tangent modulus
F_a	Axial stress permitted in the absence of bending moment
F_b	Bending stress permitted in the absence of axial force
F_e	Euler stress divided by factor of safety
F_y	Specified minimum yield stress of steel
F_c	Buckling stress of a column containing residual stresses
G	Modulus of elasticity in shear
H_0, H_h	Horizontal thrust of a pinned and built-in arch
I	Moment of inertia of a section
I_e	Moment of inertia of the elastic portion of a section
J	Polar moment of inertia, torsional constant
K	Effective length factor
K_m, K_A, K_B	Moment coefficients of a circular ring
K_d	Deflection coefficient of a circular ring
K_i, K_j, K_u	Coefficients of end moments
K_1, K_2	Torsional factors
K_3	Stress concentration factor

K_x, K_y Deflection coefficient in x and y, directions respectively

K_ϕ, K_σ Force factors in shell

K_x, K_t Stress factor in shell

K_P, K_Q Force and shear factors

K_w Displacement factor

L Span length

M Moment

M_{ij}, M_{ji} End moments

M_x, M_y Bending moments per unit length of plate or shell sections

M_{xy} Twisting moment per unit length of plate or shell sections

M_r, M_t Radial and tangential moments

M_T Torsional moment

N_x, N_y, N_{xy} Normal and shearing forces per unit length of shell sections

N_σ, N_ϕ, $N_{\sigma\phi}$ Membrane forces per unit length of shell sections

P Applied load

P_{cr} Critical load

P_e Euler load

Q_x, Q_y Shearing force per unit length of plate or shell sections

Q Shearing force per unit length of cylindrical section

R_1, R_2 Reactions

R_i, R_o Inside and outside radii of a curved beam respectively

R Radius of curvature of the neutral axis

T Membrane tension

V Static shear on beam

V_c Volume of membrane hill of circular section

V_o Volume of membrane hill of non-circular sections

V_f Shear force introduced to a flange of I beam subjected to an end torsion

U Strain energy

a, b Distances from supports to loading point, distances

c Distance

e Eccentricity

f Stress

f_a, f_b Axial and bending stress, respectively

f_{bx}, f_{by} Bending stresses induced by M_x and M_y, respectively

f'_u	A function of parameter u for plate deflection
g'_u	A function of parameter u for plate bending
k	Axial load factor for beam-column ($k^2 = P/EI$)
l	Length, span
ln	Natural logarithm
m	A parameter which gives the number of half-waves into which a plate buckles
p, q	Intensity of distributed load, pressure
r	Radius of gyration, radius, radius of curvature of shell
r_n	Radius of curvature of the middle surface of a circular plate
r_x, r_y	Radius of gyration with respect to x and y axes, respectively
s_x, s_{xy}	Shear stresses
t	Thickness
u	Axial load factor for beam-colums (u = kl), numerical factor
u, v, w	Displacements in x, y, z directions
x, y, z	Rectangular coordinates

Greek Symbols

α	Angles, coefficient of thermal expansion, numerical factors
β	Numerical factor or coefficient
γ	Shearing unit strain, specific gravity
δ	Deflection, displacement
δ_H, δ_V	Horizontal and vertical displacements
ϵ	Unit normal strain
θ	Angle, polar coordinate, the angle of twist per unit length of section
ϵ_x, ϵ_y	Unit normal strains in x and y directions
ν	Poisson's ratio
ρ	Radius of curvature
ρ_x, ρ_y	Radii of curvature of the middle surface of a rectangular plate in the xz and yz planes
γ_{xz}, γ_{yz}	Shear strains in xz and yz planes
σ	Unit normal stress
σ_x, σ_y	Unit normal stresses in x and y directions
σ_{cr}	Critical stress
τ	Unit shear stress

τ_{xy}, τ_{yz}, τ_{xz} Unit shear stresses on planes perpendicular to the x,y and z axes and parallel to the y,z and x axes

ϕ Angle, angular coordinate

χ Change of curvature in shell

χ_{xy} Twist of the middle surface of shell

ϕ Saint-Venant's torsion function

σ_{ϕ} Circumferential stress

ψ Numerical factor in calculation of reduction factor C_m for compression members braced against joint translation

Contents

Chapter 1

Flexure of Beams

BENDING OF BEAMS [1,2]

Forces or moments acting on a beam impart deflections perpendicular to the longitudinal axis of the beam and set up normal and shearing stresses on any cross section of the beam. It is convenient to imagine a beam being composed of an infinite number of fibers. The surface on the beam containing fibers that do not undergo any stress is called the *neutral surface*. The intersection of the neutral surface with any cross section of the beam perpendicular to its longitudinal axis is called the *neutral axis*. All fibers on one side of the neutral axis are in tension and those on the opposite side are in compression.

If all fibers in the beam are acting within the elastic range of the material, the following relations exist:
The bending stress at any point of a section is f_b.

$$f_b = \frac{My}{I} \tag{1-1}$$

where M is the bending moment at the section containing y, and y is the distance from the neutral axis to the point.
The shear stress at any point of a section is s_s.

$$s_s = \frac{V}{Ib}\int y\, da = \frac{VQ}{Ib} \tag{1-2}$$

where V = shear at the section
da = area of that part of the section above the point
y = distance from the neutral axis to the centroid of da
b = width of the beam
Q = first moment of the area da about the neutral axis

The radius of curvature of the elastic curve is R.

$$R = \frac{EI}{M} \tag{1-3}$$

The general differential equation of the elastic curve is:

$$M = EI \frac{d^2y}{dx^2} \tag{1-4}$$

The relations between the bending moment and the shear are:

$$V = \frac{dM}{dx} \tag{1-5a}$$

$$M = \int V \, dx \tag{1-5b}$$

The strain energy of flexure is U.

$$U = \int \frac{M^2}{2EI} \, dx \tag{1-6}$$

Example 1-1

A steel wire 1/32 in. in diameter is coiled around a pulley 20 in. in diameter; calculate the maximum bending stress set up in the wire.

$E = 30 \times 10^3$ kips/in.^{2}

Radius of curvature $R = 20/2 = 10$ in.

Normal strain $\epsilon = y/R$

Maximum strain $\epsilon_{max} = 1/640 = 0.00156$ in.

Maximum stress $\sigma_{max} = E\epsilon_{max} = 30 \times 10^3 \times 0.00156$
$= 46.88$ kips/in.^{2}

Example 1-2

If the greatest vertical shear of a rectangular beam is V, prove the maximum shear stress is 1.5 times the average shear stress.

From Equation 1-2,

$$s_s = \frac{VQ}{Ib} = \frac{12bh^2}{8b^2h^3} V = \frac{3V}{2bh} = 1.5 \frac{V}{A}$$

V/A is the average shear stress of the section with vertical force V.

ELASTIC DEFLECTION OF BEAMS [3]

The deformation of a beam is expressed in terms of the deflection of the beam from its original unloaded position. The deflection is measured from the original neutral surface to the neutral surface of the deformed beam. The configuration of the deformed neutral surface is known as the *elastic curve* of the beam.

Design specifications frequently limit the deflections as well as the stresses. It is essential that the design engineer be able to calculate deflections, and numerous methods are available for the determination of beam deflections.

Double-Integration Method

For a given beam, if the load w_x at a point x can be expressed mathematically as a function of x, and if such load condition is known for the entire beam, then

$$V_x = \int w_x \, dx \tag{1-7a}$$

$$M_x = \int V_x \, dx \tag{1-7b}$$

$$\theta_x = \int \frac{M_x}{EI} \, dx \tag{1-7c}$$

$$y_x = \int \frac{\theta_x \, dx}{EI} = \int \int \frac{M_x}{EI} \, dx \tag{1-7d}$$

It is assumed that the beam is acting in the elastic range and that the deflections caused by shearing action are negligible compared to those caused by bending action.

Example 1-3

Determine the maximum deflection of a cantilever beam subject to a uniform load of w lb per unit length, as shown in Figure 1-1.

$$M_x = wx^2/2$$

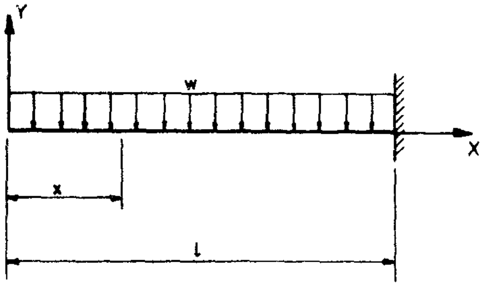

Figure 1-1. Cantilever beam for Example 1-3.

From Equations 1-7c and 1-7d,

$$EI\,\theta_x = \int M_x\,dx = wx^3/6 + C_1$$

$$EIy_x = \int\int M_x\,dx = \int (wx^3/6 + C_1)dx$$

$$EIy_x = wx^4/24 + C_1x + C_2$$

at $x = \ell$, $\theta_\ell = 0$ $C_1 = -w\ell^3/6$

at $x = \ell$, $y_\ell = 0$ $C_2 = w\ell^4/6 - w\ell^4/24 = w\ell^4/8$

at $x = 0$, $y = y_{max}$

$$y_{max} = C_2/EI = w\ell^4/(8EI)$$

Moment-Area Method

The first moment-area theorem states that the angle between the tangents at a and b of a deformed beam is equal to the area of the moment diagram between a and b, divided by EI:

$$\theta = \int_a^b \frac{M}{EI}\,dx \qquad (1\text{-}8a)$$

The second moment-area theorem states that the vertical distance from point b of the deformed beam to the tangent at point a of the beam equals the moment with respect to b of the area of the bending moment diagram between a and b, divided by EI:

$$\delta = \int_a^b \frac{M}{EI}\,x\,dx \qquad (1\text{-}8b)$$

Example 1-4

Determine the deflection of the free end of the cantilever beam in Example 1-3 using the moment-area method. The elastic curve and moment diagram are shown in Figure 1-2.

Area of the M/EI diagram = $w\ell^3/6$

Moment with respect to b = $w\ell^4/8$

$\delta = w\ell^4/8EI$

Conjugate Beam Method

In using this method, the moment diagram of the real beam is constructed, and a conjugate beam is then set up. It is loaded with the M/EI of the real beam. The vertical shear at any point of the conjugate beam equals the slope of the real beam at the same point. The bending moment at any point of the conjugate beam equals the deflection of the real beam at the same point. The boundary conditions of the conjugate beam should be selected so that the previous statements are satisfied.

Example 1-5

Determine the deflection and slope at the tip of a cantilever beam loaded by a concentrated force P, as shown in Figure 1-3.

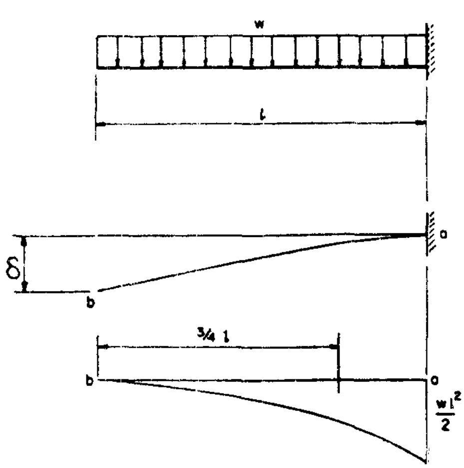

Figure 1-2. Elastic curve and moment diagram of a cantilever beam.

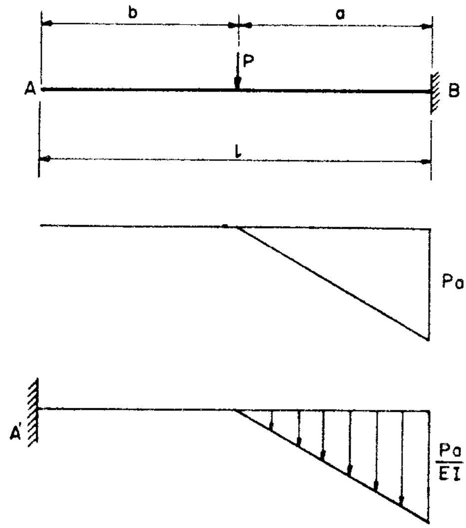

Figure 1-3. Cantilever beam for Example 1-5.

Shear at A′ $= Pa^2/(2EI)$

Moment at A′ $= \dfrac{Pa^2}{2EI}\left(\ell - \dfrac{a}{3}\right) = \dfrac{Pa^2\ell}{2EI} - \dfrac{Pa^3}{6EI}$

Slope at tip $\theta = \dfrac{Pa^2}{2EI}$

Deflection at tip $\delta = \dfrac{-Pa^2\ell}{2EI} + \dfrac{Pa^3}{6EI}$

Virtual Work Method

This method is used frequently for finding the deflection of a point on the beam. A unit virtual load is placed on the beam at the point where deflection is desired. Virtual moments caused by the unit load are determined along the beam. The internal energy of the beam after deflection is determined by integration. This is set equal to the external work done by the unit load.

Internal energy $U = \int \dfrac{M_x m_x}{EI}\, dx$

Work done $W = 1 \times y_x$

$$y_x = \int \frac{M_x m_x}{EI} dx \tag{1-9}$$

where m = virtual bending moment at any point caused by 1
M = real bending moment at the same point

Example 1-6

Solve Example 1-4 using virtual work method.

A unit load is placed at the tip of a beam as shown in Figure 1-4.

$$\text{Internal energy} = \int \frac{M_x m_x}{EI} dx = \int \frac{wx^3}{2EI} dx = \frac{w\ell^4}{8EI}$$

Work done $= 1 \times y = y$

$$y = \frac{w\ell^4}{8EI}$$

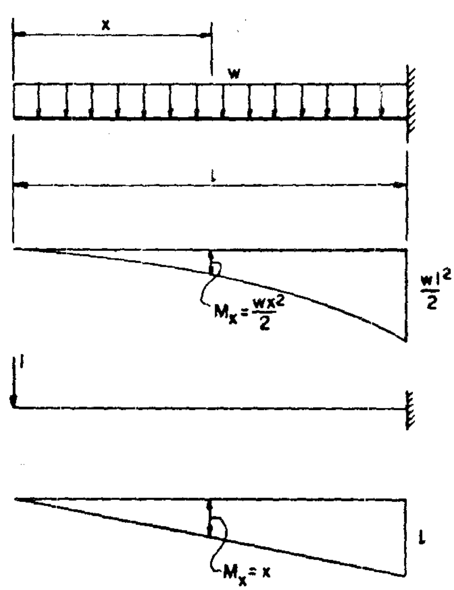

Figure 1-4. Cantilever beam for Example 1-6.